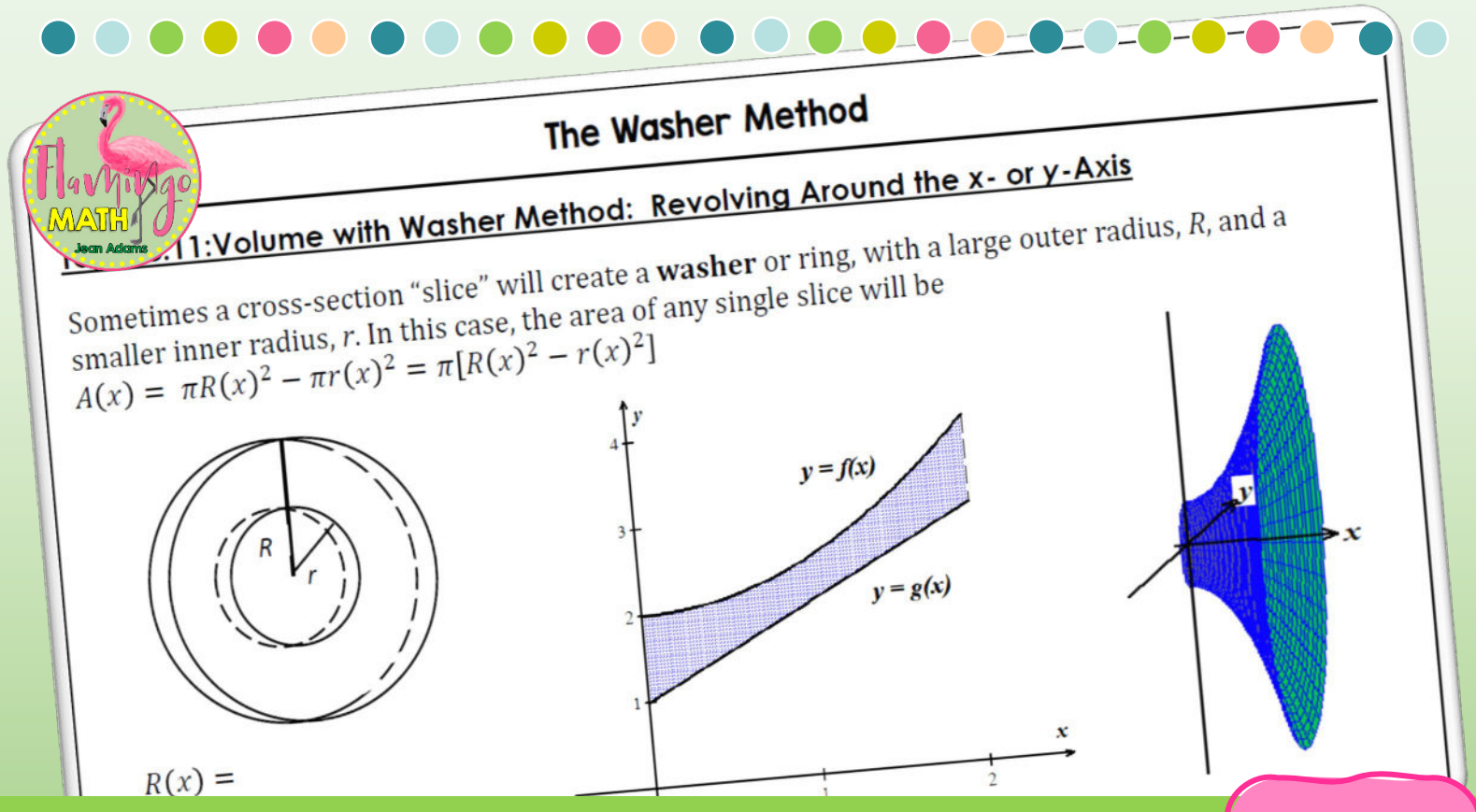
AP® Calculus

Volumes of Revolution Disc & Washer Method



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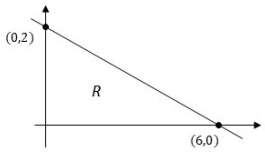
8.6

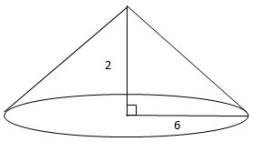
Lesson 8.6: Volumes by Disc and Washer Methods

Topic 8.9: Volume with Disc Method: Revolving Around the x- or y-Axis

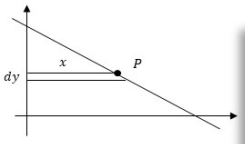
We have used integration to evaluate areas of regions. Now we will explore ways that integration can be used to find volumes of solids formed by rotating a region about some given straight line.

If a region in a plane is revolved about a line, the resulting solid is a *solid of revolution*, and the line is called the *axis of revolution*. The simplest such solid is a right circular cylinder or *disc*, which is formed by revolving a rectangle about an axis adjacent to one side of the rectangle.
For example, consider the region R, bounded by the x-axis, the y-axis, and $x + 3y = 6$. Now imagine rotating region R about the y-axis 360°. The solid formed will become a cone.





To find the volume of the cone, we could simply use the formula from geometry: $V = \frac{1}{3}\pi r^2 h$, or 24π.
Instead, let's explore the idea of a "strip" sliced through the region, and revolved around the y-axis. Remember that x would represent the coordinate of some point P on the graph and, as such, is also the length of our strip. When the "strip" is rotated or revolved, around the y-axis, it forms a thin disc.



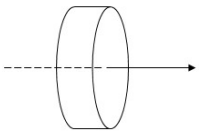
So, the cone can be created by an infinite numbe
create the cone.

EX #1: Write the volume of the cone as an integ
of the cone.

The Disc Method

When finding volumes of a solid of revolution, it is important to focus on what happens to each individual slice or disc as it is rotated around a line instead of thinking about the total volume. A thin slice or "strip" can be horizontal or vertical.

The volume of a disc sliced vertically is $\boldsymbol{V = \pi R^2 w}$, where R is the radius of the disc and w is the width.

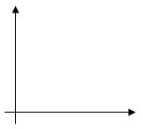


$$\text{Volume of Disk} = \left(\text{Area of Disk}\right)\left(\text{Width of Disk}\right)$$

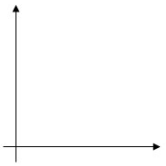
$$\boldsymbol{V = \pi R^2 w}$$

Horizontal Axis of Revolution solve for y in terms of x	$V = \pi \int_a^b [R(x)]^2\,dx$
Vertical Axis of Revolution solve for x in terms of y	$V = \pi \int_c^d [R(y)]^2\,dy$

tegral that gives the volume of the solid by revolving



EX #3: $y = \sqrt{x}$; $x = 0, x = 4$; Revolve about the x-axis.



EX #4: $y = x^2$; Revolve about y-axis from [0, 4]

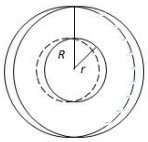


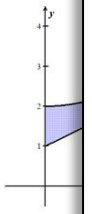
EX #5: $y = e^x$ Revolve about x-axis from [0, 2].

The Was

Topic 8.11: Volume with Washer Method: Re

Sometimes a cross-section "slice" will create a **wa**
smaller inner radius, r. In this case, the area of an
$A(x) = \pi R(x)^2 - \pi r(x)^2 = \pi[R(x)^2 - r(x)^2]$





$R(x) =$

$r(x) =$

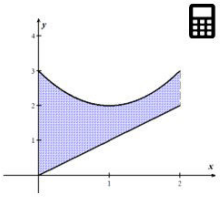
To find the volume of
each slice of the region to be rotated
use the foll

Horizontal Axis of Revolution solve for y in terms of x	$V = \pi \int_a^b \left[\left(R(x)\right)^2 - \left(r(x)\right)^2\right] dx$
Vertical Axis of Revolution solve for x in terms of y	$V = \pi \int_c^d \left[\left(R(y)\right)^2 - \left(r(y)\right)^2\right] dy$

EX #7: Find the volume of the solid formed by the region bounded by graphs of $y = \sqrt{x}$ and $y = x^2$ about the y-axis.



EX #8: Find the volume of the solid generated by rotating the region bounded by $y = (x-1)^2 + 2$, $y = x$, and $x = 0$ about the x-axis.



A. Write the area of a single slice:

B. Write an integral for finding the volume of the solid:

C. Find the volume of the solid to three decimal places.



3. Integral:



NOTES

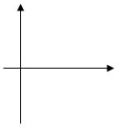
8.6 **Volumes by Disc & Washer Methods Homework**

Name _____

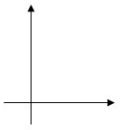
Date _____ Period _____

Problems 1 – 12, Find the volume of the region bounded by the given functions and revolved about the given axis.

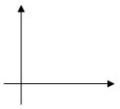
1. $y = x^2 - 4x$; $y = 0$ about the x-axis



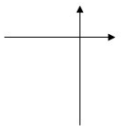
2. $y = x$; $y = 3$; $x = 0$ about the y-axis



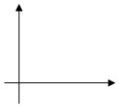
3. $y = \sqrt{9 - x^2}$; $y = 0$; $x = 0$ about the x-axis



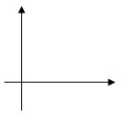
4. $y = x^3$; $x = -2$; $y = 0$ about the x-axis



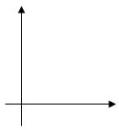
5. $y = \sqrt{x}$, $y = 3$ about the y-axis



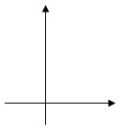
6. $y^2 = x$; $2y = x$ about the y-axis



7. $y = x^2$; $y = x + 2$; $x = 0$, $y = 0$ about the x-axis



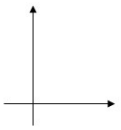
8. $y = 2x$; $y = 4x^2$ about the y-axis



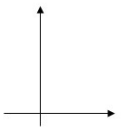
9. $y = 6 - x^2$, $x = -2$, $x = 2$, and the x-axis about the x- axis.



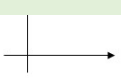
10. $y = 2x$; $y = \frac{1}{8}x^3$, $x = 0$, and $y = 0$ about the y-axis



11. Let R be the region in the first quadrant bounded by the graph of $y = 27 - x^{3/2}$, the x-axis, and the y-axis. Find an approximation of the volume of the solid generated when R is revolved about the x-axis. Show the integral used to calculate your answer.



12. Let R be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis.



Practice Sets

8.6 Volumes –The Disc Method
Form A

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $f(x) = x + 2, y = 0, x = 0, x = 3$ about the x-axis.

2. $y = x^3$, $y = 8, x = 0$ about the y-axis.

8.6 Volumes –The Disc Method
Form B

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = 2x + 1, y = 0, x = 0, x = 3$ about the x-axis.

2. $y = \sqrt{x}$, $y = 2, x = 0$ about the y-axis.

8.6 Volumes –The Disc Method
Form C

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $f(x) = 2x, y = 0, x = 0, x = 3$ about the x-axis.

2. $y = 4 - x^2, y = 0, x = 0$ about the y-axis.

8.6 Volumes –The Disc Method
Form D

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = x + 1, y = 0, x = 0, x = 3$ about the x-axis.

2. $y = x^2$, $y = 4, x = 0$ about the y-axis.

8.6 Volumes –The Washer Method
Form A

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $f(x) = 2x, y = x$, $x = 1$ about the x-axis.

2. $x = \frac{1}{2}y$, $x = \frac{\sqrt{y}}{2}$ about the y-axis.

8.6 Volumes –The Washer Method
Form B

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = 2x + 1, y = x, x = 2$ about the x-axis.

2. $y^2 = x$, $2y = x$ about the y-axis.

8.6 Volumes –The Disc Method
Form C

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $f(x) = 4 - x, y = x, x = 0$ about the x-axis.

2. $y = x^3$, $x = 1, y = 0$ about the y-axis.

8.6 Volumes –The Disc Method
Form D

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = 3 - 2x, y = x, x = 0$ about the x-axis.

2. $x = \sqrt[3]{y}$, $x = \frac{1}{4}y$ about the y-axis.

Daily Lesson Quiz