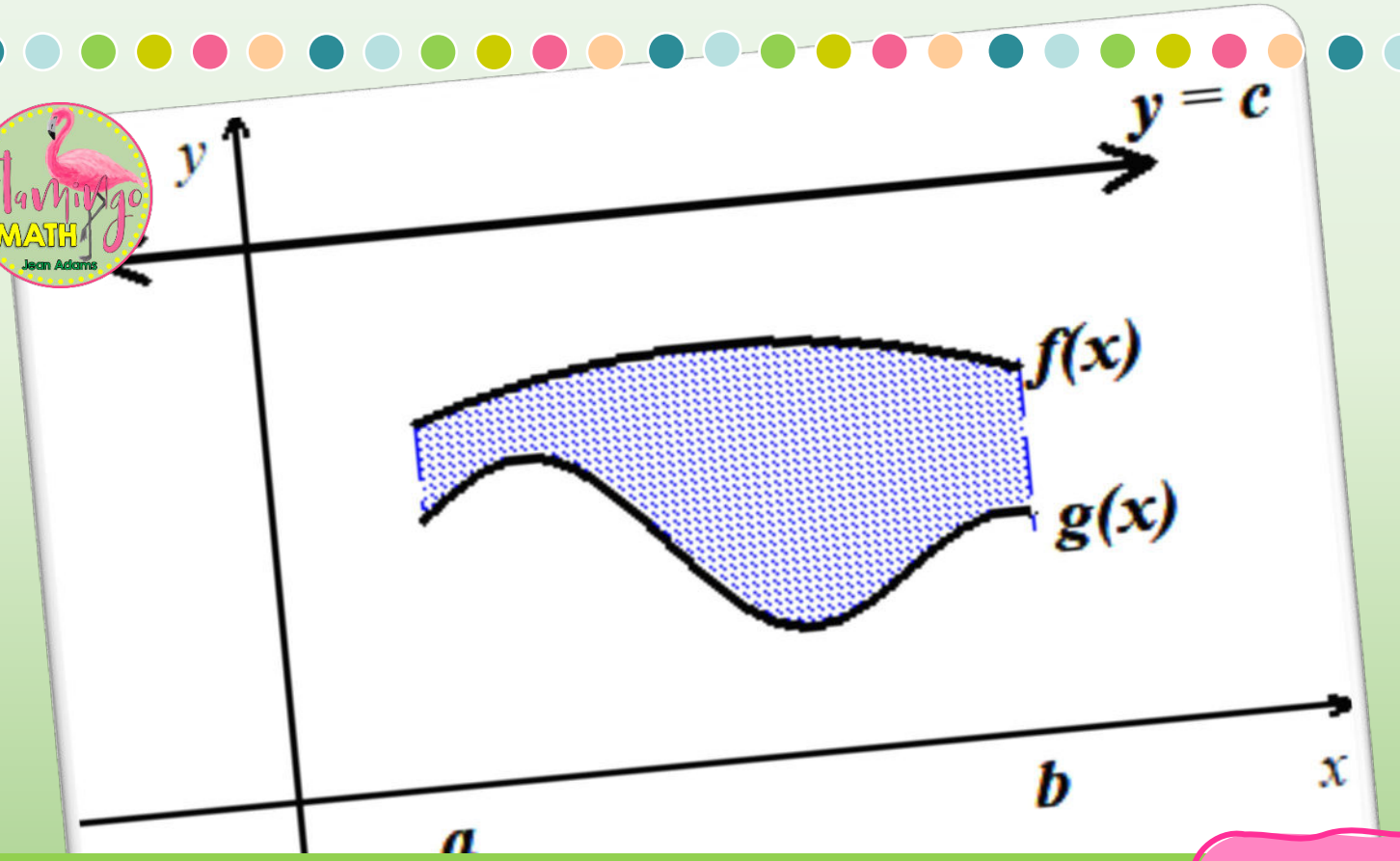
AP® Calculus

Volumes of Revolution Around Other Axes



Completely Editable 8.7

Lesson 8.7: Volumes - Revolving Around Other Axes

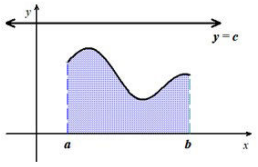
Topic 8.10: Volume with Disc Method: Revolving Around Other Axes
Topic 8.12: Volume with Washer Method: Revolving Around Other Axes

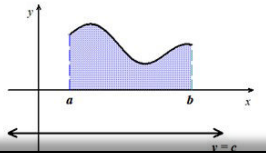
In this lesson we want to explore volumes that are revolved about any line other than the x- or y-axis. You should feel comfortable in basic understandings that if we rotate about any horizontal axis, then the cross-sectional areas will be a function of x. Similarly, if we rotate about any vertical axis, the cross sections will be a function of y. Let's generalize those findings.

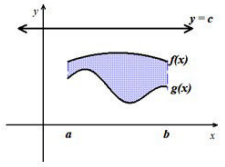
Horizontal Axis of Revolution

To rotate about a horizontal line, $y = c$, express R_{outer} and R_{inner} as functions of x and integrate along the x-axis.

EX #1: Write the general form of the volume for the area R where any cross section is **PERPENDICULAR** to the axis of revolution.





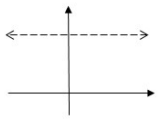


EX #2: $y = \sqrt{x}$; $y = 0, x = 4$; about the line $y = 4$

Outer radius : _____

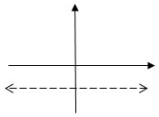
Inner radius: _____

Method: _____



EX #3: $y = 4 - x^2$, $y = 0$; about the line $y = -1$

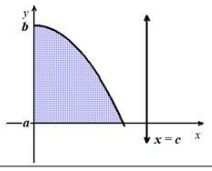
Outer radius : _____

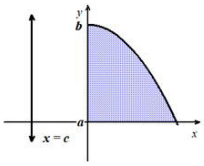


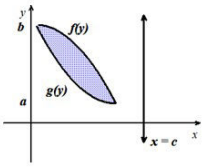
Vertical Axis of Revolution

To rotate about a vertical line, $x = c$, express R_{outer} and R_{inner} as functions of y and integrate along the y-axis.

EX #4: Write the general form of the volume for the area R where any cross section is **PERPENDICULAR** to the axis of revolution.







EX #5: Find the volume approximation of the solid gener
bounded by $y = \sqrt[3]{x}$, the x-axis and the line $x = 1$

EX #6: $y = x^2$; $y = 4$; about the line $x = 3$

Outer radius : _____

Inner radius: _____

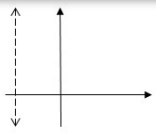
Method: _____

EX #7: $y = 2\sqrt{x}$, $y = x$; about the line $x = -2$

Outer radius : _____

Inner radius: _____

Method: _____

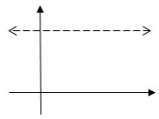


EX #8: $x = e^y$; , x = 1 ,and $y = 1$, about the line $y = 1$

Outer radius : _____

Inner radius: _____

Method: _____

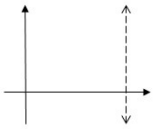


EX #9: $y = \sqrt[3]{x}$, $y = 0$, and $x = 8$ about the line $x = 8$

Outer radius : _____

Inner radius: _____

Method: _____



EX #10: $y = 2\sqrt{x}$, $y = 6$, and the y-axis, about the line $y = 6$.

NOTES

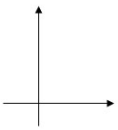
8.7 **Volumes: Revolving Around Other Axes Homework**

Name _____

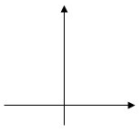
Date _____ Period _____

Problems 1- 9, Find the volume of the region bounded by the given functions and revolved about the given line.

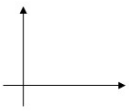
1. $y = x^2 + 1\ ;\ x = 0; y = 2$ about the line $y = 2$



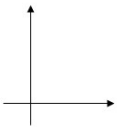
2. $y = 4 - x^2; y = 0$ about the line $y = 4$



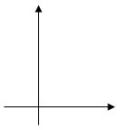
3. $y = \sqrt{x}\ ; y = 0\ ; x = 4$ about the line $y = 4$



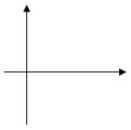
4. $y = \sqrt{x}\ ; y = 0\ ; x = 4$ about the line $x = 4$



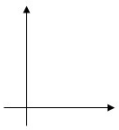
5. $y = 2x^2\ ; y = 0; x = 2$ about the $x = 2$



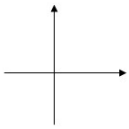
6. $y^2 = x\ ; x = 4$ about the line $x = 5$



7. $y = x^2; y = x$ about the line $y = 3$



8. $x = y^2; x = y$ about the line $y = -1$



9. $y = 2x + 1\ ,\ x = 4$ and $y = 3$ about the line $x = -3$

Practice Sets

8.7 Volumes About Other Axes **Form A**

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = x^2 + 1$ about the line y = 2.

2. $y = \sqrt{x}, x = 4, y = 0$ about the line $x = 4$.

8.7 Volumes About Other Axes **Form B**

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = 2x + 1, x = 0,$ about the line $y = 5$.

2. $y = 2\sqrt{x}, x = 4, y = 0$ about the line $x = 4$.

8.7 Volumes About Other Axes **Form C**

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = \sqrt{x}, x = 0$ about the line $y = 2$.

2. $y = 4 - x, y = 4, x = 3,$ about the line $x = 3$.

8.7 Volumes About Other Axes **Form D**

Name _____

Date _____Period _____

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = 2x + 1, x = 0,$ about the line $y = 4$.

2. $y = 3 - x, x = 2, y = 3$ about the line $x = 2$.

Daily Lesson Quiz