

Precalculus Honors

Vectors in the Plane

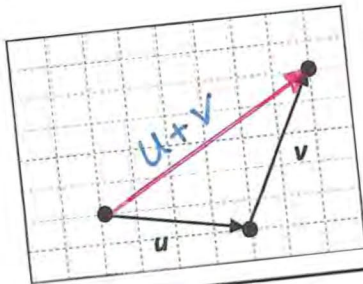


Adding Vectors Geometrically

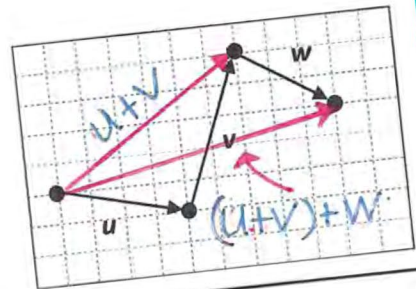
The sum of two vectors $\mathbf{u} + \mathbf{v}$ is defined by the tip-to-tail rule:
Position the vectors $\mathbf{u}$ and $\mathbf{v}$ so that the terminal point of $\mathbf{u}$ coincides with the initial point of $\mathbf{v}$.
Then the vector $\mathbf{u} + \mathbf{v}$ is formed from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

EX #2: Sketch each of the following on the grids below.

A. $\mathbf{u} + \mathbf{v}$



B. $(\mathbf{u} + \mathbf{v}) + \mathbf{w}$



C. $\mathbf{v} - \mathbf{w}$



D. $\mathbf{w} - 2\mathbf{u}$



Completely Editable

6.5

Lesson 6.5: Vectors in the Plane

In this section we will study two-dimensional vectors, vector operations, unit vectors, directional angles and applications of vectors. A **vector** (derived from the Latin *vehere*, means "to carry") is a directed line segment that has both **magnitude** and **direction** (velocity is a vector example). We use an arrow to represent a vector; the **magnitude of a vector** is the length of the line segment and the arrowhead indicates the **direction of the vector**.

Geometric Representations of Vectors

EX #1: The **linear combination** where a and b are respectively the horizontal and vertical components of the vector is $ai + bj$

Component Form: $\langle a, b \rangle$

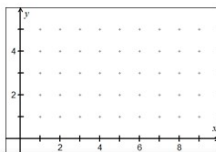
Standard Position: (position vector) initial point is the origin $(0, 0)$ and terminal point is (a, b) .

A. Given $P = (4, 1)$ and $Q = (7, 5)$
Sketch $\overrightarrow{PQ}$

B. Translate to position vector $\mathbf{z}$

C. Write $\mathbf{z}$ in component form.

D. Write as a linear combination.



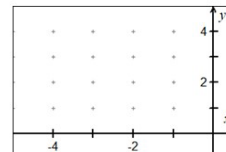
Magnitude of Vectors

The magnitude (length) of a vector $\mathbf{v} = \langle a, b \rangle$ is $\|\mathbf{v}\| = \sqrt{a^2 + b^2}$

If vector $\mathbf{v}$ is represented by the arrow from (x_1, y_1) to (x_2, y_2) , then

$$\|\mathbf{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ also if } \mathbf{v} = \langle a, b \rangle \text{ then } \|\mathbf{v}\| = \sqrt{a^2 + b^2}$$

EX #3: Sketch and find the magnitude of $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$



Properties of Vectors

The zero vector is the vector $\mathbf{0} = \langle 0, 0 \rangle$. This vector behaves the same for addition of vectors as the number 0 does for addition properties of real numbers.

Vector Addition

Multiplication by a scalar

Unit Vector

A vector $\mathbf{u}$ for which $\|\mathbf{u}\| = 1$ is called a **unit vector**.

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

$$\mathbf{i} = \langle 1, 0 \rangle \text{ and } \mathbf{j} = \langle 0, 1 \rangle$$

EX #5: Find a unit vector in the same direction as $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

Equal Vectors

Two vectors $\mathbf{v}$ and $\mathbf{w}$ are equal if and only if their corresponding components are equal.

If $\mathbf{v} = \langle a_1, b_1 \rangle$ and $\mathbf{w} = \langle a_2, b_2 \rangle$ then $\mathbf{v} = \mathbf{w}$ if and only if $a_1 = a_2$ and $b_1 = b_2$.

In many applications, a vector is described in terms of its magnitude and direction. A **velocity vector** is any vector that represents the speed and direction of an object, while a **force vector** represents the direction and amount of force that is acting on an object. In order to model velocity and force with vectors, we must be able to find the horizontal and vertical components of the vector.

Components of a Vector

Let $\mathbf{v}$ be a vector with magnitude $\|\mathbf{v}\|$ and direction θ .

Then, $\mathbf{v} = \langle a, b \rangle = a\mathbf{i} + b\mathbf{j}$ where $a = \|\mathbf{v}\| \cos \theta$ and $b = \|\mathbf{v}\| \sin \theta$

Therefore, we can express $\mathbf{v}$ as

$$\mathbf{v} = \|\mathbf{v}\| \cos \theta \mathbf{i} + \|\mathbf{v}\| \sin \theta \mathbf{j}$$

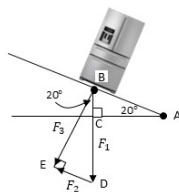
Vector Applications

EX #7: A DC-10 jet is flying on a bearing of 65° at 500 miles per hour. Find the form of the velocity of the airplane. Recall that the bearing is the angle that the line of travel makes with the north, measured clockwise.

EX #8: Find the magnitude and direction of the sum $\mathbf{w}$, of two vectors $\mathbf{u}$ and $\mathbf{v}$ of magnitudes 6.9 and 13, respectively if the angle between them measures 48° . Round magnitude to nearest whole number and the direction angle to the nearest degree.

More Vector Applications

EX #9: The delivery truck driver requires a magnitude of force of 250 pounds to push a refrigerator/freezer up a ramp at a 20° angle from the horizontal. How much does the refrigerator weigh?



Bearings and Navigation

A bearing is measured clockwise from due North, while a direction angle is measured counter-clockwise from the positive x -axis.

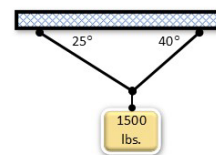
EX #10: A ship sails on a bearing of 67° east of south at 15 knots. Express the velocity of the ship as a vector. Assume no current.

An Object in Static Equilibrium

An object is said to be in **static equilibrium** if:

1. The object is at rest; and,
2. The sum of all forces acting on the object is zero. That is, if the resultant force is 0.

EX #11: A large construction generator that weighs 1500 pounds is suspended by two cables attached to the beam of a crane. What is the tension in the two cables?



NOTES

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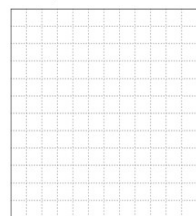
Vectors in the Plane
Homework

Name _____

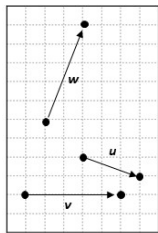
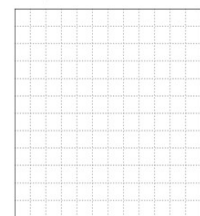
Date _____ Period _____

Problems 1–5, use the vectors in the figure at the right to graph each of the following vectors.

1. $\mathbf{v} + \mathbf{w}$



2. $\mathbf{v} + \mathbf{u}$



3. $2\mathbf{u}$



4. $\mathbf{v} - \mathbf{w}$



5. $\mathbf{v} + 2\mathbf{u} - \mathbf{w}$

Problems 24–27, find the component form of the vector $\mathbf{v}$.

24. direction angle 110° and magnitude 18

25. direction angle 18° and magnitude 12

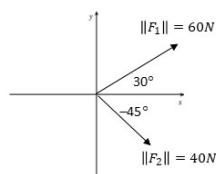
26. direction angle 35° and magnitude 29

27. direction angle 128° and magnitude 65

Problems 28–31, find the magnitude and direction angle of the vector.

28. $\langle 4, 3 \rangle$

29. $-3\mathbf{i} + 5\mathbf{j}$

36. Tommy is pulling a garden cart full of mulch with a force of 50 pounds. The handle of the cart makes an angle of 30° with the ground. Express the force vector $\mathbf{F}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.37. A man is pushing a dolly of office supplies up a ramp of 12° with a force of 95 pounds. Express the force vector $\mathbf{F}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. Round answer to nearest hundredth.38. Two forces of magnitude 60 newtons (N) and 40 newtons act on an object at angles 30° and -45° with the positive x -axis, shown at right. Find the direction and magnitude of the resultant force; $\mathbf{F}_1 + \mathbf{F}_2$.

39. Two forces of magnitude 50 newtons (N) and 80

Problems 12–15, vector $\mathbf{v}$ has initial point P and terminal point Q . Write $\mathbf{v}$ in the form of $a\mathbf{i} + b\mathbf{j}$.

12. $P = (-2, 3); Q = (5, 7)$

13. $P = (-2, -1); Q = (6, 3)$

14. $P = (4, -3); Q = (3, 3)$

15. $P = (5, -2); Q = (-4, 8)$

Problems 16–19, find the magnitude $\|\mathbf{v}\|$.

16. $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$

17. $\mathbf{v} = -5\mathbf{i} - 12\mathbf{j}$

Problems 32–42, solve.

32. A helicopter is flying on a bearing of 260° at 105 mph. Find the component form of the velocity of the helicopter.

33. A ship is heading due north at 10 mph. The current is flowing southwest at 3 mph. Find the actual bearing and speed of the ship.

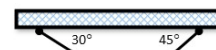
34. In a warehouse, a box is being pushed up a 12° inclined plane with a force of 5 lbs., shown in the figure. Find the component form of the force.40. A soccer ball is thrown from the side lines at a 65° angle with the ground at an initial speed of 30 feet per second.

A. Find the component form, to the nearest hundredth, of the initial velocity.

B. Explain the meaning of the horizontal and vertical components of the velocity.

41. A boat is heading due north at 15 mph. The current is flowing southeast at 5 mph. Find the actual bearing and speed of the ship.

42. A weight of 800 pounds is suspended from a steel beam by two cables, shown at right. What is the tension in the two cables?



Practice Sets

Name _____ Date _____ Period _____	
Lesson 6.5: Vectors in the Plane Day 1	
Warm-Up # 1	
1. Let $\vec{u} = (-5, 4)$ and $\vec{w} = (2, 3)$. Find the component form of the vector $3\vec{u} + \vec{w}$.	2. Use $\vec{v} = (6, -3)$, find the magnitude of the vector.

Name _____ Date _____ Period _____	
Lesson 6.5: Vector Applications Day 2	
Warm-Up # 1	
1. A plane is flying on a bearing of 56° east of north at 500 mph. Express the velocity of the plane as a vector. Assume that there is no wind.	2. Find the components of the vector $\vec{v}$ with direction angle 175° and length of 10. Round your answer to two decimal places.

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 6.5: Vectors in the Plane Day 1	
Warm-Up # 2	
1. Let $\vec{u} = (-5, 4)$ and $\vec{w} = (2, 3)$. Find the component form of the vector $\vec{u} + 2\vec{w}$.	2. Use $\vec{v} = (4, -5)$, find the magnitude of the vector.
3. Find the direction angle of the vector from #2.	4. Find the unit vector in the direction of the given vector $\langle 4, -5 \rangle$

Name _____ Date _____ Period _____	
Lesson 6.5: Vector Applications Day 2	
Warm-Up # 2	
1. A plane is flying on a bearing of 53° east of north at 485 mph. Express the velocity of the plane as a vector. Assume that there is no wind.	2. Find the components of the vector $\vec{v}$ with direction angle 215° and length of 9. Round your answer to two decimal places.
3. The magnitude and direction of two forces acting on an object are 41 pounds, $N42^\circ E$, and 63 pounds, $S55^\circ E$, respectively. Find the magnitude to the nearest hundredth of a pound.	

6.5 Vectors in the Plane	Form A
Name _____	
Date _____ Period _____	
1. Let $\vec{u} = (-4, 3)$ and $\vec{w} = (1, 5)$. Find the component form of the vector $2\vec{u} + \vec{w}$	
For questions 2 - 4: Use $\vec{v} = \langle 3, -5 \rangle$.	
2. Find the magnitude of the vector.	
3. Find the direction angle of the vector.	

6.5 Vectors in the Plane	Form B
Name _____	
Date _____ Period _____	
1. Let $\vec{u} = (6, -3)$ and $\vec{w} = (-1, 2)$. Find the component form of the vector $\vec{u} + 3\vec{w}$.	
For questions 2 - 4: Use $\vec{v} = \langle 4, -2 \rangle$.	
2. Find the magnitude of the vector.	
3. Find the direction angle of the vector.	

6.5 Vector Applications	Form A
Name _____	
Date _____ Period _____	
1. A plane is flying on a bearing of 42° east of north at 475 mph. Express the velocity of the plane as a vector. Assume that there is no wind.	
2. Find the components of the vector $\vec{v}$ with direction angle 240° and length 8. Round your answer to two decimal places.	
3. The magnitude and direction of two forces acting on an object are 46 pounds, $N35^\circ E$, and 72 pounds, $S42^\circ E$, respectively. Find the magnitude, to the nearest hundredth of a pound.	

6.5 Vector Applications	Form B
Name _____	
Date _____ Period _____	
1. A plane is flying on a bearing of 68° east of north at 550 mph. Express the velocity of the plane as a vector. Assume that there is no wind.	
2. Find the components of the vector $\vec{v}$ with direction angle 178° and length 7. Round your answer to two decimal places.	
3. The magnitude and direction of two forces acting on an object are 36 pounds, $N45^\circ E$, and 70 pounds, $S52^\circ E$, respectively. Find the magnitude, to the nearest hundredth of a pound.	

Daily Lesson Quiz