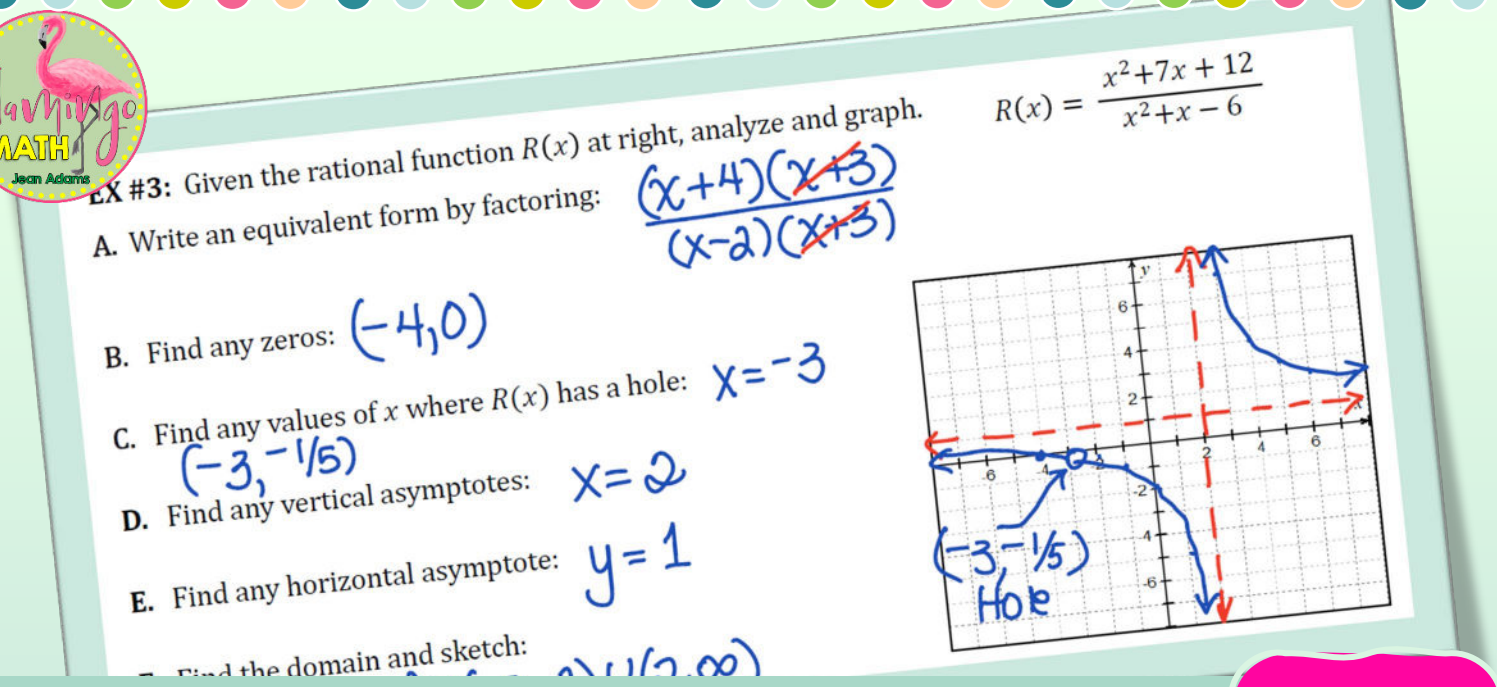
AP® Precalculus

Polynomial and Rational Functions



Guided Notes 1

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AP PRECALCULUS CURRICULUM

Scope and Sequence

Unit 1: Polynomials and Rational Functions

Lesson/Topic	Skills & Learning Objectives
[1.1] Change in Tandem	▪ Find the domain and range of a relation ▪ Determine whether a relation is a function ▪ Find the domain of a function ▪ Evaluate a function using both explicit and implicit forms of a function ▪ Identify intervals of increase, decrease or constant ▪ Find points of inflection and intervals of concavity ▪ Find x-intercepts and y-intercepts
[1.2] Rates of Change	▪ Find average rate of change of a function over an interval ▪ Find rate of change at a point using small intervals containing the point ▪ Describe the relationship between the rate of change and the behavior of the function ▪ Find velocity of application problems
[1.3] Rates of Change in Linear & Quadratic Functions	▪ Find the average rate of change over a closed interval ▪ Find the slope of the secant line over a closed interval ▪ Use average rates of change to determine concavity ▪ Write a linear equation for consecutive equal-length input-intervals that represent the average rate of change of a quadratic function
[1.4] Polynomial Functions Rates of Change	▪ Identify turning points of a polynomial ▪ Find relative and absolute extrema ▪ Describe relationships of extrema related to even/odd functions ▪ Find points of inflection by using key characteristics of polynomials

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Scope and Sequence

Unit I: Polynomials and Rational Functions

Lesson/Topic	Skills & Learning Objectives
[1.5] Polynomial Functions Complex Zeros	▪ Use long division and synthetic division to find zeros of polynomials ▪ Factor polynomials over real numbers ▪ Factor polynomials over complex numbers ▪ Use Conjugate Root Theorem to find zeros of polynomials ▪ Find the degree of a polynomial using successive differences of output value over equal-interval input values ▪ Determine whether a polynomial is even or odd ▪ Solve polynomial inequalities
[1.6] Polynomial Functions and End Behavior	▪ Use leading coefficient test to describe end behavior ▪ Use multiplicity of zeros to describe the behavior of the graph at the x-axis ▪ Use Intermediate Value Theorem to verify zeros of a polynomial ▪ Use limit notation to describe end behavior ▪ Analyze polynomial functions and sketch based on key characteristics
[1.7] Rational Functions and End Behavior	▪ Read and use arrow notation ▪ Find domain of rational functions ▪ Find vertical and horizontal asymptotes ▪ Determine the quotient of two polynomials using long division to find slant asymptotes ▪ Sketch rational functions ▪ Use limit notation to describe end behavior of rational functions
[1.8] Rational Functions and Zeros	▪ Rewrite polynomial and rational expressions in equivalent forms ▪ Analyze a rational function completely ▪ Write a rational function given the graph ▪ Solve rational inequalities

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Scope and Sequence

Unit I: Polynomials and Rational Functions

Lesson/Topic	Skills & Learning Objectives
[1.9-1.10] Rational Functions Vertical Asymptotes & Holes	▪ Find vertical asymptotes ▪ Determine coordinates of holes in a graph ▪ Write equations of a rational function given limit properties ▪ Analyze rational function given the graph
[1.11] Equivalent Representations of Polynomial and Rational Expressions	▪ Rewrite polynomial and rational expressions in equivalent forms ▪ Use the division algorithm and polynomial long division to find equations for slant asymptotes ▪ Use the binomial theorem to expand binomial expressions
[1.12] Transformations of Functions	▪ Use transformations of functions to graph ▪ Construct a function that is an additive and/or multiplicative transformation of another function ▪ Analyze transformations of functions from tables and symbolically
[1.13-1.14] Function Models and Applications	▪ Draw scatter plots to find a trend line equation ▪ Identify an appropriate function type to construct a function model given a scenario ▪ Describe underlying assumptions about a model for a given scenario ▪ Construct a linear, polynomial, or piecewise-defined function to model a scenario ▪ Construct a rational function model based on a context ▪ Apply a function model to answer questions about a data set ▪ Use technology to find and compare regression equations

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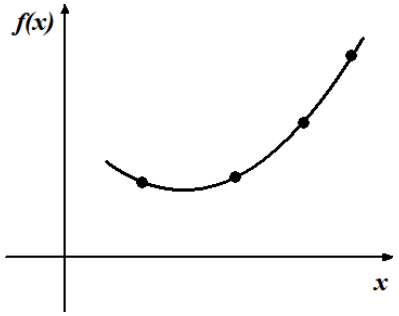
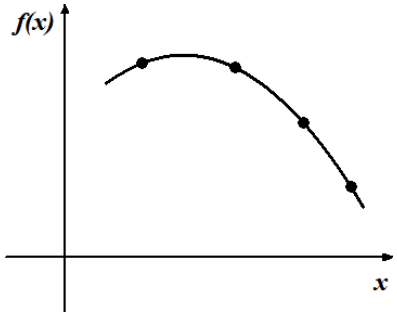
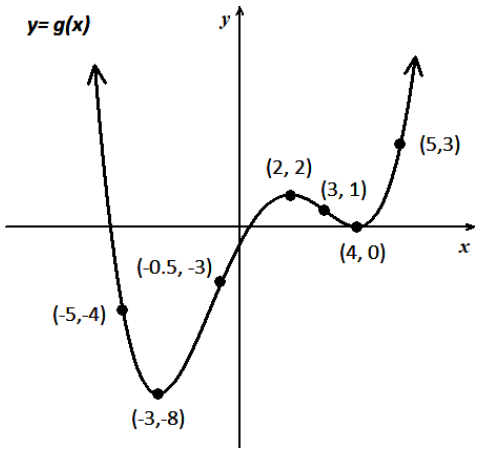
Lesson I-B: Change in Tandem

Concavity and Inflection Points

An **inflection point** is where a curve changes from **concave upward** to **concave downward** (or vice versa). We need Calculus to algebraically find where a curve goes from concave up to concave down.

Concave Up	Concave Down
	
Rate of change is increasing	**Rate of change is decreasing**



EX #4: The graph of $g(x)$ is shown above. Use the graph of g to find intervals where g has the following behaviors.

A. g is increasing

B. g is decreasing

C. g is concave up

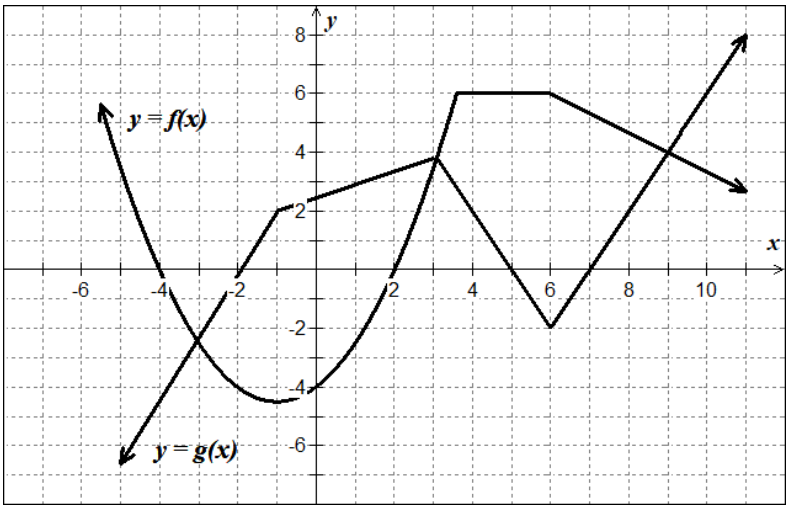
D. g is both increasing and concave up

E. g is concave down

F. g is both decreasing and concave down

Comparing Functions Graphically

EX #9: Use the graphs of functions $f(x)$ and $g(x)$, shown at right, to answer the following.



A. The domain for each function.

$f(x)$

$g(x)$

B. The value(s) of x such that

$f(x) = 0$

$g(x) = 0$

C. The value(s) of x such that $f(x) = g(x)$

D. The value(s) of x such that $f(x) > g(x)$

E. The value(s) of x such that $f(x) < g(x)$

F. The value(s) of x such that $g(x) > 0$

G. The value(s) of x such that $f(x) < 0$

H. The value(s) of x such that $f(x)$ increases

I. The value(s) of x such that $f(x)$ is constant

J. The value(s) of x such that $g(x)$ decreases

K. The value(s) of x such that $f(x) > 0$

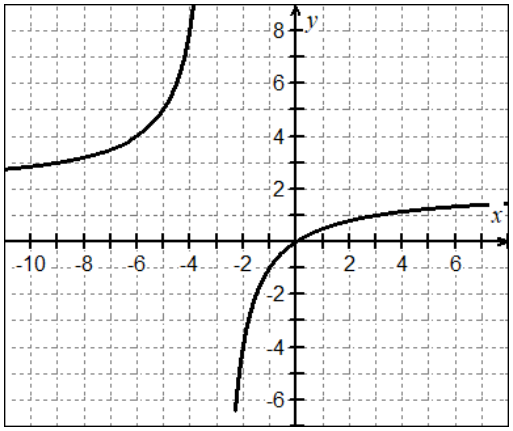
L. The value(s) of x such that $f(x)$ decreases

Horizontal and Vertical Asymptotes

Vertical Asymptotes: _____

The graph behavior will approach the vertical line and either increase or decrease without bound. That is,

$\lim_{x \to a^-} f(x) =$ or $\lim_{x \to a^+} f(x) =$



Horizontal Asymptotes: _____

The graph behavior will approach the horizontal line as x

tends to _____ or _____.

That is,

$\lim_{x \to -\infty} f(x) =$ or $\lim_{x \to \infty} f(x) =$

Note: It is possible to cross a horizontal asymptote. The graph does not need to have same behavior on both sides.

Finding Horizontal Asymptotes

Case:	Example:
(1) degree of numerator < degree of denominator The line _____ is the horizontal asymptote.	
(2) degree of numerator = degree of denominator The _____ of the leading coefficients is the horizontal asymptote. That is, $y =$ ———	
(3) degree of numerator > degree of denominator The graph has _____ horizontal asymptote.	

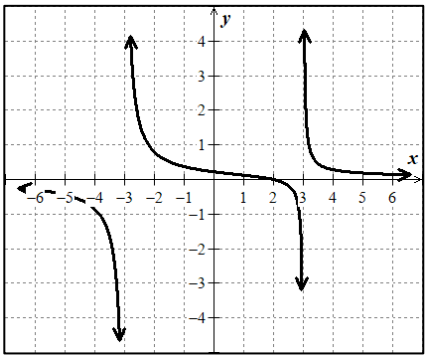
Solving Rational Inequalities

When solving rational inequalities, it is important to locate **BOTH** the **zeros** and the **undefined values**. We can use a procedural approach to solve rational inequalities.
Remember: $f(x) > 0$ means the graph of f is above the x-axis and $f(x) < 0$ means the graph of f lies below the x-axis.

1. **Isolate** the inequality so that 0 is on the right.
2. Be sure the left side of the inequality is a single rational function $R(x) = \frac{N(x)}{D(x)}$.
3. **Factor** the expressions $N(x) = 0$ and $D(x) = 0$, use solutions to find values for the sign chart.
4. Create a **sign chart** with the values from **Step 3**. Mark the values where the denominator is zero. ***These values will never be included in the solution.***
5. **Test values** in each interval to determine **positive or negative** values for the interval.
6. **Interpret** the sign chart results to answer the given inequality.
7. Write your answer in **interval notation**, consider the **endpoints**.

EX #3: Solve the inequality $R(x) \geq 0$ by using the graph of $R(x)$ shown below.

$$R(x) = \frac{x-2}{x^2-9}$$



EX #4: Solve each inequality algebraically.

A. $(x-1)(x-2)(x-4) \leq 0$

Lesson II-B: Equivalent Representations of Polynomial and Rational Expressions

The Binomial Theorem and Pascal's Triangle

When writing out the *binomial expression* $(a+b)^n$, where n is a positive integer, the pattern known as ***Pascal's Triangle*** appears, as well as other interesting facts to be discovered. In order to expand the binomial expression, we need to know how to find the binomial coefficient.

Definition of a Binomial Coefficient

The expression $\binom{n}{r}$ (read "*n above r")* is called a **binomial coefficient** is defined by

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

For nonnegative integers n and r, with $n \geq r$.

The symbol ${}_nC_r$ is often used in place of $\binom{n}{r}$ notation.

You **DO NOT** need to know this formula for the AP Precalculus Exam.

EX #9: Evaluate each of the following.

A. $\binom{6}{2}$ **B.** $\binom{9}{4}$ **C.** $\binom{3}{3}$

Pascal's Triangle and Binomial Expansion

Pascal's Triangle can be used to find the values of the binomial coefficients in expanding expressions in an organized and efficient manner.

Pascal's Triangle	Binomial Expansion
Row 0: **1**	$(x+y)^0 = 1$
Row 1: **1 1**	$(x+y)^1 = x + y$
Row 2: **1 2 1**	$(x+y)^2 = x^2 + 2xy + y^2$
Row 3: **1 3 3 1**	$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
1 4 6 4 1	
1 5 10 10 5 1	

Lesson I2-A: Transformations of Functions

Topic 1.12: Transformations of Functions

The simplest form in a set of functions that form a family, namely, $F(x) = a \cdot f(x-h) + k$, is commonly called the **parent function. A change made to at least one one of the values** a, h, and k, is called a **transformation.** There are four types of transformations: ***dilation, reflection, rotation,*** and ***translation.***

RIGID TRANSFORMATIONS:

Keep the original shape and size of the graph unchanged

Notation	Changes to Parent Function	Ordered Pair
$g(x) = f(x) + k$	Vertical translation $f(x)$ k units **(up)**	
$g(x) = f(x) - k$	Vertical Translation $f(x)$ $-k$ units **(down)**	
$g(x) = f(x-h)$	Horizontal translation $f(x)$ h units **(right)**	
$g(x) = f(x+h)$	Horizontal translation $f(x)$ $-h$ units **(left)**	
$y = -f(x)$	Reflect function across x −axis	
$y = f(-x)$	Reflect function across y −axis	

NON-RIGID TRANSFORMATIONS:
Stretch and Shrink Factors

Distorts the shape of a graph

Notation	Changes to Parent Function	Ordered Pair
$y = \lvert f(x) \rvert$	Reflect negative y −values across x −axis	
$y = a \cdot f(x), a > 1$	Vertical dilation (stretch) factor of $\lvert a \rvert$	
$y = a \cdot f(x), 0 < a < 1$	Vertical compression (shrink) factor $\lvert a \rvert$	
$y = f(b \cdot x)\,, b > 1$	Horizontal compression (shrink) factor $\left\lvert \frac{1}{b} \right\rvert$	
$y = f(b \cdot x)\,, 0 < b < 1$	Horizontal dilation (stretch) factor $\left\lvert \frac{1}{b} \right\rvert$	

Lesson 1.13-1.14A: Function Models and Applications

Topic 1.13: Function Model Selection and Assumption Articulation
Topic 1.14: Function Model Construction and Application

This lesson will help us to construct linear, polynomial or piecewise-defined function models and applications given a real-world scenario.

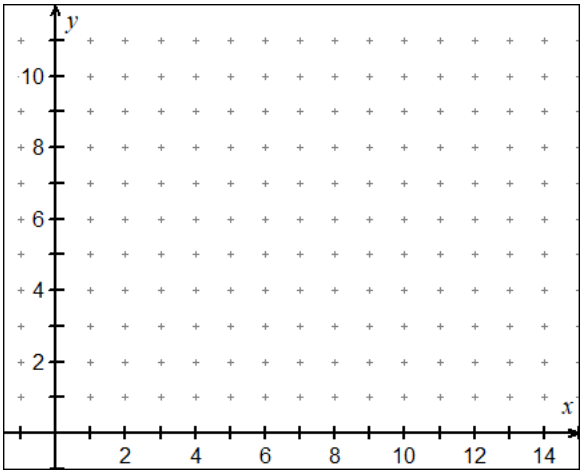
Finding Trend Lines

EX #1: An antique dealer visited an auction house and recorded an estimated selling price of six different antique chests. Then, he checked the actual selling price after the auction ended. The data points (estimate, actual) show the results, where each number is in thousands of dollars.

A. Draw a scatter plot and trend line.

B. Write the equation of the line.

estimate	12.5	8	10.5	5	9	7
actual	11	9.5	12	3.9	10	6.5



Creating Linear Regression Models

Let's use a graphing utility to draw a scatter plot then create a line of best fit to analyze the data in the table below. (*Note: Instructions for TI-84 or TI-Nspire included.*)

EX #2: Write the linear regression model for the price of oranges in dollars per box.

Fresh Market Price of Florida Oranges in Dollars/Box							
Year	2006	2007	2008	2009	2010	2011	2012
Price per Box	$8.40	$15.99	$10.75	$8.80	$12.75	$12.91	$13.12

SOURCE: http://www.nass.usda.gov/Data_and_Statistics/Quick_Stats/

A. Use your linear function to estimate the price for a box of oranges in 2014 and 2015.