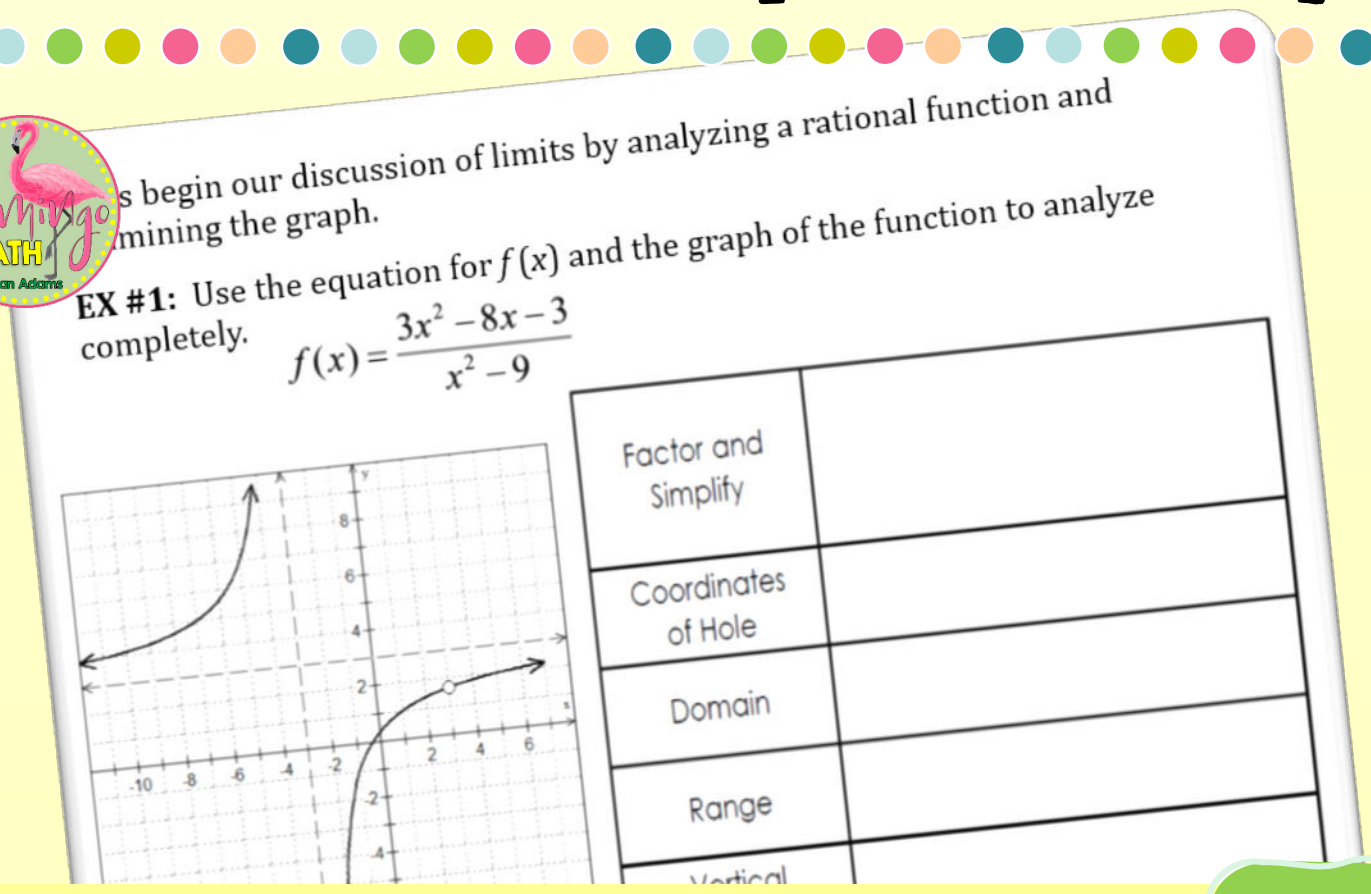
AP® Calculus

Understanding Limits Graphically



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Lesson 2: Understanding Limits Graphically and Numerically

Topic 1.2: Defining Limits and Using Limit Notation

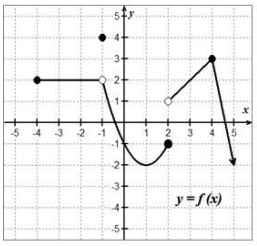
Limits are the "backbone" of understanding that connect algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific x-value. Recall from Pre-Calculus that you evaluated three types of limits. Complete the table below:

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	**PROPER NOTATION**	**VERBALLY:**
Right-hand limit		
Left-hand limit		
General limit		

Topic 1.3: Estimating Limit Values from Graphs

Consider the function shown at right.

Say you want to find $\lim_{x \to 4^+} f(x)$, the positive sign in the limit notation indicates a right-hand limit. If you think of the function as a highway and imagine you are traveling along the graph of $f(x)$ toward $x = 4$ **FROM THE RIGHT, NOT TO THE RIGHT**, and you stop at the vertical line $x = 4$, the y-value where you stop is 3. Therefore, $\lim_{x \to 4^+} f(x) = 3$.



EX #3: Use the graph above to evaluate each of the following limits:

A. $f(2)$	**B.** $f(-1)$
C. $\lim_{x \to 4^-} f(x)$	**D.** $\lim_{x \to 2^+} f(x)$
E. $\lim_{x \to 2^-} f(x)$	**F.** $\lim_{x \to -1^+} f(x)$
G. $\lim_{x \to -1^-} f(x)$	**H.** $\lim_{x \to -4^+} f(x)$
I. $\lim_{x \to -4^-} f(x)$	**J.** $\lim_{x \to -1} f(x)$

Topic 1.4: Estimating Limit Values from Tables

EX #5: Now, consider the function $f(x) = \frac{x-3}{x^2+2x-15}$. Complete the table below to find the limit as $x \to 3$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

Based on your analysis, what are the values of each of the limits below?

$\lim_{x \to 3^-} f(x) =$	$\lim_{x \to 3^+} f(x) =$	$\lim_{x \to 3} f(x) =$

By evaluating certain types of functions at a particular value, we may not necessarily have a sufficient understanding of the function's behavior at a specific point. This is especially true for rational functions that contain discontinuities.

This is why the idea of a ***limit is so important!*** Rather than just going directly to some x-value using direct substitution , we can **approach a point from either side** to get some sense of behavior in the **neighborhood**.

Look at the photo of the arch support construction for the Hoover Dam Bypass Bridge (2009). We can tell where the missing section is going to be. The actual height at that point would be the limit. With limits, we can discuss behavior at a point whether the point exists or not. With limits, we are looking at the y-value the graph approaches, not the actual y-value at that point.



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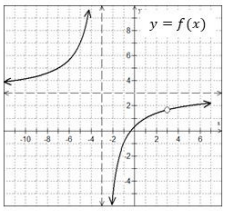
where L is a real number.

Verbally: The limit as x approaches c on $f(x)$ will exist if and only if the limit as x approaches c from the left is equal to the limit as x approaches c from the right.

Understanding Limit Notation

Let's revisit the notation we learned in Pre-Calculus. We need to convert these old methods of explaining extreme behaviors into limit notation for use in calculus.

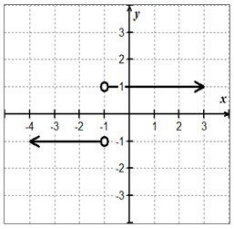
EX #2: Use the graph to complete the table below.



Pre-Calculus Notation vs. Calculus Notation	
As $x \to -\infty$, the graph of $f(x) \to$ _____	As $x \to \infty$, the graph of $f(x) \to$ _____

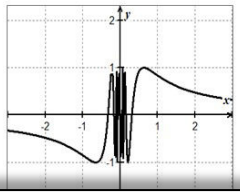
EX #4: Limits can fail to exist in three situations:

CASE 1: _____



Justify why the limit does not exist at $x = -1$ for $f(x) = \frac{|x+1|}{x+1}$

CASE 2: _____



Justify why the limit does not exist at $x = 0$ for $f(x) = \sin\left(\frac{1}{x}\right)$

EX #6: YOU OWN IT! In the box below, complete the sentence in your own words.

In order for the GENERAL LIMIT to exist, the function:

EX #7: Sketch a graph to satisfy each set of conditions.

1. $f(a)$ is undefined
2. $x = a$ is a point discontinuity
3. $\lim_{x \to a} f(x)$ exists

1. $\lim_{x \to a} f(x)$ DNE
2. $x = a$ is a jump discontinuity
3. $f(a)$ is defined

NOTES

1.2 **Understanding Limits Graphically & Numerically**

Name _____

Date _____ Period _____

Problems 1 – 4, complete the table and use the result to estimate the limit. Graph the function and verify your result.

1. $\lim_{x \to 2} \frac{x-2}{2x^2-9x+10}$

x	1.99	1.999	2	2.001	2.01
$f(x)$					

2. $\lim_{x \to 0} \frac{\sin x}{x}$

x	−0.01	−0.001	0	0.001	0.01
$f(x)$					

3. $\lim_{x \to 4} \frac{x^2-16}{x-4}$

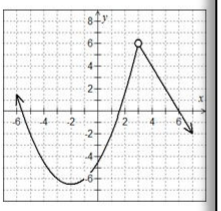
x	3.99	3.999	4	4.001	4.01
$f(x)$					

4. $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

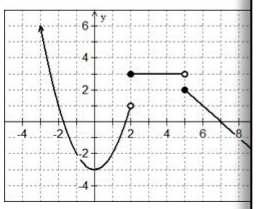
x	2.99	2.999	3	3.001	3.01
$f(x)$					

Problems 5 – 8, use the graph to fin

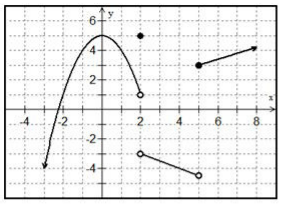
5.



6.

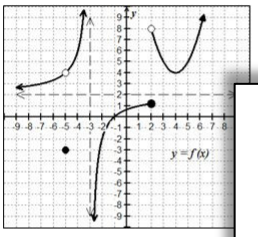


7.



A. $\lim_{x \to 5^-} f(x)$	**B.** $\lim_{x \to 2} f(x)$
C. $\lim_{x \to 0} f(x)$	**D.** $\lim_{x \to \infty} f(x)$
E. $\lim_{x \to 4} f(x)$	**F.** $\lim_{x \to 2^+} f(x)$

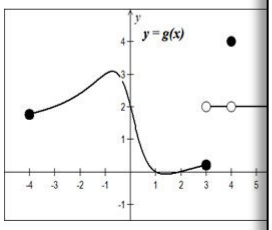
8.



A. $\lim_{x \to -3^-} f(x)$	**B.** $\lim_{x \to -3^+} f(x)$

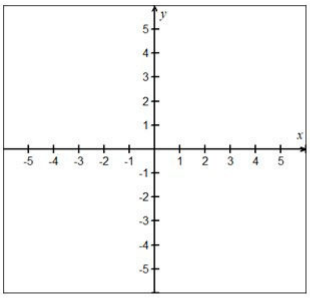
G. $\lim_{x \to -3} f(x)$

Problems 9 – 11, Use the graph of $g($
If false, explain why.

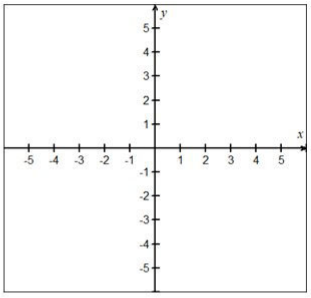


Problems 12 – 13, Sketch a graph of a function that satisfies each of the following conditions.

12. $\lim_{x \to 1^-} f(x) = 2$ $\lim_{x \to 1^+} f(x) = -3$ $f(1) = 4$



13. $\lim_{x \to -3^-} f(x) = 1$ $\lim_{x \to -3^+} f(x) = -4$ $\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$



14. Use the table below of the rational function $y = H(x)$ to find the indicated limits. For limits that do not exist, write D.N.E.

x	−1000	−4.001	−4	−3.999	1.999	2	2.001	1000
$H(x)$	−0.998	0.666	undefined	0.666	−4497	undefined	4504	3.0002

A. $\lim_{x \to -4^-} H(x) =$ **B.** $\lim_{x \to -4^+} H(x) =$ **C.** $\lim_{x \to -\infty} H(x) =$

G. $\lim_{x \to 2} H(x) =$ **H.** $\lim_{x \to \infty} H(x) =$

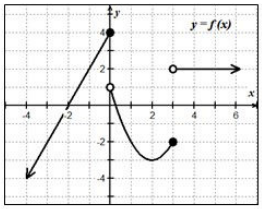
Practice Sets

1.2 Understanding the Limit — Form A

Name _____

Date _____ Period _____

Use the graph below to find the limits.



1. $\lim_{x \to 0^-} f(x) =$
2. $\lim_{x \to 0^+} f(x) =$
3. $\lim_{x \to 3^+} f(x) =$
4. $\lim_{x \to 3^-} f(x) =$
5. $\lim_{x \to \infty} f(x) =$
6. $\lim_{x \to -\infty} f(x) =$

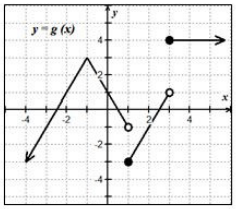
Grade:

1.2 Understanding the Limit — Form B

Name _____

Date _____ Period _____

Use the graph below to find the limits.



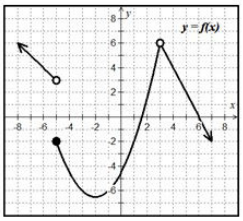
1. $\lim_{x \to 1^-} g(x) =$
2. $\lim_{x \to 1^+} g(x) =$
3. $\lim_{x \to \infty} g(x) =$

1.2 Understanding the Limit — Form C

Name _____

Date _____ Period _____

Use the graph below to find the limits.



1. $\lim_{x \to -5^+} f(x) =$
2. $\lim_{x \to -5^-} f(x) =$
3. $\lim_{x \to 3} f(x) =$
4. $\lim_{x \to \infty} f(x) =$

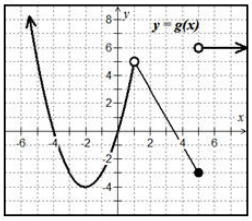
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1.2 Understanding the Limit — Form D

Name _____

Date _____ Period _____

Use the graph below to find the limits.



1. $\lim_{x \to 1^+} g(x) =$
2. $\lim_{x \to 5^-} g(x) =$
3. $\lim_{x \to 5^+} g(x) =$
4. $\lim_{x \to \infty} g(x) =$

Grade:

Daily Lesson Quiz