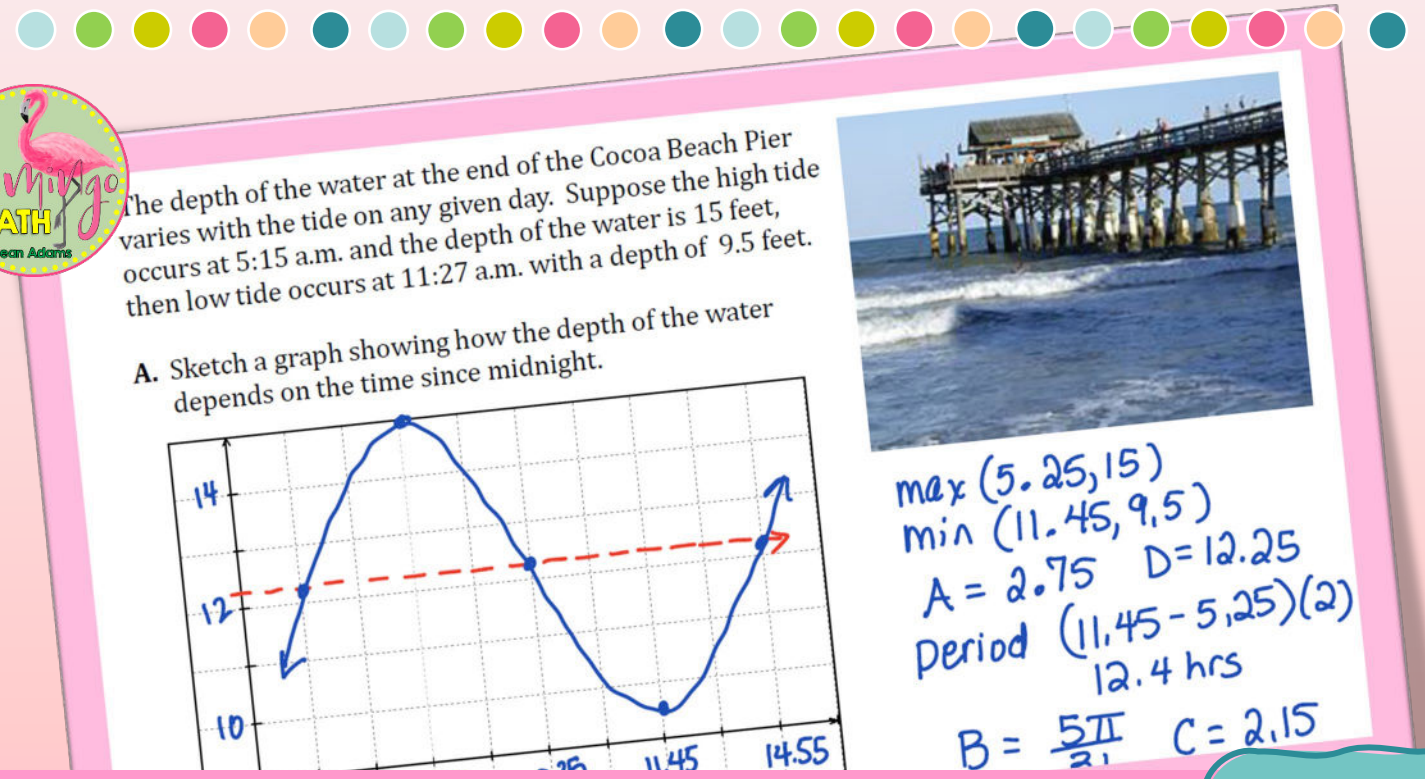
AP® Precalculus

Trigonometric and Polar Functions



Homework 3

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Pre-AP PRECALCULUS CURRICULUM

Scope and Sequence

Unit 3: Trigonometric and Polar Functions

Lesson	Skills & Learning Objectives
[3.1] Periodic Phenomena	▪ Understand periodic behavior and identify periodic functions ▪ Find amplitude and period for one cycle of a periodic function ▪ Sketch periodic functions based on verbal descriptions ▪ Describe key characteristics of period functions
[3.2] Sine, Cosine, and Tangent	▪ Name and draw angles in standard position ▪ Use the 16-point unit circle ▪ Find exact values using the unit circle ▪ Find and use coterminal angles ▪ Understand properties and behavior of positive and negative angles ▪ Make a connection between the 30-60-90 and 45-45-90 triangles and a unit circle and ratios (SOH-CAH-TOA)
[3.3] Sine and Cosine Function Values	▪ Define trigonometric functions using isosceles right and equilateral triangles ▪ Define trigonometric functions on a circle of radius *r* ▪ Find coordinates (x, y) given angle and radius of an angle in standard position.
[3.4] Sine and Cosine Function Graphs	▪ Understand periodic behavior and identify oscillating behavior. ▪ Find amplitude and period for one cycle of a periodic function. ▪ Sketch graphs of sine and cosine functions using the unit circle.

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Scope and Sequence

Unit 3: Trigonometric and Polar Functions

Lesson	Skills & Learning Objectives
[3.5] Sinusoidal Functions	▪ Find equivalent functions using transformations of sine and cosine. ▪ Use symmetry properties for even and odd functions. ▪ Find period, amplitude and midline for a periodic function. ▪ Find intervals of change in concavity for sinusoidal functions. ▪ Understand frequency of a sinusoidal function is the reciprocal of the period.
[3.6] Sinusoidal Function Transformations	▪ Identify key characteristics of periodic functions ▪ Sketch periodic functions based on transformations of parent functions ▪ Write equations for sinusoidal functions given a graph
[3.7] Sinusoidal Function Context and Data Modeling	▪ Solve real-world problems related to periodic behavior. ▪ Use curve fitting techniques to sketch sinusoidal functions. ▪ Construct sinusoidal function models using key points.
[3.8] The Tangent Function	▪ Understand tangent function using unit circle. ▪ Find amplitude and period for one cycle of the tangent function. ▪ Sketch graphs of tangent functions and transformations. ▪ Describe key characteristics of the tangent function.

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Scope and Sequence

Unit 3: Trigonometric and Polar Functions

Lesson	Skills & Learning Objectives
[3.9] Inverse Trigonometric Functions	▪ Understand and use the inverse sine, cosine, and tangent functions. ▪ Find the exact value of expressions involving the inverse sine, cosine, and tangent functions. ▪ Use a calculator to evaluate inverse trigonometric functions. ▪ Find exact values of composite functions with inverse trigonometric functions.
[3.10] Trigonometric Equations and Inequalities	▪ Find general solutions for trigonometric equations. ▪ Solve trigonometric equations of multiple angles. ▪ Solve trigonometric equations in quadratic form. ▪ Use factoring techniques and technology to solve trigonometric equations and inequalities. ▪ Solve trigonometric inequalities.
[3.11] The Secant, Cosecant, and Cotangent Functions	▪ Graph secant, cosecant and cotangent functions ▪ Find amplitude and period for one cycle of a periodic function ▪ Describe key characteristics of periodic reciprocal functions ▪ Graph transformations of reciprocal functions.
[3.12] Equivalent Representations of Trigonometric Functions	▪ Recognize and write identities ▪ Use fundamental identities to simplify expressions ▪ Prove identities analytically ▪ Evaluate trig functions using identities ▪ Apply identities for sum and difference formulas ▪ Apply identities for multiple angles ▪ Solve trigonometric equations using identities

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Scope and Sequence

Unit 3: Trigonometric and Polar Functions

Lesson	Skills & Learning Objectives
[3.13] Trigonometry and Polar Coordinates	▪ Plot points using polar coordinates. ▪ Convert coordinates from polar to rectangular form and vice versa. ▪ Transform equations between polar and rectangular forms. ▪ Represent complex numbers in the complex plane. ▪ Write complex numbers in trigonometric form.
[3.14] Polar Function Graphs	▪ Graph polar equations. ▪ Convert polar equations to rectangular form and vice versa. ▪ Identify types of polar graphs from equations ▪ Find the domain for completing one cycle of a polar graph. ▪ Find intervals where the *r*-values are negative. ▪ Find points of intersection of two polar graphs and identify any common points.
[3.15] Rates of Change in Polar Functions	▪ Determine intervals on which a polar function is positive and increasing or negative and decreasing and vice versa. ▪ Determine intervals on which the distance between a polar function and the origin is increasing or decreasing. ▪ Find relative extremum on a polar function where function changes from increasing or decreasing or vice versa. ▪ Find average rate of change at which the radius is changing per radian. ▪ Use average rate of change of r with respect to θ to estimate values of the function on an interval. ▪ Justify conclusions about the behavior of a function based on the rate of change.

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3.1

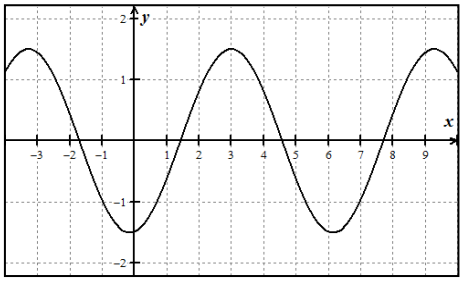
Periodic Phenomena
Practice Set 1

Name _____

Date _____ Period _____

Problems 1 – 4, determine whether each function *is* or *is not* periodic. If it is, find the period. If not, explain why.

1.



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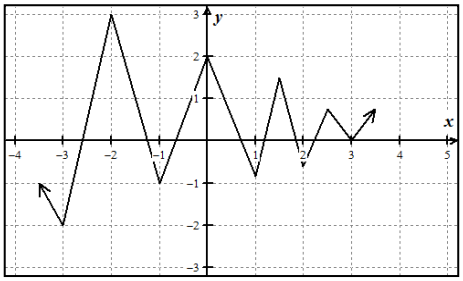

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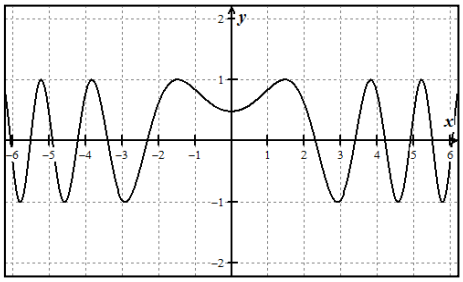

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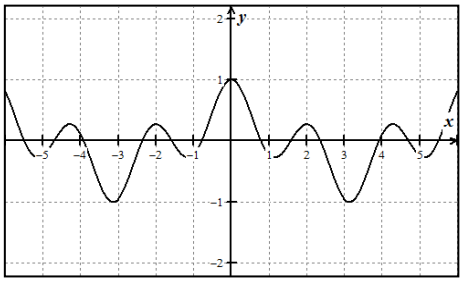
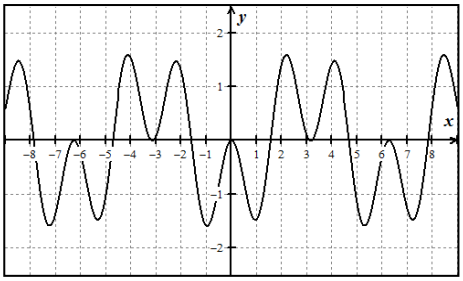


Problems 5 – 6, identify one cycle in two different ways. Then determine the amplitude and period of the function. State maximum and minimum values for each graph.

5.



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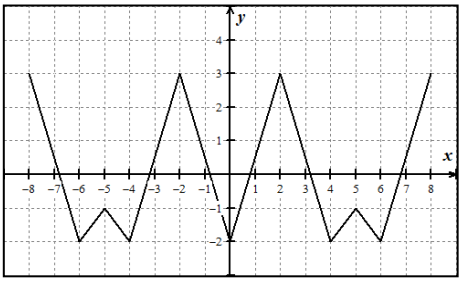
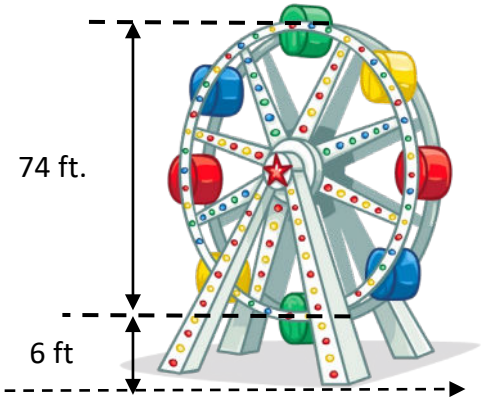


Free Response Question

Kevin and Alice are the last two seated on the Ferris wheel. Once the ride begins, the wheel moves at a constant speed. It takes 48 seconds to complete one revolution.

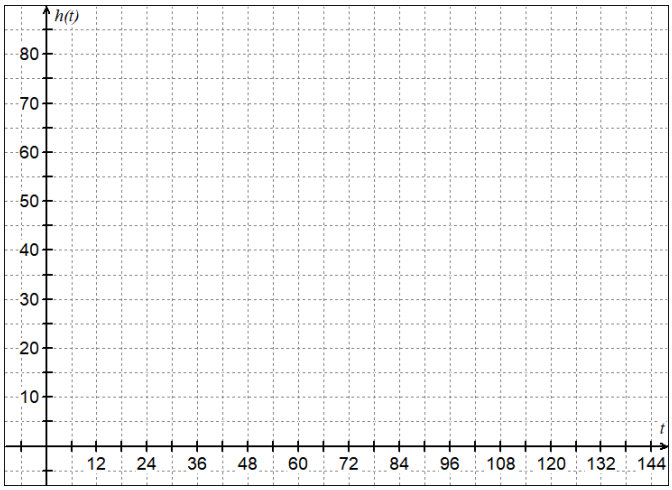


A. How high are the two friends when the ride starts?

B. At what height are they at 12 seconds? 24 seconds? 36 seconds?

C. What is their height at 168 seconds? How many revolutions have they made? Show work to support your answer.

D. Sketch a graph showing the relationship between the height above the ground and the time since the ride began. Use $0 \leq t \leq 144$ for the domain, where $t = 0$ is the time at which the ride starts.



E. Predict where the riders will be at 4 minutes.

F. How far (in feet) have the two friends traveled after one revolution of the wheel? How far have you traveled at 144 seconds?

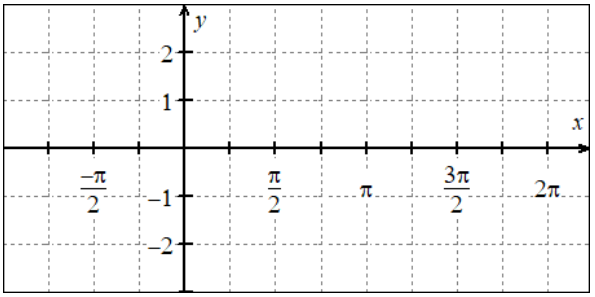
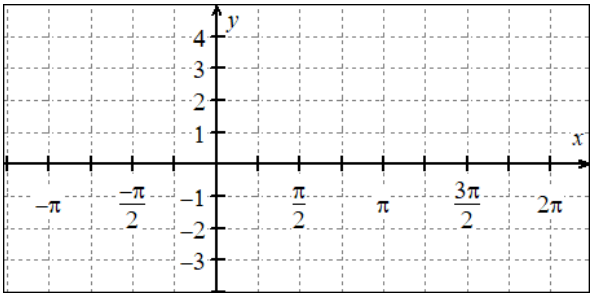
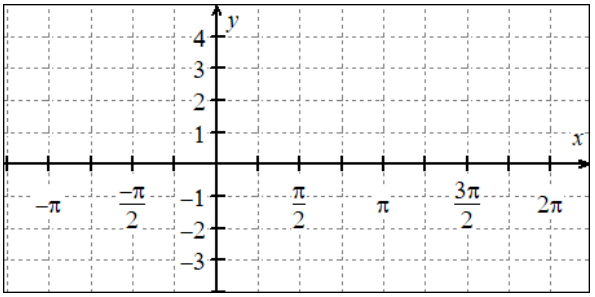
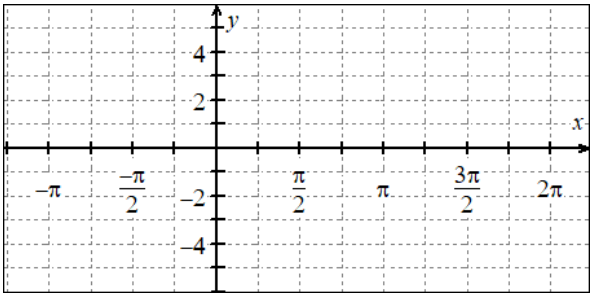
3.6

Sinusoidal Function Transformations Practice Set 1

Name _____

Date _____ Period _____

Problems 1-8, Graph each equation.

Equation	Characteristics	Graph
1. $y = 2\cos\left(x + \frac{\pi}{2}\right)$	Amplitude: Period: Phase Shift:	
2. $y = -3\sin(x - \pi)$	Amplitude: Period: Phase Shift:	
3. $y = 2\cos\left(x - \frac{\pi}{2}\right)$	Amplitude: Period: Phase Shift:	
4. $y = 4\sin[2(x - \pi)]$	Amplitude: Period: Phase Shift:	

3.7 Sinusoidal Function Context and Data Modeling Practice Set 1

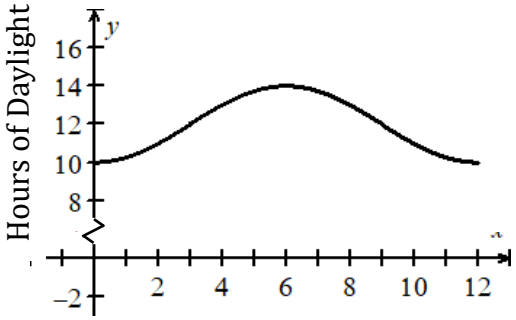
Name _____

Date _____ Period _____

Problems 1- 10, solve each problem using sinusoidal models.

1. The figure at right shows the number of hours of daylight in St. Augustine, Florida. Hours of daylight are at a minimum of 10 hours in December. Hours of daylight are at a maximum of 14 hours in June.

Let x represent the month of the year, where Jan. =1, Feb. = 2, and Dec. = 12. If y represents the number of hours of daylight in month x, use a sine function in the form $y = A\sin[B(x - C)] + D$ to model the hours of daylight.



Month (Jan = 1, Feb = 2, etc.)

Explain the meaning of the points of inflection in context to the scenario.

2. The London Eye Ferris wheel has a diameter of 135 meters. A ride on the Eye will complete one rotation every 30 minutes. Passengers board from a platform that is 2 meters above the round. Write an equation that models the height above ground as a function of time in minutes.

3. A person's blood pressure can be modeled by the function $f(t) = 20\sin(160\pi t) + 100$, where $f(t)$ represents the blood pressure at time t, measured in minutes. Interpret the function in terms of period and frequency.

3.7 Sinusoidal Function Context and Data Modeling Practice Set 2

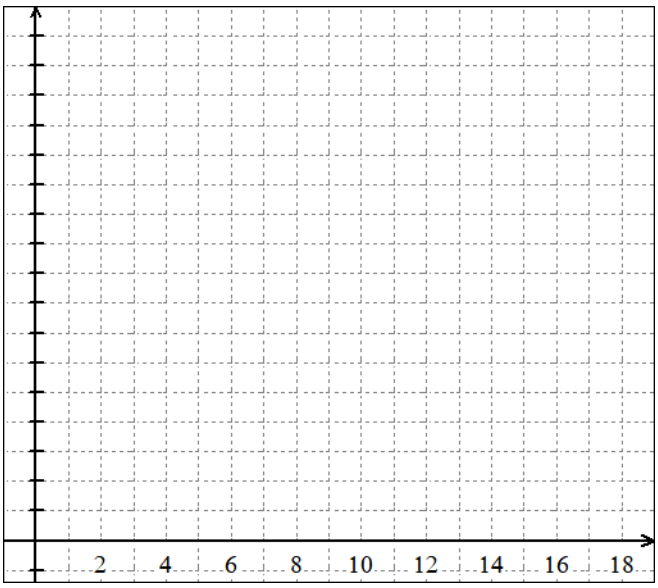
Name _____

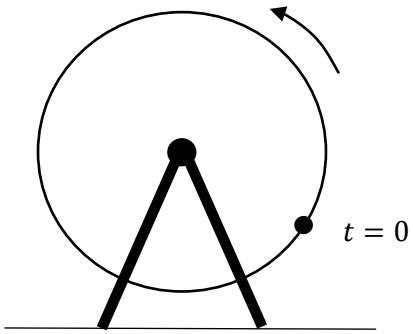
Date _____ Period _____

Problems 1- 8, solve each problem using sinusoidal models.

1. The Ferris Wheel at the Florida State Fair in Tampa is a popular ride that soars to a maximum height of 45 feet and a low point of 3 feet. When the last seat is filled and the Ferris Wheel begins to revolve, your seat is at the position marked in the figure below at *t=0*. Your stopwatch records that it takes you 4 seconds to reach the top, and that the wheel makes a complete revolution once every 12 seconds. The wheels diameter is 42 feet.

A. Graph the sinusoid.





B. Write an equation that models the behavior of the Ferris Wheel ride.

C. Use your equation to find the height above the ground at your starting location $t = 0$.

D. What is the value of t the second time that you are 15 feet above the ground?

3.9

Inverse Trigonometric Functions Practice Set 1

Name _____

Date _____ Period _____

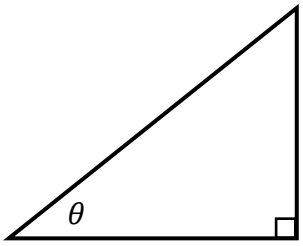
Problems 1-12, Find the value of each expression.

1. $\sin^{-1}(-1)$	**2.** $\cos^{-1}(0)$
3. $\tan^{-1}(0)$	**4.** $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$
5. $\tan^{-1}\left(-\sqrt{3}\right)$	**6.** $\cos^{-1}\left(-\frac{1}{2}\right)$
7. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$	**8.** $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$
9. $\cos^{-1}(1)$	**10.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
11. $\tan^{-1}(-1)$	**12.** $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Problems 13– 20, Use a calculator to find the value of each expression rounded to two decimal places.

13. $\cos^{-1}(0.75)$	**14.** $\tan^{-1}(4)$
15. $\sin^{-1}\left(\frac{3}{8}\right)$	**16.** $\arccos\left(\frac{5}{8}\right)$
17. $\arctan(-0.4)$	**18.** $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$
19. $\arcsin(0.75)$	**20.** $\tan^{-1}(12.4)$

21. Use a right triangle to write the trigonometric expression $\sin(\cos^{-1}(2x))$ as an algebraic expression for $0 \leq x \leq \frac{1}{2}$.



Problems 22 - 24, solve.

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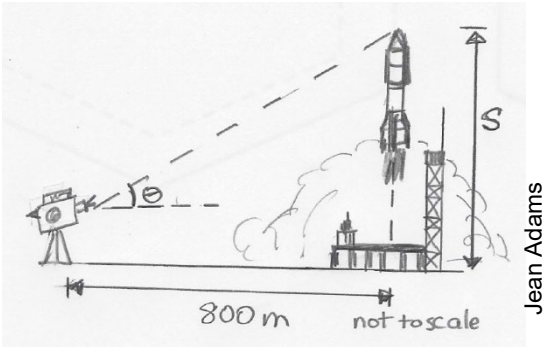
22. A remote camera at ground level is filming the lift-off of a Falcon rocket at a point 800 meters from the launch pad. Let θ be the angle of elevation to the rocket and let s be the height of the rocket.
Find θ when $s = 1500$ meters.



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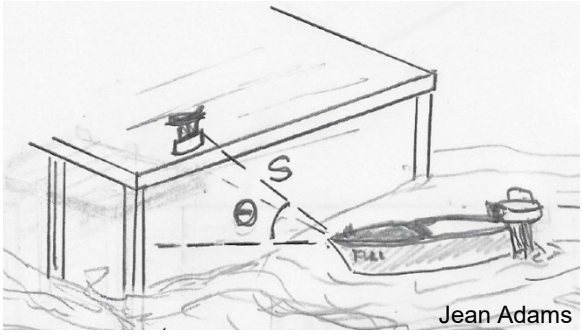
23. A motorboat is being pulled into a seawall dock by an electric pulley at a dock that is 8 feet above the deck of the boat. Let θ be the angle of elevation from the boat to the pulley and let s be the length of the rope from the pulley to the boat. Findθ when $s = 30$ feet.



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Problems 21 – 26, Consider the behavior of the function as $x \to c$.

21. $f(x) = \tan x$

A.) $x \to \frac{\pi}{2}^+$, the value of $f(x) \to$

B.) $x \to \frac{\pi}{2}^-$, the value of $f(x) \to$

C.) $x \to -\frac{\pi}{2}^+$, the value of $f(x) \to$

D.) $x \to -\frac{\pi}{2}^-$, the value of $f(x) \to$

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22. $f(x) = \sec x$

A.) $x \to \frac{\pi}{2}^+$, the value of $f(x) \to$

B.) $x \to \frac{\pi}{2}^-$, the value of $f(x) \to$

C.) $x \to -\frac{\pi}{2}^+$, the value of $f(x) \to$

D.) $x \to -\frac{\pi}{2}^-$, the value of $f(x) \to$

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23. $f(x) = \csc x$

A.) $x \to 0^+$, , the value of $f(x) \to$

B.) $x \to 0^-$, the value of $f(x) \to$

C.) $x \to \pi^+$, the value of $f(x) \to$

D.) $x \to \pi^-$, the value of $f(x) \to$

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24. $f(x) = \cot x$

A.) $x \to 0^+$, the value of $f(x) \to$

B.) $x \to 0^-$, the value of $f(x) \to$

C.) $x \to \pi^+$, the value of $f(x) \to$

D.) $x \to \pi^-$, the value of $f(x) \to$

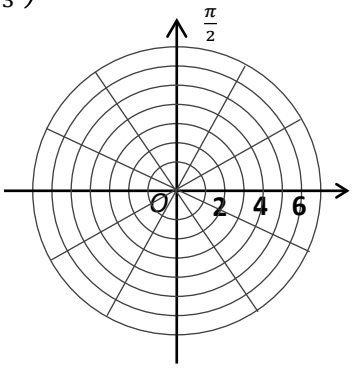
3.13 **Trigonometry and Polar Coordinates Practice Set 1**

Name _____

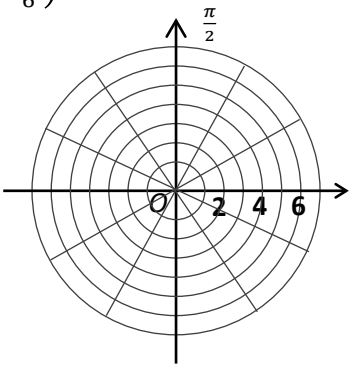
Date _____ Period _____

Problems 1-6, plot each point given in polar coordinates.

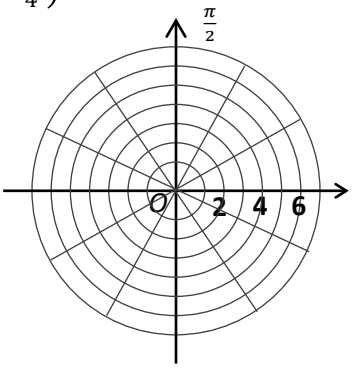
1. $\left(4, \frac{4\pi}{3}\right)$



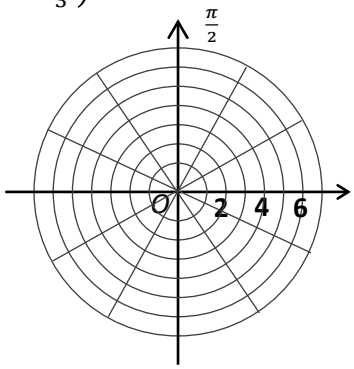
2. $\left(-2, \frac{5\pi}{6}\right)$



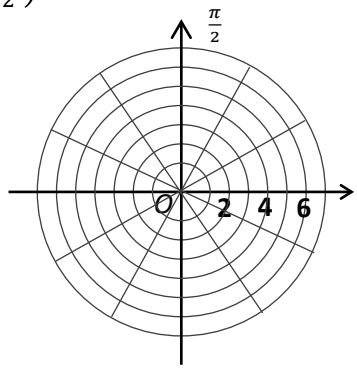
3. $\left(3, -\frac{3\pi}{4}\right)$



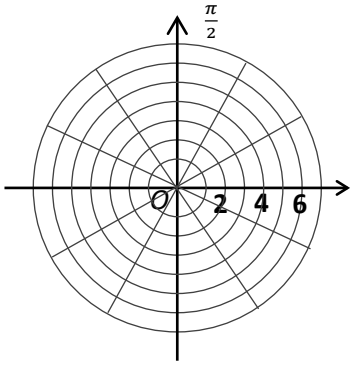
4. $\left(-5, -\frac{5\pi}{3}\right)$



5. $\left(6, \frac{3\pi}{2}\right)$



6. $(-4, -\pi)$



Problems 7 –10, the polar coordinates of a point are given. Find its rectangular coordinates.

7. $\left(3, -\frac{\pi}{3}\right)$

8. $\left(-2, \frac{7\pi}{6}\right)$

9. $\left(-5, -\frac{2\pi}{3}\right)$

10. $\left(-4, -\frac{11\pi}{6}\right)$

Problems 9 – 12, find the product of $z \cdot w$. Leave your answer in trigonometric form.

9. $z = 6\left(\cos\frac{\pi}{9} + i\ \sin\frac{\pi}{9}\right)$
$w = 3\left(\cos\frac{\pi}{4} + i\ \sin\frac{\pi}{4}\right)$

10. $z = \cos\frac{\pi}{3} + i\ \sin\frac{\pi}{3}$
$w = \cos\frac{5\pi}{12} + i\ \sin\frac{5\pi}{12}$

11. $z = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$
$w = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

12. $z = 5\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$
$w = 3\left(\cos\frac{3\pi}{16} + i\sin\frac{3\pi}{16}\right)$

Problems 13 – 16, find the trigonometric form of the quotient.

13. $\dfrac{3\left(\cos\frac{5\pi}{18} + i\ \sin\frac{5\pi}{18}\right)}{9\left(\cos\frac{\pi}{9} + i\ \sin\frac{\pi}{9}\right)}$

14. $\dfrac{8\left(\cos\frac{11\pi}{9} + i\ \sin\frac{11\pi}{9}\right)}{2\left(\cos\frac{11\pi}{18} + i\ \sin\frac{11\pi}{18}\right)}$

3.15

Rates of Change in Polar Functions Practice Set 2

Name _____

Date _____ Period _____

Problems 1 – 4, Given each polar function,

A. Graph the curve

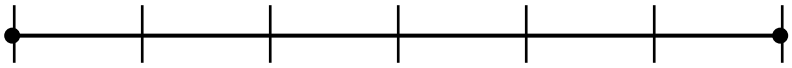
B. State value of θ where $f(\theta) = 0$.

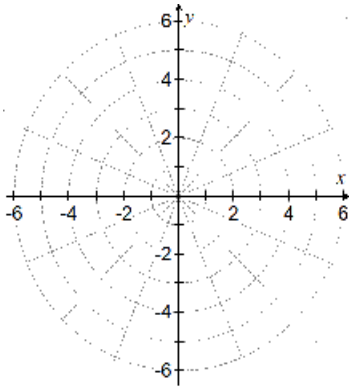
C. Find intervals where r is positive or negative

D. Find intervals where r is increasing or decreasing.

E. State intervals where the distance r from the pole is increasing or decreasing.

1. $r = f(\theta) = 5\sin(2\theta)$





2. $r = f(\theta) = -3\cos(3\theta)$

