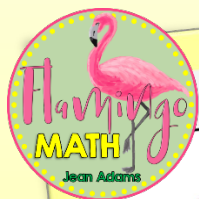


Precalculus Honors

Trigonometric Functions



Ratios on the Unit Circle

EX #1: In the sketch, at left, θ is in standard position and $P(x, y)$ is a point on the unit circle that corresponds to θ . Complete the following in terms of (x, y) :

$\sin \theta = y$	$\cos \theta = x$	$\tan \theta = \frac{y}{x}$
$\csc \theta = \frac{1}{y}$	$\sec \theta = \frac{1}{x}$	$\cot \theta = \frac{x}{y}$

Domain and Range of the Trigonometric Functions

	Domain	Range
	$[-\pi, 2\pi)$	

Guided Notes

4

Completely Editable

PRECALCULUS CURRICULUM

Scope and Sequence

Unit 4: Trigonometric Functions

Lesson	Skills & Learning Objectives
[4.1] Angles and Their Measure	☐ Convert from decimal degree to DMS ☐ Find reference angles and coterminal angles ☐ Convert between degrees and radians ☐ Naming and drawing angles in standard position
[4.2] Right Triangle Trigonometry	☐ Evaluate trigonometric functions of special triangles ☐ Use technology to find approximate values of trigonometric functions ☐ Solve real-world problems using right triangles
[4.3] Arc Length and Circular Motion	☐ Find arc length and area of a sector ☐ Linear speed and angular speed
[4.4] The Unit Circle	☐ Use the 16-point unit circle ☐ Define trigonometric functions using right triangles ☐ Find exact values using the unit circle
[4.5] Properties of Trigonometric Functions	☐ Find domain and range of trigonometric functions ☐ Determine signs and values given an angle or a quadrant ☐ Use even and odd properties ☐ Use fundamental identities
[4.6] Graphs of Sine and Cosine	☐ Find period, amplitude, midline of sine and cosine curves ☐ Graph transformations.

PRECALCULUS CURRICULUM

Scope and Sequence

Unit 4: Trigonometric Functions

Lesson	Skills & Learning Objectives
[4.7] Graphs of Other Trig Functions	☐ Graph tangent, cotangent, secant, and cosecant functions ☐ Graph transformations
[4.8] Inverse Trigonometric Functions	☐ Relate inverse properties to trigonometric functions
[4.9] More Inverse Trig Functions	☐ Relate inverse properties for cosecant, secant and cotangent to trigonometric functions
[4.10] Sinusoidal Curve Fitting	☐ Solve real-world problems related to periodic behavior ☐ Use curve fitting techniques to sketch sinusoidal functions
[4.11] Solving Problems with Trigonometry	☐ Solve more right triangle problems including navigation and simple harmonic motion

Lesson I: Angles and Their Measure

Engineers in the ancient world discovered ratios within right triangles, as they learned fundamental engineering skills. The secrets and patterns necessary to understanding circles and spheres helped in the development of trigonometry. Our chapter begins the journey into trigonometry with an introduction into angles and their measures.

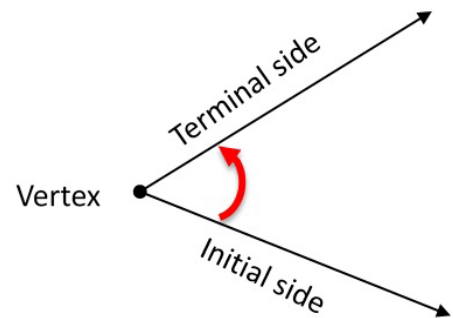
Angle Characteristics

An angle is determined by rotating a **ray** about its **endpoint**.

The _____ is the endpoint of the ray.

The _____ side is the starting position.

The _____ side is the position after rotation.

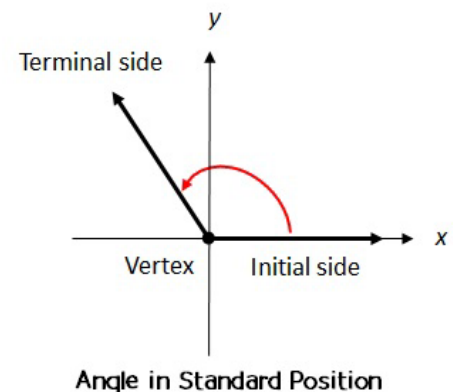


Standard Position Angles

Angles that are in standard position are placed on a coordinate plane, with the vertex at the origin and the initial side on the positive x-axis.

_____ angles are determined by a counterclockwise rotation.

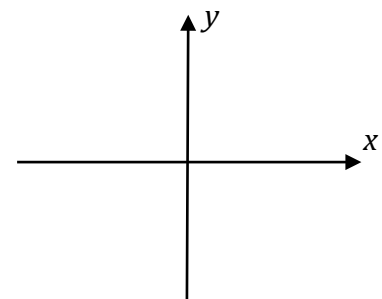
_____ angles are determined by a clockwise rotation.



Coterminal Angles

Two angles in standard position that have the same terminal side are called **coterminal** angles. We can find an angle that is coterminal to a given angle by adding or subtracting **one revolution**. So, any given angle has many coterminal angles. For example $\beta = 36^\circ$ is coterminal to all of the following: $396^\circ, 756^\circ, -324^\circ, -684^\circ$

EX #1: Draw $\theta = 220^\circ$ and label θ' as the angle that is coterminal.



Radians and Central Angles

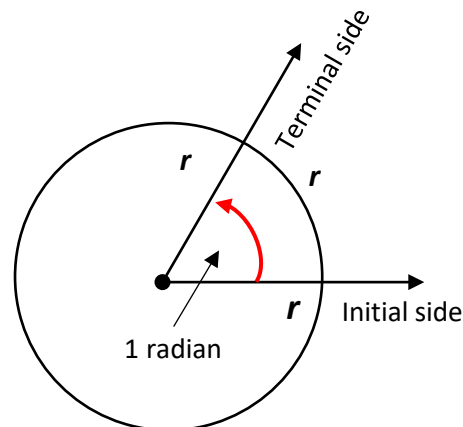
Radians

Another way to measure angles is in _____.

In the late 1800's mathematicians began to see a need for another unit of measure, because they needed angle measurements to "cancel out." This type of measure is needed in calculus to help simplify certain mathematical and physical formulas.

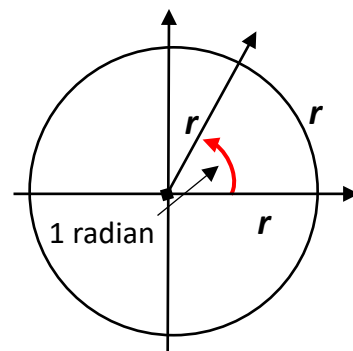
- ☐ Radians have no units
- ☐ One complete circular revolution is _____
- ☐ 1 radian is approximately 57.296°

The definition of a radian is based on the concept of a **unit circle**, which is a circle of radius 1 unit, with its center at the origin of a coordinate system. The radian measure of an angle is based on the length of an arc on the unit circle.



Central Angles on the Unit Circle

We use a **central angle** of a unit circle, shown at left, where the vertex is the center of the circle. One radian is the measure of the central angle that subtends (intercepts) an arc equal in length to the radius of the circle. The circumference of any circle is _____, where r is the radius measure. So the circumference of a unit circle where $r = 1$ is _____ units. Therefore, an angle representing one complete revolution of the circle measures _____ radians.

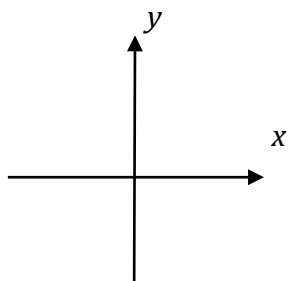


If θ is a central angle in a circle of radius r , and if θ is measured in radians, then the length s of the intercepted arc is given by $s = r\theta$.

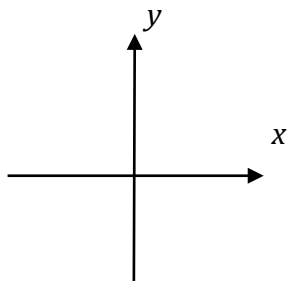
Common Radian Measures

Sketch each radian measure on the coordinate plane below.

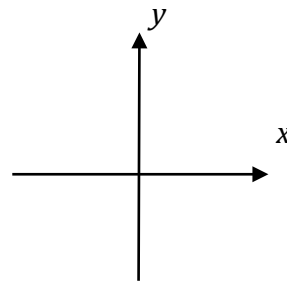
$-\frac{\pi}{4}$ radians



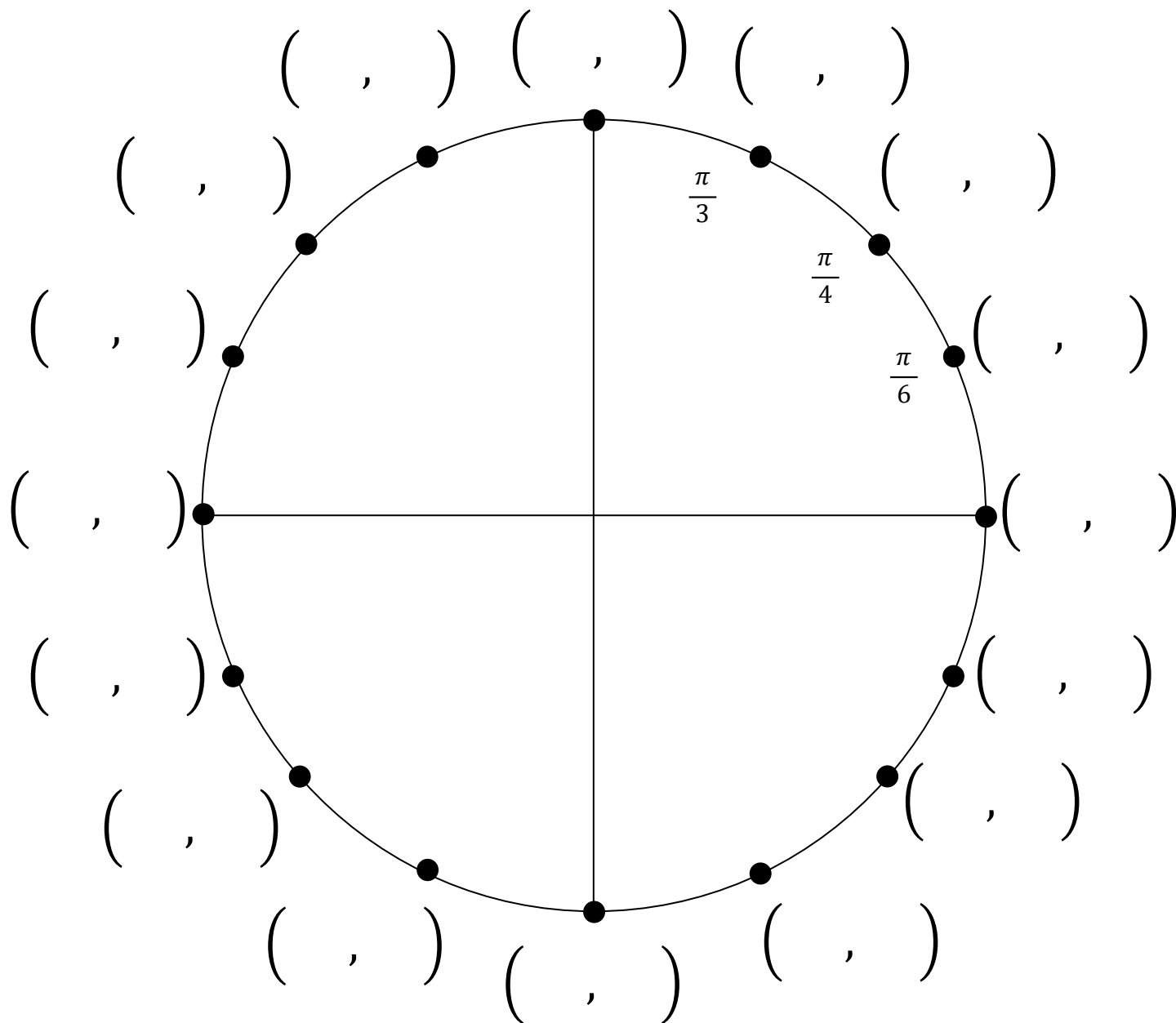
$\frac{\pi}{2}$ radians



π radians



The 16-Point Unit Circle



Finding Exact Values of the Trigonometric Functions

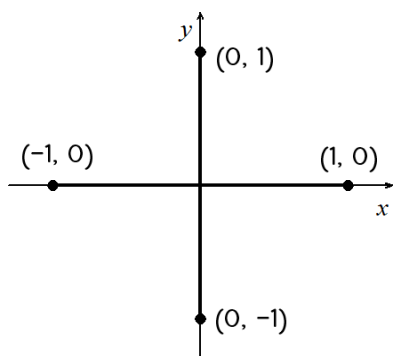
EX #2: Summarize the exact values for 30° , 60° , and 90° angles (rationalize denominators).

θ Degrees	θ Radians	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
30°	$\frac{\pi}{6}$						
45°	$\frac{\pi}{4}$						
60°	$\frac{\pi}{3}$						

Quadrantal Angles

A quadrantal angle is an angle whose terminal side lies along one of the coordinate axes.

EX #3: Evaluate the six trig functions at the four quadrant angles.



Note:

There is no need to memorize the table, simply draw the angle and apply the definition for any given function.

θ degrees	0°	90°	180°	270°
radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin \theta$				
$\cos \theta$				
$\tan \theta$				
$\csc \theta$				
$\sec \theta$				
$\cot \theta$				

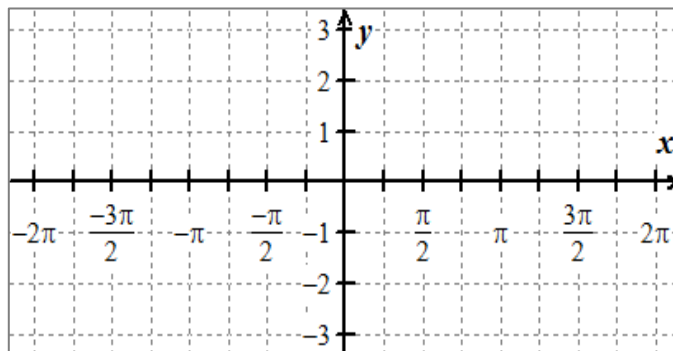
Lesson 7: Graphs of Other Trigonometric Functions

The graphs of tangent, cotangent, secant, and cosecant functions are also periodic. However, tangent and cotangent will have a shorter cycle (period) than the other four functions. We just need to define the graph over some interval with a length π .

The Graph of $y = \tan x$

The graph of the tangent function will complete one full cycle in the interval of $[0, \pi]$ so the **period** of $y = \tan \theta$ is π . However, there is a vertical asymptote at $\theta = \frac{\pi}{2}$. On what continuous interval can the entire behavior of the function be shown?

$$Y = A \tan (Bx - C)$$

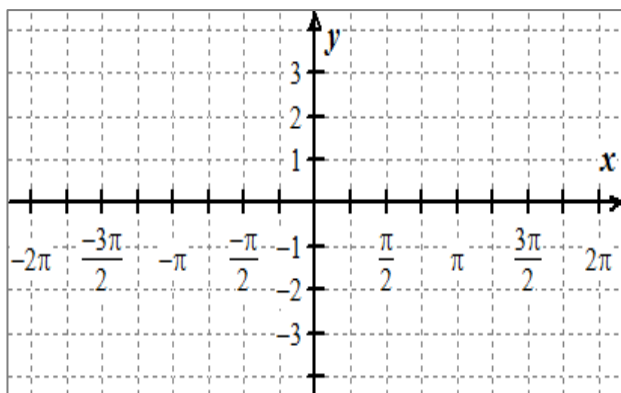


Characteristics of the Tangent Function

Domain		Range	
x-intercepts		y-intercept	
Odd Symmetry		Asymptotes	

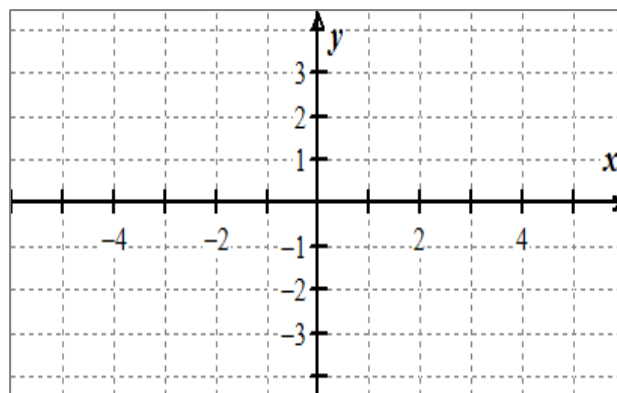
EX #2: Use transformations, graph each function, state period and show all asymptotes.

A. Graph $y = -2 \tan \left(\frac{x}{2} \right)$



Period: _____

B. Graph $y = \tan \frac{\pi}{4} x + 2$



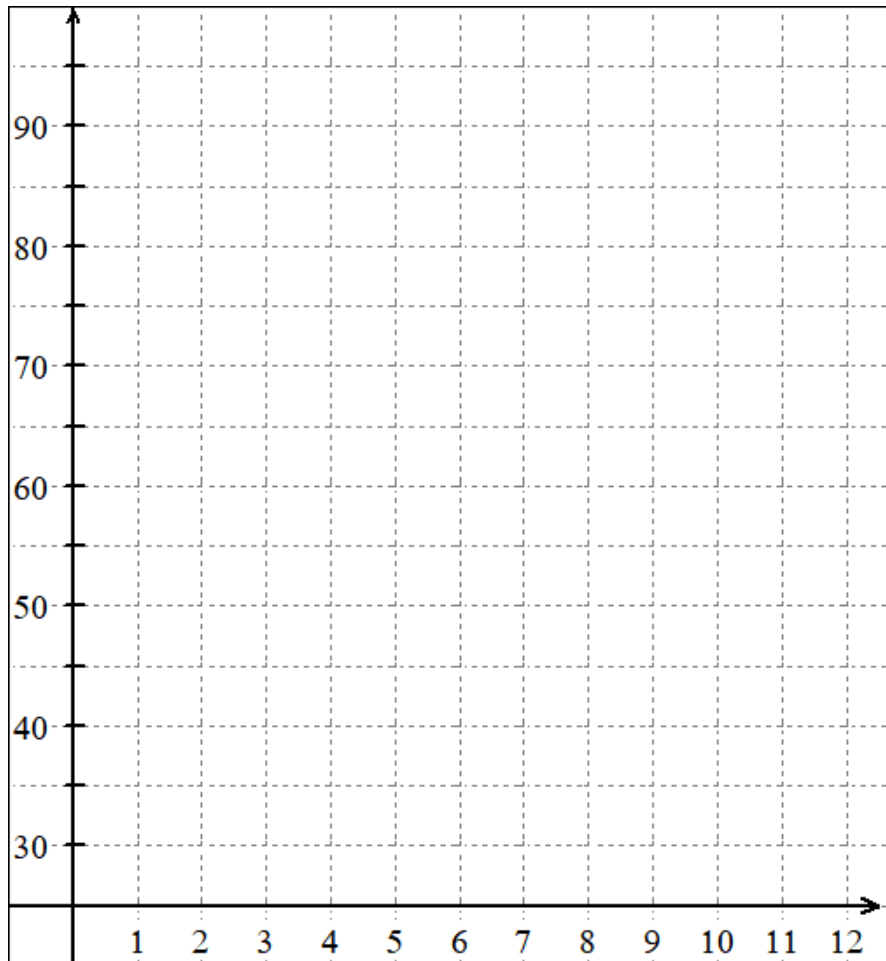
Period: _____

Finding Sinusoidal Functions from Data

EX #5: The table below shows the average temperature per month for Atlanta, Georgia. Information was found at the World Climate website (www.worldclimate.com).

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
$^{\circ}F$	51.6	55.9	63.7	72.7	80.1	86.2	88.3	87.4	82.0	72.7	62.6	54.0

A: Plot the data below, and notice that they approximate sinusoidal behavior.



B. Why does it make sense that average monthly temperatures can be modeled by a periodic function?