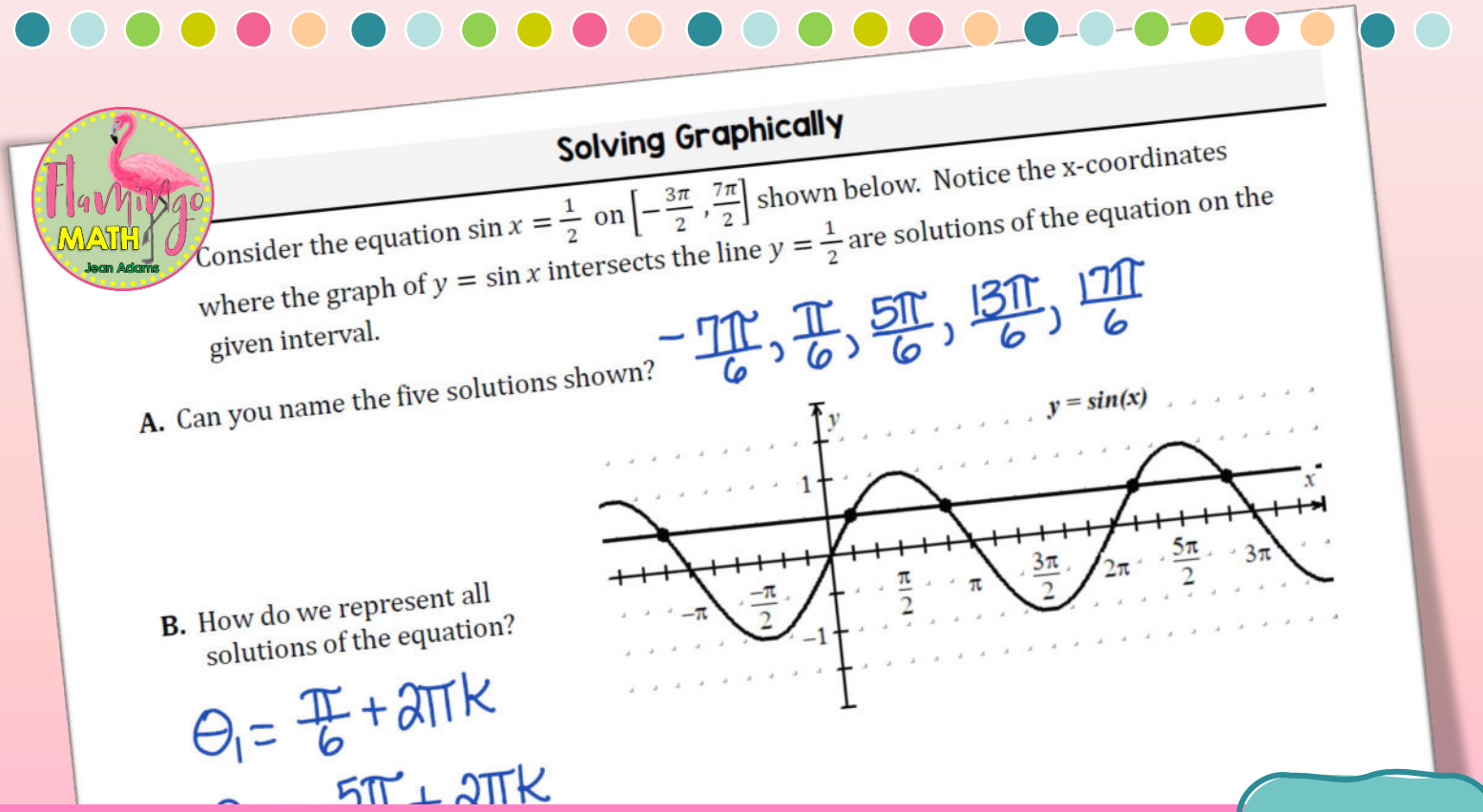
AP® Precalculus

Trigonometric Equations & Inequalities



Solving Graphically

Consider the equation $\sin x = \frac{1}{2}$ on $\left[-\frac{3\pi}{2}, \frac{7\pi}{2}\right]$ shown below. Notice the x-coordinates where the graph of $y = \sin x$ intersects the line $y = \frac{1}{2}$ are solutions of the equation on the given interval.

A. Can you name the five solutions shown? $-\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

B. How do we represent all solutions of the equation?

$\theta_1 = \frac{\pi}{6} + 2\pi k$

$\theta_2 = \frac{5\pi}{6} + 2\pi k$

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Lesson 10-A: Trigonometric Equations and Inequalities

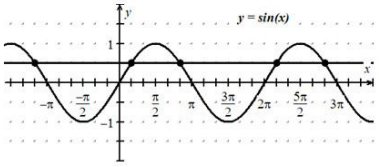
In this lesson, we consider **trigonometric equations** and **inequalities**. Equations that contain trigonometric functions can be used to answer periodic behavior in our world. We can predict the tides, moon phases, seasons, breathing intervals, and more. A **trigonometric equation** is an equation that contains a trigonometric expression with a variable, that are satisfied only by some values of the variable (or, possibly, have no solutions,). Inverse trigonometric functions are useful in solving equations and inequalities involving trigonometric functions, but solutions may need to be modified due to domain restrictions.

Solving Graphically

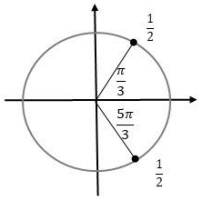
EX #1: Consider the equation $\sin x = \frac{1}{2}$ on $\left[-\frac{3\pi}{2}, \frac{7\pi}{2}\right]$ shown below. Notice the x-coordinates where the graph of $y = \sin x$ intersects the line $y = \frac{1}{2}$ are solutions of the equation on the given interval.

A. Can you name the five solutions shown?

B. How do we represent all solutions of the equation?



EX #2: Solve the equation by using the unit circle. Give the general solutions.
$2\cos x - 1 = 0$



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Finding General Solutions

EX #3: Solve each equation. Give a general formula for all solutions.

A. Solve. $\sin x = -\frac{\sqrt{2}}{2}$

B. Solve. $18\cos x - 7 = 14\cos x - 5$

Using Technology

EX #4: Use a calculator to solve each equation. Round your answers to two decimal places.

A. Solve. $\tan x = 5$

B. Solve. $\cos\theta = 0.6$

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Lesson 10-B: Trigonometric Equations and Inequalities

Now we will turn our focus on finding solutions of trigonometric equations for $0 \le x < 2\pi$. Let's begin with some factoring examples for single trigonometric functions and then move to multiple angle equations.

Various Techniques

EX #5: Solve by the square root technique on $[0, 2\pi)$

A. $4\sin^2 x - 1 = 0$

B. $3\tan^2 x - 1 = 0$

Solve Quadratics by Factoring

EX #6: Solve on $[0, 2\pi)$

A. $2\sin^2 x - \sin x - 1 = 0$

B. $6\cos^2 x - \cos x - 2 = 0$

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As we continue solving more difficult equations, remember that you can use a graphing utility to check your solutions of these equations. By graphing the left side as Y_1 and the right side as Y_2, the solutions are the x-coordinates of the points where the graphs intersect.
Let's consider equations that include multiple angles. Here is some examples.

$\sin 2x = 1$ $\qquad$ $\tan\frac{x}{2} = \sqrt{3}$

Solving an Equation with a Multiple Angle

EX #8: Solve on $[0, 2\pi)$ $\tan 2x = \sqrt{3}$

EX #9: Solve on $[0, 2\pi)$. $\sin\frac{x}{3} = \frac{1}{2}$

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Guided Notes

Lesson 10-C: Trigonometric Equations and Inequalities

Trigonometric Inequalities

Finally, we want to look at solving trigonometric inequalities. You will need to consider using the inverse trigonometric properties when values for x are not exact.

EX #10: Find an interval that will satisfy the inequality: $\cos(2x) > 0$

EX #11: Solve $\sin\left(x - \frac{\pi}{3}\right) \geq \sin x$ if $[0, 2\pi)$

EX #12: Solve $\cos x + \frac{1}{3} \geq 0$ given $0 \leq x \leq 2\pi$

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Trigonometric Inequalities

EX #13: Solve the inequality $4\,|\tan x\,| - 1 < 0$ for $[0, \pi)$

Application of a Trigonometric Inequality

EX #14: When breathing, a normal cycle takes place about every 5 seconds. Velocity of air flow, y, measured in liters per second, after x seconds is modeled by $y = 0.6 \sin\left(\frac{2\pi}{5}\right)$. Velocity of air flow is positive when we inhale and negative when we exhale.

A. For one cycle of breathing, find an interval of time where $0.6 \sin\left(\frac{2\pi}{5}x\right) \geq 0.3$

B. Explain what this means in context to the scenario.

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Application of a Trigonometric Inequality

EX #15: The number of hours of daylight on campus at Harvard University is given by

$$y = 3 \sin\left[\frac{2\pi}{365}(x - 79)\right] + 12$$

A. Find the interval where Harvard University will have no more than 10 hours of daylight within a year.

B. Find the interval where the campus will have more than 14 hours of daylight within a year.

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Guided Notes

3.10 **Trigonometric Equations and Inequalities Practice Set 1**

Name _____

Date _____ Period _____

Problems 1 – 8, Find all solutions of each equation.

1. $\sin(\theta) = -\frac{\sqrt{3}}{2}$

2. $2\cos\theta - \sqrt{3} = 0$

3. $\tan\theta + 1 = 0$

4. $1 - \sin\theta = \frac{1}{2}$

5. $3 + 2\sin\theta = 2$

6. $\sqrt{3}\tan\theta + 1 = 0$

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7. $(\sqrt{3}\tan x + 1)(\cos x - 1) = 0$

8. $(\tan x - 1)(\sin x - 1) = 0$

Problems 9 – 12, Use a graphing utility to solve each equation on $[0, 2\pi)$. Round answers to two decimal places.

9. $\sin x = 0.2$

10. $4\tan x + 2 = 0$

11. $5\cos x = 2$

12. $\cos x = 2\sin x$

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3.10 **Trigonometric Equations and Inequalities Practice Set 2**

Name _____

Date _____ Period _____

Problems 1 – 18, Solve each equation on the interval $0 \le \theta \le 2\pi$.

1. $\cos x - 2\sin x\cos x = 0$

2. $\tan^2 x\cos x = \tan^2 x$

3. $\sin x + 2\sin x\cos x = 0$

4. $\cos(\theta) + 3 = 5\cos\theta$

7. $1 - \cos\theta = \frac{1}{2}$

8. $4\cos^2\theta = 1$

9. $\sqrt{3}\tan x + 1 = 0$

10. $4\sin^2\theta - 3 = 0$

11. $2\sin^2\theta - \sin\theta - 1 = 0$

12. $2\cos^2\theta + \cos\theta = 0$

Practice Sets

13. $\cos(2\theta) = -\frac{1}{2}$	**14.** $\tan(2\theta) = -1$
15. $\sin\frac{\theta}{2} = \frac{\sqrt{3}}{2}$	**16.** $\tan\frac{\theta}{2} = \sqrt{3}$
17. $\cos\left(2\theta - \frac{\pi}{2}\right) = -1$	**18.** $\sin\left(3\theta + \frac{\pi}{6}\right) = 1$

3.10 **Trigonometric Equations and Inequalities Practice Set 3**

Name _____

Date _____ Period _____

Problems 1 – 10, Use a graphing utility to solve each equation on $0 \le \theta \le 2\pi$. Round answers to two decimal places.

1. $x + 5\cos x = 0$	**2.** $x^2 + 3\sin x = 0$
3. $\sin x - \cos x = x$	**4.** $x^2 = x + 3\cos(2x)$

Problems 5-10, Find all values of x on $0 \le x \le 2\pi$ that satisfy the inequality.

5. $\sin x > \cos x$	**6.** $\tan x \ge 1$

7. $1 + \sin x > \sqrt{3}\cos x$	**8.** $\sin x + \frac{1}{4} \le 0$
9. $3\lvert\sin x\rvert - 1 > 0$	**10.** $\frac{\cos x - 1}{\tan x} > 0$

Problems 11- 13, Solve each question.

11. Graph $f(x) = 2\cos\left(\frac{x}{2}\right) + 3$ and $g(x) = 4$ on the same coordinate plane for the interval $[0, 4\pi]$.

A. Solve for $f(x) = g(x)$ on the interval.

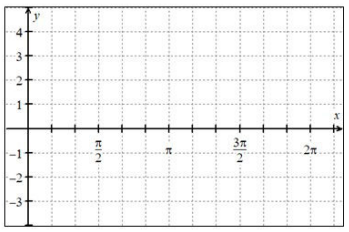
B. Solve for $f(x) < g(x)$ on the interval.

12. Graph $f(x) = 2\sin x$ and $g(x) = -2\sin x + 2$ on the interval $[0, \pi]$.

A. Solve for $f(x) = g(x)$ on the interval.

B. Solve for $f(x) > g(x)$ on the interval.

C. Shade the region found in part B on the graph.



Practice Sets

3.10 Trigonometric Equations And Inequalities Set 1 **Form A**

Name _____

Date _____ Period _____

Find all solutions of each equation.

1. $2\cos\theta + 1 = 0$

2. $3 + \sqrt{2}\sin\theta = 2$

3. $\tan\theta - \sqrt{3} = 0$

4. $(\cos x + 1)(\tan x - 1) = 0$

Grade:

3.10 Trigonometric Equations And Inequalities Set 1 **Form B**

Name _____

Date _____ Period _____

Find all solutions of each equation.

1. $2\sin\theta + 1 = 0$

2. $3 + \sqrt{2}\cos\theta = 2$

3. $3\tan\theta - \sqrt{3} = 0$

4. $(\sin x + 1)(\tan x - 1) = 0$

Grade:

3.10 Trigonometric Equations And Inequalities Set 2 **Form A**

Name _____

Date _____ Period _____

Solve each equation on the interval $0 \le x \le 2\pi$

1. $\sqrt{3}\cos x - 2\sin x\cos x = 0$

2. $2\sin^2 x - 5\sin x - 3 = 0$

3. $3\tan^2 x - 1 = 0$

Grade:

3.10 Trigonometric Equations And Inequalities Set 2 **Form B**

Name _____

Date _____ Period _____

Solve each equation on the interval $0 \le x \le 2\pi$

1. $\sqrt{3}\sin x - 2\sin x\cos x = 0$

2. $2\cos^2 x - 5\cos x - 3 = 0$

3. $\tan^2 x - 3 = 0$

Grade:

3.10 Trigonometric Equations And Inequalities Set 3 **Form A**

Name _____

Date _____ Period _____

Problems 1 and 2, Use a graphing utility to solve each equation on $0 \le x \le 2\pi$. Round answers to three decimal places.

1. $x - 4\sin(x) = 0$

2. $x^2 = \cos(2x)$

Find all values on $0 \le x \le 2\pi$ that satisfy the inequality.

3. $2\sin(x) - 1 \ge 0$

4. $2\cos(x) - \sqrt{2} < 0$

Grade:

3.10 Trigonometric Equations And Inequalities Set 3 **Form B**

Name _____

Date _____ Period _____

Problems 1 and 2, Use a graphing utility to solve each equation on $0 \le x \le 2\pi$. Round answers to three decimal places.

1. $x - 2\cos(x) = 0$

2. $x^2 = \sin(2x) + 1$

Find all values on $0 \le x \le 2\pi$ that satisfy the inequality.

3. $2\cos(x) - 1 \ge 0$

4. $2\sin(x) - \sqrt{2} < 0$

Grade:

Daily Lesson Quiz