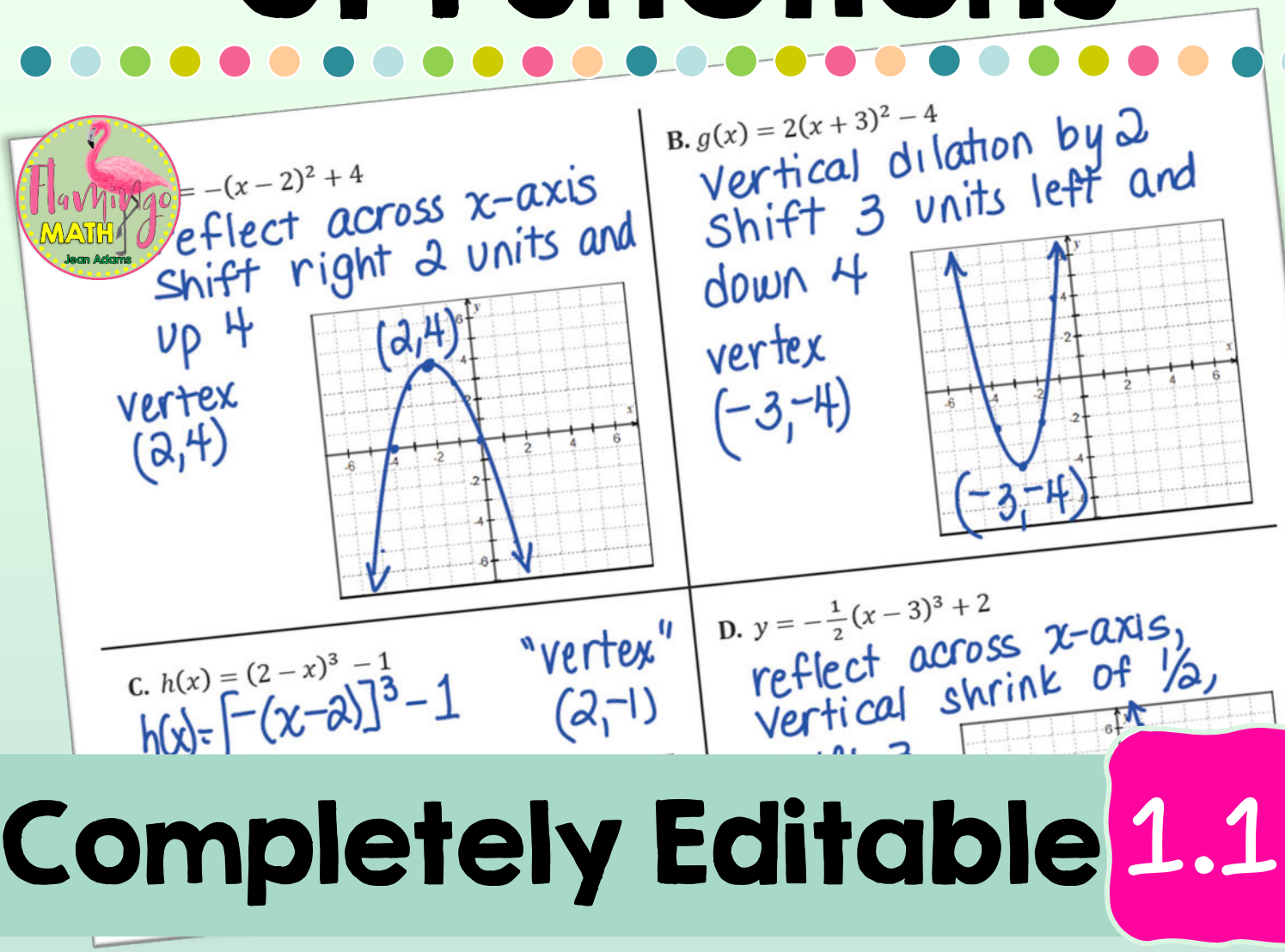
AP® Precalculus

Transformations of Functions



Completely Editable

1.12

Lesson 12-A: Transformations of Functions

Topic 1.12: Transformations of Functions

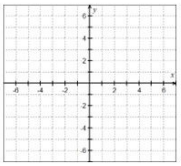
The simplest form in a set of functions that form a family, namely, $F(x) = a \cdot f(x-h) + k$, is commonly called the **parent function. A change made to at least one one of the values** a, h, and k, is called a **transformation.** There are four types of transformations: ***dilation, reflection, rotation,*** and ***translation.***

RIGID TRANSFORMATIONS:
Keep the original shape and size of the graph unchanged

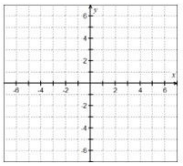
Notation	Changes to Parent Function	Ordered Pair
$g(x) = f(x) + k$	Vertical translation $f(x)$ k units **(up)**	
$g(x) = f(x) - k$	Vertical Translation $f(x)$ $-k$ units **(down)**	

EX #1: Use the parent functions for polynomials to describe how the graphs of each of the following functions will be transformed. State the types of transformations and graph the function.

A. $f(x) = -(x-2)^2 + 4$



B. $g(x) = 2(x+3)^2 - 4$



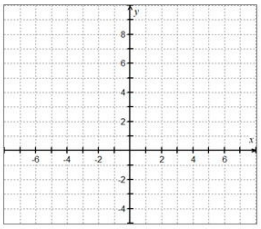
C. $h(x) = (2-x)^3 - 1$

D. $y = -\frac{1}{2}(x-3)^3 + 2$

EX #3: The graph of $y = f(x)$ can be drawn by connecting the points in the table below with line segments.

A. In the grid at right, graph $f(x)$.

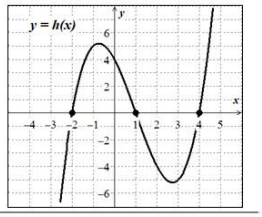
B. The graph of $h(x) = -2f(x-3) + 4$. Describe the transformations that are performed on $y = f(x)$ to obtain the graph of $h(x)$.



C. Complete the table, and graph $y = h(x)$ on the same coordinate grid.

x	$f(x)$	x-coordinate for $h(x)$	y-coordinate for $h(x)$	Ordered pairs for $h(x)$
-4	-2			
-2	3			
-1	0			
1	4			
4	2			

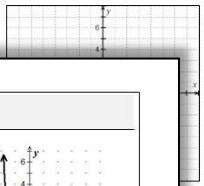
EX #5: The function h is a continuous function that is defined on the closed interval from $(-\infty, \infty)$. Use the graph to complete the table of h.



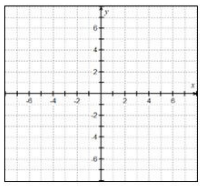
x	-2.5	-2	-1	0	1	2	3	4	4.5
$h(x)$									

On the axes provided below sketch the entire graph of each transformation for $y = h(x)$

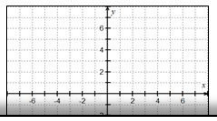
A. $y = -h(x)$



B. $y = |h(x)|$



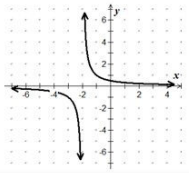
D. $y = h\left(\frac{1}{2}x\right)$



Transformations of Rational Functions

EX #10: The graph of $f(x)$ is shown below.

A. Write the equation for the rational function f. State the domain and range of $f(x)$.



B. Use the grid to sketch the graph of g as a transformation of f.
Given: $g(x) = -2f(x-4) + 3$

C. Describe the transformations that changed $f(x)$ t

D. State the domain and range for $g(x)$.

EX #11: State the vertical and horizontal asymptotes for equation for a rational function.

$$y = \frac{a}{(x-h)} + k$$

	Rational Function	Vertical Asym
A.	$f(x) = \frac{1}{x-3}$	
B.	$f(x) = \frac{3}{x+4} - 5$	
C.	$f(x) = \frac{-2}{5-x} + 3$	
D.	$f(x) = 4 - \frac{6}{12-x}$	

Lesson 12-B: Transformations of

We can also use the transformation rules for polynomia sketch rational functions.

EX #6: State the domain, asymptotes, intercepts and sk

A. $f(x) = \frac{1}{x+2}$

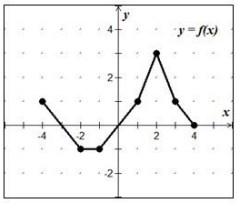
B. $g(x) = \frac{2}{x+1} + 3$

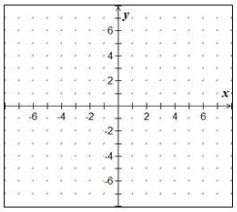
C. $h(x) = \frac{-1}{x-3} + 2$



Transformations of a Piecewise Function

EX #7: The graph $y = f(x)$ consisting of six line-segments is shown for $-4 \le x \le 4$. Sketch the graph of $g(x)$ using the following transformation $g(x) = f(x+2) - 3$





Applying Transformations

EX #8: The function f is given by $f(x) = 2x^2 + 5x + 8$. Write the equation for the graph of the function k that is a multiplicative transformation of f by a factor of 3.

EX #9: The table gives values for a polynomial function f at selected values of x.
Let $g(x) = af(bx) + c$, where a, b, and c are positive constants. In the xy-plane, the graph of g is constructed by applying three transformations to the graph of f in this order:

What is the value of $g(-6)$?

Guided Notes

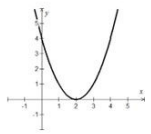
1.12 **Transformations of Functions Practice Set 1**

Name _____

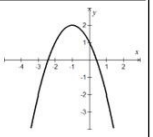
Date _____ Period _____

Problems 1 - 4, Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown,

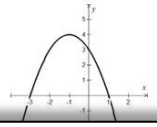
1.



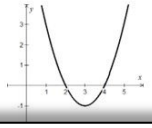
2.



3.

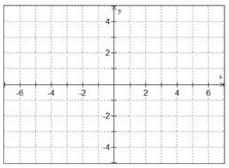


4.



Problem 12, The function $g(x)$ is a continuous function that is defined on the closed interval $[-5, 5]$. Use the grid below to draw the graph of g.

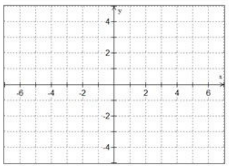
x	-5	-3	-1	0	1	3	5
$g(x)$	-4	-1	2	1	-2	0	2



On the axes provided below, sketch the entire graph of each transformation for $y = g(x)$.

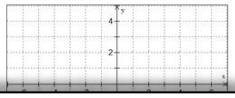
A. $y = g(|x|)$

x	-5	-3	-1	0	1	3	5
$g(\|x\|)$							



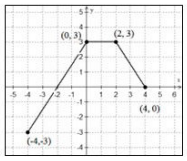
B. $y = |g(x)|$

x	-5	-3	-1	0	1	3	5
$\|g(x)\|$							

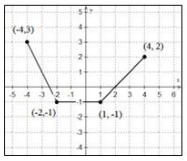


Problems 5 - 6, Use the graph of f to sketch each translation.

5. $y = f(x) - 2$



6. $y = -f(x - 2)$



Problems 7 - 10, Write the function that is finally graphed after the following transformations are applied ***in the order listed*** to the designated parent function.

7. $y = x^2$

1.) Horizontal translation -3

2.) Reflect about the y -axis

3.) Shift up 1 unit

8. $y = x^3$

1.) Reflect about the x-axis

2.) Horizontal translation 4

3.) Vertical translation -3

9. $y = \frac{1}{x}$

1.) Reflect about the x -axis

2.) Vertically dilation by a factor of 2

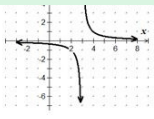
3.) Vertical translation -2

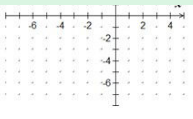
10. $y = \frac{1}{x^3}$

1.) Horizontal translation 5

2.) Reflect about the x-axis

3.) Vertical compression by a factor of $\frac{1}{2}$





Problems 9 - 11, Each o f the table of values below lie on the graph of a continuous function $f(x)$. State the transformations that occur on the coordinates of $f(x)$ to obtain a new set of ordered pairs on the transformed function.

9. $g(x) = -f(x + 3) - 4$

x	-3	-1	1	2	4
$f(x)$	5	3	0	-2	1

x-coordinate on $g(x)$	y-coordinate on $g(x)$	New Ordered Pairs on $g(x)$

10. $h(x) = 2f(x - 1) + 3$

x	-4	-2	0	3	5
$f(x)$	-5	-1	2	-4	-7

x-coordinate on $h(x)$	y-coordinate on $h(x)$	New Ordered Pairs on $h(x)$

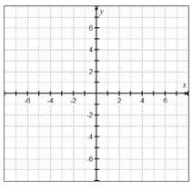
1.12 **Transformations of Functions Practice Set 2**

Name _____

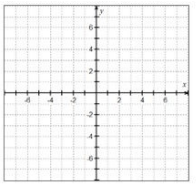
Date _____ Period _____

Problems 1 – 4, State the domain, range, asymptotes, intercepts and sketch the transformation.

1. $f(x) = \frac{1}{x - 3}$



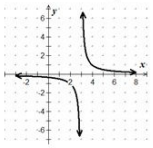
2. $g(x) = \frac{2}{x + 2} - 1$

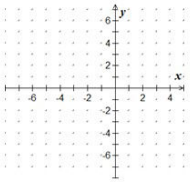


3. $h(x) = \frac{-1}{\quad} + 2$

4. $k(x) = \frac{1}{\quad} + 1$

12. Use the graph of the rational function f below, to sketch the transformation g. Given: $g(x) = -\frac{1}{2}f(x + 2) - 3$





Problems 13 - 16, Write the equation for the graph of the function transformation. State the domain of the transformed function $g(x)$.

13. Given $f(x) = x^2 + 3x - 18$
The function g is a multiplicative transformation of the function f that results in a vertical dilation of the graph of f by a factor of $a = -3$.

14. Given $f(x) = x^3 - 5x$.
The function g is an additive transformation of the function f that results in a horizontal translation of the graph of f by $h = -2$.

Practice Sets

1.12 Transformations of Functions **Form A**
Set 1

Name _____

Date _____ Period _____

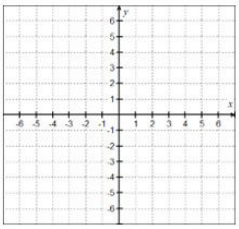
The parent function for problems 1-2.

$f(x) = x^2$

1. State the types of transformations for g.

$g(x) = -f(x+1)^2 + 4$

2. Graph the function g and state the domain and range.



3. Let f be a cubic polynomial, state the transformations for $p(x) = -2f[3(x-4)]$

Grade:

1.12 Transformations of Functions **Form B**
Set 1

Name _____

Date _____ Period _____

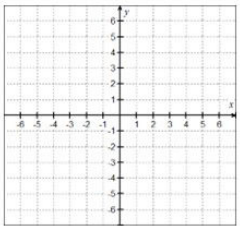
The parent function for problems 1-2.

$f(x) = x^2$

1. State the types of transformations for g.

$g(x) = -2f(x+4)^2 + 3$

2. Graph the function g and state the domain and range.



3. Let f be a cu
transformati

Grade:

1.12 Transformations of Functions **Form A**
Set 2

Name _____

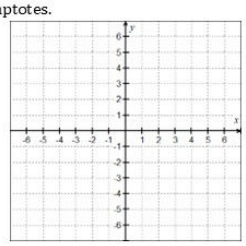
Date _____ Period _____

The function $f(x) = \frac{1}{x}$

$g(x) = -1f(x+2) - 1$

1. Write the equation for the rational function $g(x)$ in terms of x.

2. Sketch the graph of $g(x)$ and label the asymptotes.



3. Use the table for f to find the average rate of change for $p(x) = -2f(x+3)$ over the interval $[-3, 1]$.

x	-3	-2	0	1	4
$f(x)$	9	2	1	-7	-15

Grade:

1.12 Transformations of Functions **Form B**
Set 2

Name _____

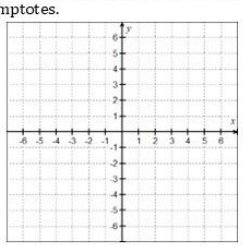
Date _____ Period _____

The function $f(x) = \frac{1}{x}$

$g(x) = -1f(x-2) + 1$

1. Write the equation for the rational function $g(x)$ in terms of x.

2. Sketch the graph of $g(x)$ and label the asymptotes.



3. Use the table for f to find the average rate of change for $p(x) = -3f(x-2)$ over the interval $[-2, 1]$.

x	-4	-2	-1	1	4
$f(x)$	3	1	-4	-7	-12

Grade:

Daily Lesson Quiz