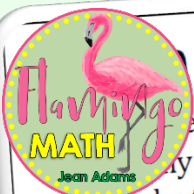


Precalculus Honors

Transformations of Functions



6: Transformations of Functions

The simplest form in a set of functions that form a family, namely, $F(x) = a \cdot f(x - h) + k$, is called the **parent function**. A change made to at least one of the values a , h , and k , is called a **transformation**.

There are four types of transformations:

1. _____
2. _____
3. _____
4. _____

_____ transformations keep the original shape and size of the graph unchanged.

_____ transformations distort the shape of a graph.

RIGID TRANSFORMATIONS:

Keep the original shape and size of _____ unchanged

NON-RIGID TRANSFORMATIONS:

Distort the shape of a graph

Completely Editable

1.6

Lesson 6: Transformations of Functions

The simplest form in a set of functions that form a family, namely, $F(x) = a \cdot f(x - h) + k$, is commonly called the **parent function**. A change made to at least one of the values a , h , and k , is called a **transformation**.

There are four types of transformations:

- _____
- _____
- _____
- _____

RIGID TRANSFORMATIONS:

Keep the original shape and size of the graph unchanged

Let c be any real number . . .

$y = f(x) + c$	Vertical translation up " c "	$(x, y + c)$
$y = f(x) - c$	Vertical translation down " c "	$(x, y - c)$
$y = f(x - c)$	Horizontal translation right " c "	$(x + c, y)$
$y = f(x + c)$	Horizontal translation left " c "	$(x - c, y)$
$y = f(x) $	Reflect negative y -values	(x, y)
$y = -f(x)$	Reflect across x -axis	$(x, -y)$
$y = f(-x)$	Reflect across y -axis	$(-x, y)$

NON-RIGID TRANSFORMATIONS: Stretch and Shrink Factors

Distort the shape of a graph

Let c be any real number . . .

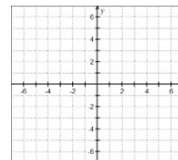
$y = c \cdot f(x), c > 1$	Vertical dilation (stretch) by a factor of " c "	(x, cy)
$y = c \cdot f(x), 0 < c < 1$	Vertical dilation (compression) by a factor of " c "	(x, cy)
$y = f(c \cdot x), c > 1$	Horizontal dilation (compression) by a factor of $\frac{1}{c}$	$(\frac{x}{c}, y)$
$y = f(c \cdot x), 0 < c < 1$	Horizontal dilation (stretch) by a factor of $\frac{1}{c}$	$(\frac{x}{c}, y)$

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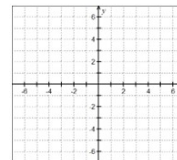
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EX #1: Use the parent functions from our previous lesson to describe how the graphs of each of the following functions will be transformed. State the types of transformations and graph the function.

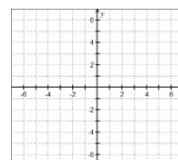
A. $f(x) = -(x - 2)^2 + 4$



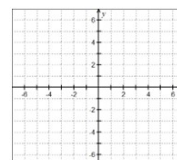
B. $g(x) = 2|x + 3| - 5$



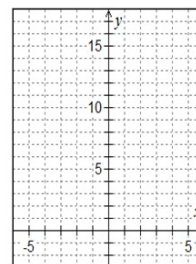
C. $h(x) = \sqrt{2 - x} - 3$



D. $y = -\frac{1}{2}(x - 3)^3 + 2$



EX #2: Use a graphing utility to explore the transformations of $f(x) = x^2$ by graphing $f(x)$, $f(2x)$ and $f(\frac{1}{2}x)$ on the axes below. Explain the transformations.



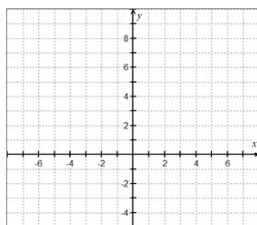
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EX #3: The graph of $y = f(x)$ can be drawn by connecting the points in the table below with line segments.

A. In the grid at right, graph $f(x)$.

B. The graph of $h(x) = -2f(x - 3) + 4$. Describe the transformations that are performed on $y = f(x)$ to obtain the graph of $h(x)$.



C. Complete the table, and graph $y = h(x)$ on the same coordinate grid.

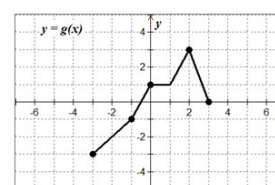
x	$f(x)$	x -coordinate for $h(x)$	y -coordinate for $h(x)$	Ordered pairs for $h(x)$
-4	-2			
-2	3			
-1	0			
1	4			
4	2			

EX #4: Compare and contrast the graph of $f(x) = \sqrt{x}$ with the following transformations. Consider changes to the domain in each case. Sketch the parent and each function.

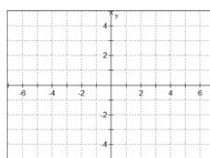
EX #5: The function g is a continuous function that is defined on the closed interval from $[-3, 3]$. Use the graph to complete the table of g .

x	-3	-2	-1	0	1	2	3
$g(x)$							

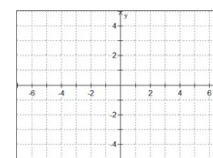
On the axes provided below sketch the entire graph of each transformation for $y = g(x)$



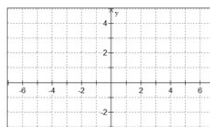
A. $y = g(|x|)$



B. $y = |g(x)|$



C. $y = g(-x)$



D. $y = g(\frac{1}{2}x)$



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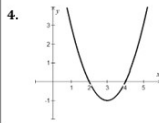
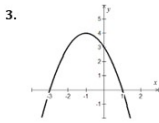
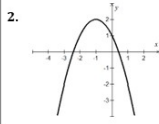
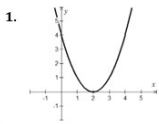
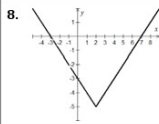
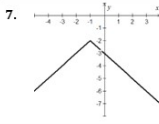
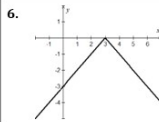
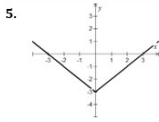
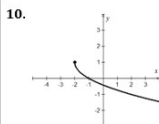
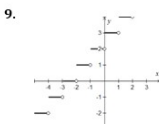
NOTES

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Name _____

Date _____ Period _____

Problems #1 - 4. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown.**Problems #5 - 8.** Use the graph of $f(x) = |x|$ to write an equation for each function whose graph is shown.**Problems #9 - 10.** Identify the parent function and write an equation for the function shown.**Problems #11 - 14.** Write the function that is finally graphed after the following transformations are applied to the designated parent function.

11. $y = \sqrt{x}$

- 1.) Shift 3 units left
- 2.) Reflect about the y-axis
- 3.) Shift up 1 unit

12. $y = x^3$

- 1.) Reflect about the x-axis
- 2.) shift right 4 units
- 3.) shift down 3 units

13. $y = x^2$

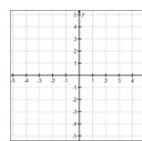
- 1.) Reflect about the x-axis
- 2.) Vertically stretch by a factor of 2
- 3.) Shift down 2 unit

14. $y = |x|$

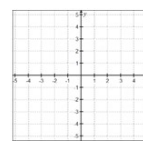
- 1.) shift 5 units right
- 2.) Reflect about the x-axis
- 3.) Vertically shrink by a factor of $\frac{1}{2}$

Problems #15 - 18. Describe the transformation from a parent function and sketch the graph.

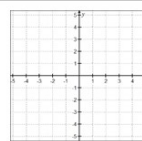
15. $f(x) = (x - 2)^2 - 1$



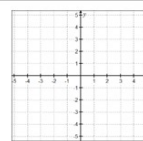
16. $f(x) = 4 - |x + 2|$



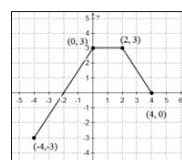
17. $f(x) = -(x + 1)^3$



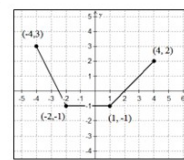
18. $f(x) = -\frac{1}{3}x$

**Problems #19 - 20.** Use the graph of f to sketch each translation

19. $y = f(x) - 2$



20. $y = -f(x - 2)$

**Problems #21 - 23.** Each of the table of values below lie on the graph of a continuous function $f(x)$. State the transformations that occur on the coordinates of $f(x)$ to obtain a new set of ordered pairs on the transformed function.

21. $g(x) = -f(x + 3) - 4$

x	-3	-1	1	2	4
$f(x)$	5	3	0	-2	1

x-coordinate on $g(x)$	y-coordinate on $g(x)$	New Ordered Pairs on $g(x)$

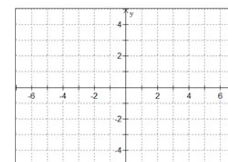
22. $h(x) = 2f(x - 1) + 3$

x	-4	-2	0	3	5
$f(x)$	-5	-1	2	-4	-7

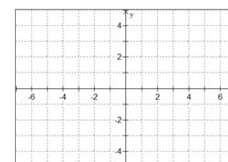
x-coordinate on $h(x)$	y-coordinate on $h(x)$	New Ordered Pairs on $h(x)$

Problem #24. The function $g(x)$ is a continuous function that is defined on the closed interval $[-5, 5]$. Use the grid below to draw the graph of g .

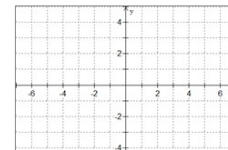
x	-5	-3	-1	0	1	3	5
$g(x)$	-4	-1	2	1	-2	0	2

On the axes provided below, sketch the entire graph of each transformation for $y = g(x)$.

A. $y = g(|x|)$



B. $y = |g(x)|$



Practice Sets

Lesson Warm-Ups

1.6 Graph Transformations

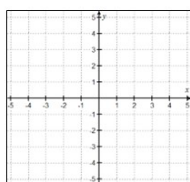
Form A

Name _____

Date _____ Period _____

- Describe the transformation from the common function that occurs in the function:
 $f(x) = (x - 1)^2 - 2$

- Graph the function from question 1.



- State the domain and range for the graph above.

Domain:

Range:

- Write an equation for the shape of $f(x) = x^3$ but moved two units to the left, five units up, and reflected across the y-axis.

Grade:

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1.6 Graph Transformations

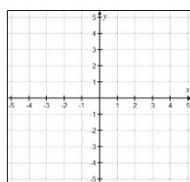
Form B

Name _____

Date _____ Period _____

- Describe the transformation from the common function that occurs in the function:
 $f(x) = -|x - 1| + 3$

- Graph the function from question 1.



- State the domain and range for the graph above.

Domain:

Range:

- Write an equation for the shape of $f(x) = x^2$ but moved three units to the left, four units down, and reflected across the x-axis.

Grade:

Name _____ Date _____ Period _____

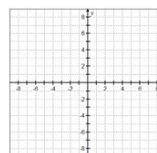
Lesson I.6: Graph Transformations

Warm-Up # 1

- Describe the transformation from the parent function $f(x) = (x + 1)^2 - 3$

- State the domain and range of the graph of $f(x)$.

- Graph the function $f(x)$



- Write an equation $g(x)$, for the shape of $y = x^4$ but moved one unit right and up two units and reflect over the y-axis.

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Name _____ Date _____ Period _____

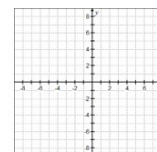
Lesson I.6: Graph Transformations

Warm-Up # 2

- Describe the transformation from the parent function $g(x) = |x - 2| + 4$

- State the domain and range of the graph of $g(x)$.

- Graph the function $g(x)$



- Write an equation $h(x)$, for the shape of $y = x^3$ but moved one unit right and up two units and reflect over the x-axis.

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Daily Lesson Quiz