

## Precalculus Honors

# The Unit Circle



### Finding Exact Values of the Trigonometric Functions

Task #2: Summarize the exact values for  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$  angles (rationalize denominators).

| $\theta$<br>Degrees | $\theta$<br>Radians | $\sin \theta$        | $\cos \theta$        | $\tan \theta$        | $\csc \theta$         | $\sec \theta$         | $\cot \theta$        |
|---------------------|---------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|----------------------|
| $30^\circ$          | $\frac{\pi}{6}$     | $\frac{1}{2}$        | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | 2                     | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$           |
| $45^\circ$          | $\frac{\pi}{4}$     | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1                    | $\sqrt{2}$            | $\sqrt{2}$            | 1                    |
| $60^\circ$          | $\frac{\pi}{3}$     | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$        | $\sqrt{3}$           | $\frac{2\sqrt{3}}{3}$ | 2                     | $\frac{\sqrt{3}}{3}$ |

Quadrantal Angles

# Completely Editable

4.4

## Lesson 4: The Unit Circle

Now we are ready to explore trigonometric functions. We will use the unit circle approach. The **unit circle** that we will develop is the most useful tool in trigonometry. It provides an easy way to know and recall trigonometric values of the most popular angles. To be successful in this class, and then later in calculus and beyond, you must understand it and **memorize it!**

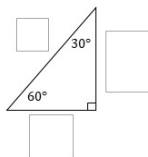
### A Geometry Connection

First, let's remember the important ratio relationships for two famous triangles.

#### 30°-60°-90° Triangle

The lengths of the legs of the triangle correspond to the angles 30°: 60°: 90° as  $x: x\sqrt{3}: 2x$  respectively. In the figure at right,

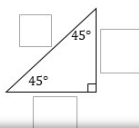
Name the lengths of the sides if the hypotenuse is 1.



#### 45°-45°-90° Triangle

The length of the legs of the triangle correspond to the angles 45°: 45°: 90° as  $x:x: x\sqrt{2}$ , respectively. In the figure at right,

Name the lengths of the sides if the hypotenuse is 1.

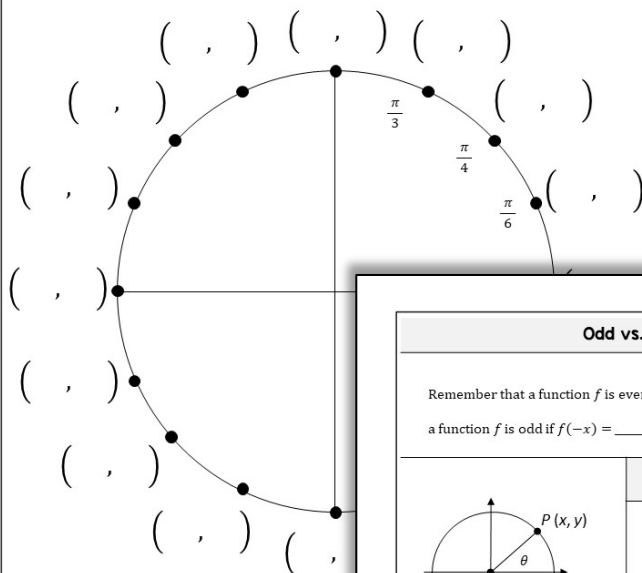


### The Unit Circle

The unit circle is a circle of radius 1 centered at the origin.



### The 16-Point Unit Circle



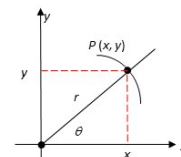
## Trigonometric Functions of Any Angle

Let  $\theta$  be an angle in standard position with  $(x, y)$  a point on the terminal side of  $\theta$  and

$$r = \sqrt{x^2 + y^2} \neq 0$$

**EX #1:** Use the figure at right, to name the six basic trigonometric functions in terms of  $x, y$ , and  $r$ , where  $r$  is the radius of a circle.

|                 |                 |
|-----------------|-----------------|
| $\sin \theta =$ | $\csc \theta =$ |
| $\cos \theta =$ | $\sec \theta =$ |
| $\tan \theta =$ | $\cot \theta =$ |



$$r = \sqrt{x^2 + y^2}$$

... Where Trigonometric Functions are Positive ...

| Quadrant II | Quadrant III | Quadrant IV |
|-------------|--------------|-------------|
|             |              |             |
|             |              |             |
|             |              |             |

### Finding Exact Values of the Trigonometric Functions

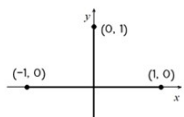
**EX #2:** Summarize the exact values for 30°, 60°, and 90°.

| $\theta$<br>Degrees | $\theta$<br>Radians | $\sin \theta$ | $\cos \theta$ |
|---------------------|---------------------|---------------|---------------|
| 30°                 | $\frac{\pi}{6}$     |               |               |
| 45°                 | $\frac{\pi}{4}$     |               |               |
| 60°                 | $\frac{\pi}{3}$     |               |               |

### Quadrantal Angles

A quadrantal angle is an angle whose terminal side lies on one of the axes.

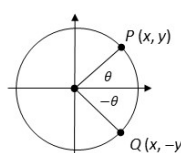
**EX #3:** Evaluate the six trig functions at the four quadrantal angles.



| $\theta$ degrees | 0° | 90°             | 180°  | 270°             |
|------------------|----|-----------------|-------|------------------|
| radians          | 0  | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
| $\sin \theta$    |    |                 |       |                  |

### Odd vs. Even Trigonometric Functions

Remember that a function  $f$  is even if  $f(-x) = f(x)$  for all  $x$  in the domain of  $f$  and a function  $f$  is odd if  $f(-x) = -f(x)$  for all  $x$  in the domain of  $f$ .



#### Even Trig Functions

$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

#### Odd Trig Functions

$$\sin(-\theta) = -\sin(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

**EX #4:** Find the exact value of each of the following.

A.  $\cos(-60^\circ)$

B.  $\sin\left(-\frac{\pi}{2}\right)$

C.  $\tan(-\pi)$

D.  $\cot\left(-\frac{11\pi}{2}\right)$

# NOTES

for any given function.

$\cot \theta$

## 4.4

The Unit Circle  
Homework

Name \_\_\_\_\_

Date \_\_\_\_\_ Period \_\_\_\_\_

**Problems 1 – 8,** Find the exact value of each expression. Do not use a calculator.

1.  $\sin\left(\frac{13\pi}{2}\right)$

2.  $\cos(9\pi)$

3.  $\tan(-3\pi)$

4.  $\csc\left(\frac{13\pi}{2}\right)$

5.  $\sec\left(\frac{9\pi}{2}\right)$

6.  $\cot\left(\frac{7\pi}{2}\right)$

7.  $\sin(-5\pi)$

8.  $\cos\left(-\frac{3\pi}{2}\right)$

**Problems 9 – 14,** Find the exact value of each expression. Do not use a calculator.

9.  $\cos 60^\circ + \tan 45^\circ$

10.  $\sin 30^\circ - \cos 45^\circ$

11.  $\cot 45^\circ \sec 30^\circ$

12.  $\tan 60^\circ \csc 45^\circ$

13.  $5 \sin 270^\circ - 3 \cos 180^\circ$

14.  $\tan 135^\circ \cos 30^\circ$

**Problems 15 – 22,** Use a calculator to approximate each expression. Use the  $\angle$  key for degrees. Round to three decimal places. Give answers correct to three decimal places.

15.  $\sin 38^\circ$

16.  $\csc 59^\circ$

17.  $\tan 3^\circ$

18.  $\tan 3$

19.  $\sin\left(\frac{5\pi}{9}\right)$

20.  $\cos\left(\frac{7\pi}{18}\right)$

21.  $\tan\left(-\frac{5\pi}{12}\right)$

22.  $\sec\left(-\frac{2\pi}{11}\right)$

**Problems 23 – 26,** Find the exact value of the six trigonometric functions of the given angle. If any are not defined, say "undefined." Do not use a calculator.

23.  $\frac{5\pi}{6}$

24.  $\frac{7\pi}{3}$

25.  $-\frac{3\pi}{4}$

26.  $-\frac{5\pi}{2}$

**Problems 27 – 30,** A point on the terminal side of an angle  $\theta$  is given. Find the exact values of the six trigonometric functions of  $\theta$ .

27.  $(3, -4)$

28.  $(-1, 2)$

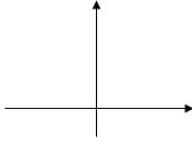
29.  $(-1, -1)$

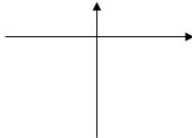
# Practice Sets

# Lesson Warm-Ups

|                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |               |               |               |               |               |               |
|--------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Name _____ Date _____ Period _____                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |               |               |               |               |               |               |
| <b>Lesson 4.4 - The Unit Circle</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |               |               |               |               |               |               |
| <b>Warm-Up # I</b>                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |               |               |               |               |               |               |
| <p>1. Create the reference triangle for <math>150^\circ</math>.</p>                                                                                    | <p>2. Find the six trig. functions for the triangle in #1.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: center;"><math>\sin \theta</math></td> <td style="width: 50%; text-align: center;"><math>\csc \theta</math></td> </tr> <tr> <td style="text-align: center;"><math>\cos \theta</math></td> <td style="text-align: center;"><math>\sec \theta</math></td> </tr> <tr> <td style="text-align: center;"><math>\tan \theta</math></td> <td style="text-align: center;"><math>\cot \theta</math></td> </tr> </table> | $\sin \theta$ | $\csc \theta$ | $\cos \theta$ | $\sec \theta$ | $\tan \theta$ | $\cot \theta$ |
| $\sin \theta$                                                                                                                                          | $\csc \theta$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |               |               |               |               |               |               |
| $\cos \theta$                                                                                                                                          | $\sec \theta$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |               |               |               |               |               |               |
| $\tan \theta$                                                                                                                                          | $\cot \theta$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |               |               |               |               |               |               |
| <p>3. Find <math>\cos \theta</math> and <math>\tan \theta</math> given <math>\sin \theta = \frac{3}{5}</math> and <math>\tan \theta &lt; 0</math>.</p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |               |               |               |               |               |               |

|                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                       |               |               |               |
|--------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|---------------|---------------|
| Name _____ Date _____                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                       |               |               |               |
| <b>Lesson 4.4 - The Unit Circle</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                       |               |               |               |
| <p>1. Create the reference triangle for <math>240^\circ</math>.</p>                                                                                    | <p>2. Find the</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: center;"><math>\sin \theta</math></td> </tr> <tr> <td style="text-align: center;"><math>\cos \theta</math></td> </tr> <tr> <td style="text-align: center;"><math>\tan \theta</math></td> </tr> </table> | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
| $\sin \theta$                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                       |               |               |               |
| $\cos \theta$                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                       |               |               |               |
| $\tan \theta$                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                       |               |               |               |
| <p>3. Find <math>\cos \theta</math> and <math>\tan \theta</math> given <math>\csc \theta = \frac{5}{4}</math> and <math>\tan \theta &lt; 0</math>.</p> |                                                                                                                                                                                                                                                                                                                                       |               |               |               |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                 |                 |                 |                 |                 |                 |                 |                 |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| <b>4.4 The Unit Circle</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                 | <b>Form A</b>   |                 |                 |                 |                 |                 |                 |
| Name _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                 |                 |                 |                 |                 |                 |                 |                 |
| Date _____ Period _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                 |                 |                 |                 |                 |                 |                 |                 |
| <p>1. Create the reference triangle for <math>120^\circ</math></p> <div style="text-align: center;">  </div>                                                                                                                                                                                                                                                                                                                                                                                                                   |                 |                 |                 |                 |                 |                 |                 |                 |
| <p>2. Find the six trigonometric functions for triangle above.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33.33%; text-align: center;"><math>\sin \theta =</math></td> <td style="width: 33.33%; text-align: center;"><math>\cos \theta =</math></td> <td style="width: 33.33%; text-align: center;"><math>\tan \theta =</math></td> </tr> <tr> <td style="text-align: center;"><math>\csc \theta =</math></td> <td style="text-align: center;"><math>\sec \theta =</math></td> <td style="text-align: center;"><math>\cot \theta =</math></td> </tr> </table> |                 |                 | $\sin \theta =$ | $\cos \theta =$ | $\tan \theta =$ | $\csc \theta =$ | $\sec \theta =$ | $\cot \theta =$ |
| $\sin \theta =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\cos \theta =$ | $\tan \theta =$ |                 |                 |                 |                 |                 |                 |
| $\csc \theta =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\sec \theta =$ | $\cot \theta =$ |                 |                 |                 |                 |                 |                 |
| <p>3. Find <math>\cos \theta</math> and <math>\tan \theta</math> given <math>\sin \theta = \frac{5}{13}</math> and <math>\tan \theta &lt; 0</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                 |                 |                 |                 |                 |                 |                 |                 |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                 |                 |                 |                 |                 |                 |                 |                 |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| <b>4.4 The Unit Circle</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                 | <b>Form B</b>   |                 |                 |                 |                 |                 |                 |
| Name _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                 |                 |                 |                 |                 |                 |                 |                 |
| Date _____ Period _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                 |                 |                 |                 |                 |                 |                 |                 |
| <p>1. Create the reference triangle for <math>210^\circ</math></p> <div style="text-align: center;">  </div>                                                                                                                                                                                                                                                                                                                                                                                                                 |                 |                 |                 |                 |                 |                 |                 |                 |
| <p>2. Find the six trigonometric functions for triangle above.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33.33%; text-align: center;"><math>\sin \theta =</math></td> <td style="width: 33.33%; text-align: center;"><math>\cos \theta =</math></td> <td style="width: 33.33%; text-align: center;"><math>\tan \theta =</math></td> </tr> <tr> <td style="text-align: center;"><math>\csc \theta =</math></td> <td style="text-align: center;"><math>\sec \theta =</math></td> <td style="text-align: center;"><math>\cot \theta =</math></td> </tr> </table> |                 |                 | $\sin \theta =$ | $\cos \theta =$ | $\tan \theta =$ | $\csc \theta =$ | $\sec \theta =$ | $\cot \theta =$ |
| $\sin \theta =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\cos \theta =$ | $\tan \theta =$ |                 |                 |                 |                 |                 |                 |
| $\csc \theta =$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\sec \theta =$ | $\cot \theta =$ |                 |                 |                 |                 |                 |                 |
| <p>3. Find <math>\cos \theta</math> and <math>\tan \theta</math> given <math>\csc \theta = \frac{17}{8}</math> and <math>\tan \theta &lt; 0</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                 |                 |                 |                 |                 |                 |                 |                 |

# Daily Lesson Quiz

|              |              |
|--------------|--------------|
| Grade: _____ | Grade: _____ |
|--------------|--------------|