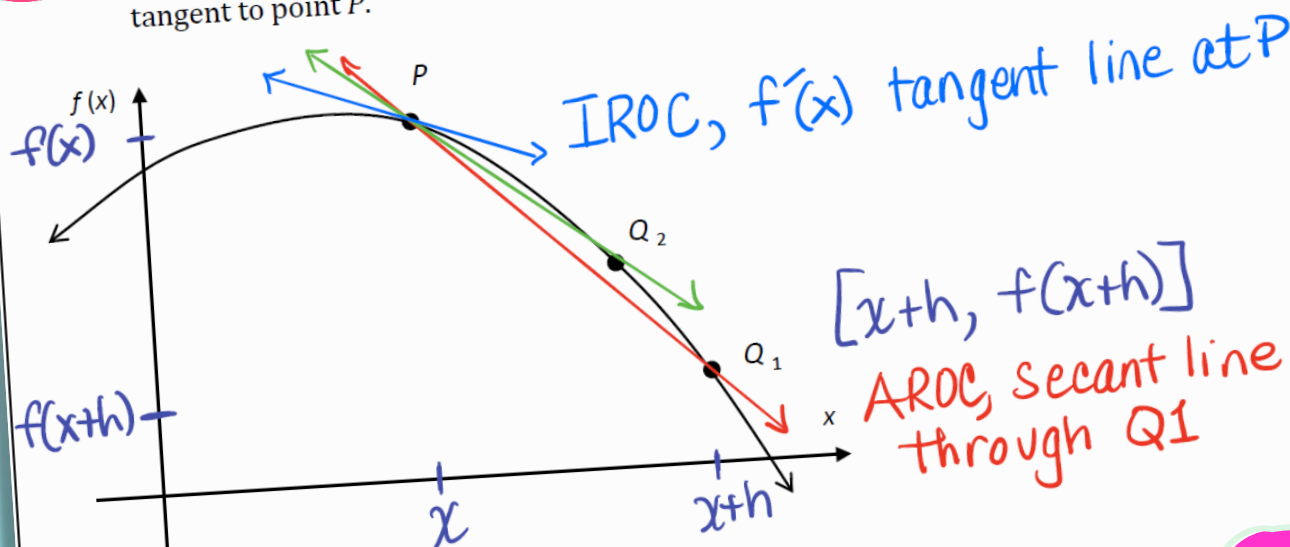


Precalculus Honors

Derivative and Tangent Lines

Slope and the Limit Process

Sketch the secant line through PQ point. Then imagine point Q slides closer and closer to point P until it coincides at point P . Sketch this movement until you have a line that is tangent to point P .



Completely Editable 10.4

Lesson 10.4: Tangent Lines and the Derivative

In this lesson we will explore how limits can be used to find the "instantaneous rate of change" of a function and the slope of the tangent line at some given x -value.

Vocabulary	Visual Vocabulary
Average Rate of Change of y with respect to x on the interval (a, b) is $AROC = m_{sec} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$. Geometrically, this is the slope of the secant line through the points $(a, f(a))$ and $(b, f(b))$.	
Tangent Line in simple terms, this is the line that best approximates the slope of the graph at a single point. $y - f(c) = m_{tan}(x - c)$	

The **derivative** of the function f at $x = c$, is the slope of the tangent line at a single x -value $x = c$. Often called the **instantaneous rate of change** and is denoted by $f'(c)$.

$$f'(x) = m_{tan} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Average Velocity is the change in position divided by the change in time.

$$m_{sec}$$

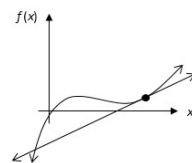
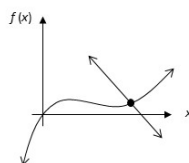
Instantaneous Velocity is the velocity at a single instant in time.

$$m_{tan}$$

Defining a Tangent Line

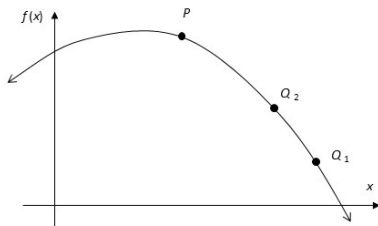
In geometry, we know that a line is tangent to a circle if the line intersects the circle at only one point. However, in noncircular graphs, tangent lines can intersect the graph at more than one point.

EX #1: Compare and contrast.



Slope and the Limit Process

EX #3: Sketch the secant line through PQ point. Then imagine point Q slides closer and closer to point P until it coincides at point P . Sketch this movement until you have a line that is tangent to point P .



The Difference Quotient

EX. #4: Complete the expressions below.

A. $f(x) = 3x - 4$

$$f(x+h)$$

$$f(x+h) - f(x)$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

EX. #4 (cont.) Complete the expressions below.

C. $f(x) = 5x - x^2$

D. $f(x)$

$$f(x+h)$$

$$f(x)$$

$$f(x+h) - f(x)$$

$$f(x)$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

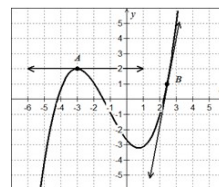
$$\lim_{h \rightarrow 0}$$

Finding a Derivative at a Point

EX #5: Using the results from EX 4, find the derivative for each function at the indicated point.

4A. Find $f'(1)$

4B. Find $f'(-3)$



Point B:

EX #6: In the equation from #4B: $f(x) = x^2 + x$, the point of tangency is $(-3, 6)$, the slope of the tangent line at this point is _____. Write the equation of the tangent line at this point.

Particle Motion and Velocity

EX #7: A water balloon is thrown upward from the top of an 80-foot building with a velocity of 64-feet per second. The height (in feet) of the balloon can be modeled by the position function $s(t) = -16t^2 + 64t + 80$, where t is the time in seconds from when the balloon was thrown.

A. Find the average velocity of the balloon after the first three seconds.

B. Find the instantaneous rate of change of the balloon.

NOTES

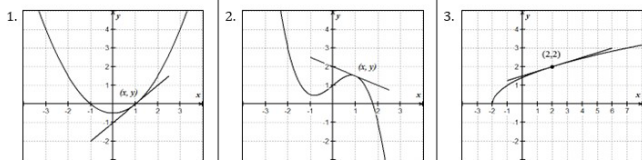
10.4

The Derivative and Tangent Lines Homework

Name _____

Date _____ Period _____

Problems 1 – 3, use the figure to approximate the slope of the curve at the indicated point.



Problems 4 – 7, use the limit process to find the slope of the graph of the function at the specified point.

4. $f(x) = 3 - 2x$; $(2, -1)$

5. $f(x) = x^2 - 3x$; $(-2, 10)$

6. $g(x) = \sqrt{x - 4}$; $(8, 2)$

7.

Problems 19 – 20, find $f'(x)$ for each function below. Then write the equation for the tangent line at the given x -value. $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$

19. (a) $f(x) = \sqrt{x - 3}$ at $x = 4$

20. (a) $f(x) = \frac{2}{x-1}$ at $x = 3$

(b) Find the equation of the line tangent to the graph.

(b)

Problems 8 – 13, find the derivative of the function.

8. $f(x) = 5x - 6$

9. $f(x) = 3x^2 - x$

10. $g(x) = x^3 + x$

11. $h(x) = x^2 - 3x + 2$

13. $g(x) = \frac{1}{x-2}$

Problems 21 – 25, find average velocity and numeric derivative.

21. An object travels along a line so that its position s is given by $s(t) = t^3 - 2t^2 + 3t - 1$.A. What is the average velocity on the interval $3 \leq t \leq 5$?B. What is the average velocity on the interval $3 \leq t \leq 5$?C. Find the instantaneous velocity of the object using $t = 3$ seconds.22. A bacteria culture is growing so that it has a mass m given by $m(t) = 100e^{0.05t}$.A. How much did the culture grow during the interval $5 \leq t \leq 6$?B. How much did the culture grown during the interval $5 \leq t \leq 5.01$?23. A spherical balloon is being inflated with helium for your birthday party. The volume V is approximated by the formula $V(r) = \frac{4}{3}\pi r^3$, where r is the radius.A. Find the derivative of V with respect to r when the radius is 4 inches using the formula for derivative at a point given below.

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

24. Olympia High School students had a virus outbreak. The school nurse estimates that t days after the beginning of the outbreak, the number of students that contracted the virus is modeled by $v(t) = 60t^2 - t^3$ for days $0 \leq t \leq 21$. Find the rate at which the virus is spreading at time $t = 10$.

Practice Sets

Lesson Warm-Ups

3. Write the equation for the tangent line of $f(x) = x^2 - 4$ at $x = -2$.	

Name _____ Date _____ Period _____	
Lesson 10.4: The Derivative and Tangent Lines Warm-Up # 2	
1. Find the derivative of the function by the limit process. $f(x) = 4x - 3$	2. Use $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ to find the derivative for $f(x) = x^2 + 4$ at $x = -4$.
3. Write the equation for the tangent line of $f(x) = x^2 + 4$ at $x = -4$.	

Derivative and Tangent Lines	Form A
Name _____	
Date _____ Period _____	
1. Find the derivative of the function by the limit process. $f(x) = 5x - 3$	
2. Use $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ to find the derivative for $f(x) = (x^2 + 3)$ at $x = -3$	
3. Write the equation for the tangent line of $f(x) = x^2 + 3$ at $x = -3$	

Derivative and Tangent Lines	Form B
Name _____	
Date _____ Period _____	
1. Find the derivative of the function by the limit process. $f(x) = 5x - 3$	
2. Use $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ to find the derivative for $f(x) = (x^2 + 3)$ at $x = -3$	
3. Write the equation for the tangent line of $f(x) = x^2 + 3$ at $x = -3$	

Daily Lesson Quiz

Grade: _____	Grade: _____
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