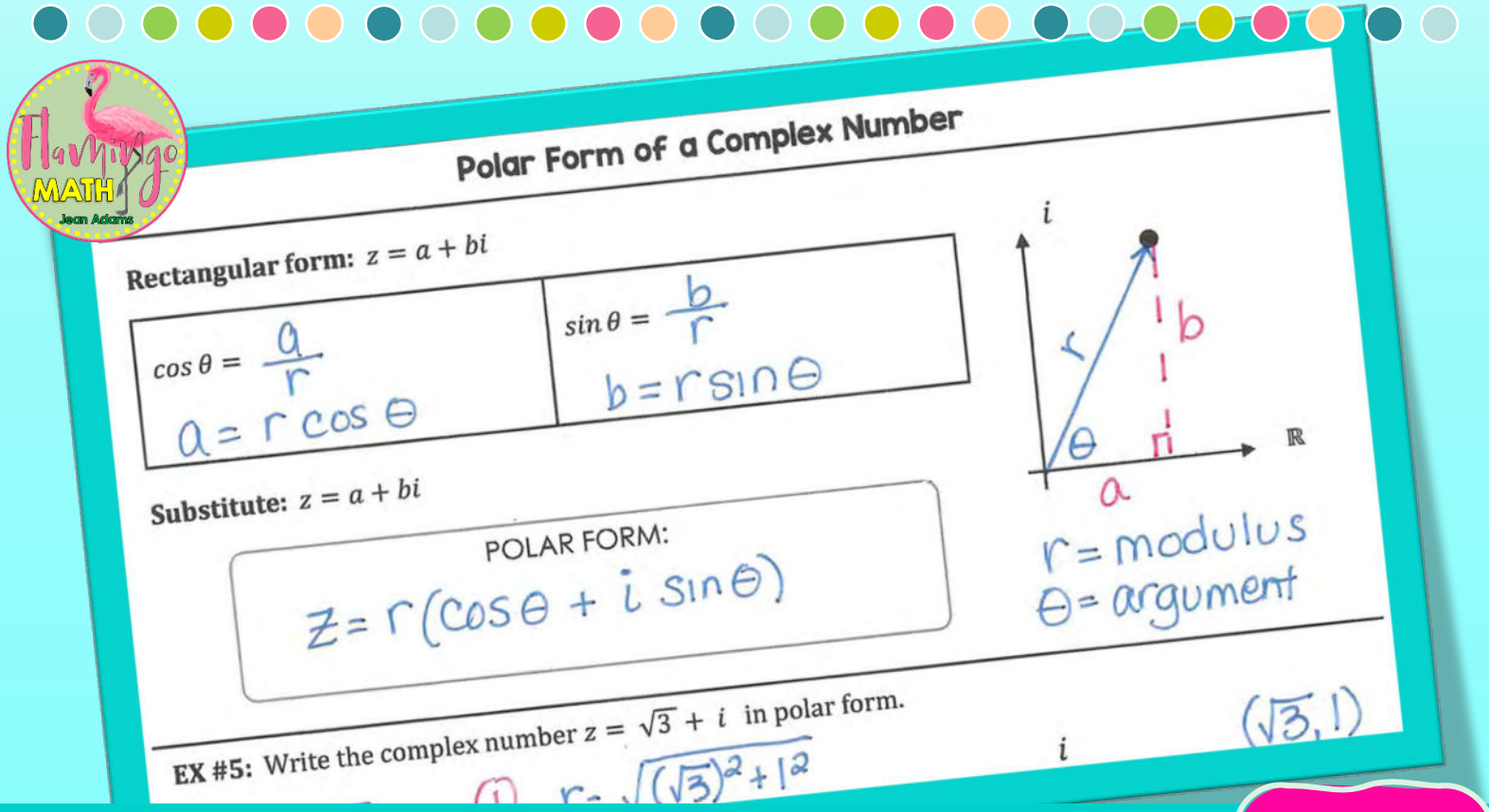
Precalculus Honors

The Complex Plane
DeMoivre's Theorem



Completely Editable

6.4

The Complex Plane: DeMoivre's Theorem

Lesson 6.4 The Complex Plane: DeMoivre's Theorem is a two-day lesson designed for a Precalculus Honors course. The resources are **editable** and **customizable** to provide teachers with the ability to customize examples and practice sets needed for their particular learning environment.

Product Overview

The lesson includes fully-editable notes and practice sets

- Guided Student Notes
- Lesson Warm-Up – 2 versions
- Daily Lesson Quiz – 2 forms
- Homework/Practice assignment
- Google Slides®
- Fully-editable SMART Board® Slides (You **MUST** have SMART Notebook software to open these slides)
- YouTube Lesson Video link
- Answer keys and full solution sets

Learning Objectives & Essential Knowledge

★ Represent complex numbers in the complex plane
★ Write complex numbers in trigonometric form
★ Find the conjugate of a complex number and moduli
★ Find product and quotients of complex numbers
★ Apply De Moivre's Theorem to find nth root of complex numbers

Standards Alignment

BEST: MAFS.912.F-IF.3.8; MAFS.912.F-IF.2.4; MAFS.912.F-IF.3.7
CCSS: HSF-IF.B.4; HSF-IF.C.7; HSF-IF.C.8
TEKS: MA.9-12.P.2.P, MA.9-12.P.3.D

Lesson 6.4: The Complex Plane; DeMoivre's Theorem

This lesson will show how to represent complex numbers in polar (trigonometric) form. We first need to learn how to represent complex numbers graphically. Complex numbers $z = a + bi$ have two components, the real part, a; and, the imaginary part, bi. We will use a **complex plane** with two axes in order to graph complex numbers, the **real axis** and the **imaginary axis**. To graph the complex number $a + bi$, we will plot the ordered pair of numbers (a, b) in this plane.

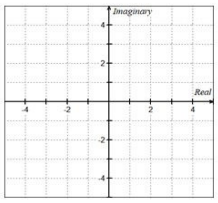
Graphing Complex Numbers

EX #1: Sketch each point on grid at right.

$A = 3 + 4i$ $\quad D = -4 + 0i$

$B = 0 + 2i$ $\quad F = -1 - 2i$

$C = -2 + 3i$ $\quad G = 3 - i$



Absolute Value of Complex Numbers

EX #2: OP has magnitude (length) and direction

The **modulus** (or **absolute value**) of the complex number $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$

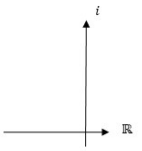
$|a + bi| = \sqrt{a^2 + b^2}$

$tan\,\theta = \frac{b}{a} \quad a \neq 0$

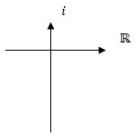
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ if $a > 0$

NOTE: If $a < 0$, then $\theta = \tan^{-1}\left(\frac{b}{a}\right) + \pi$

EX #3: Find $|z|$ and θ. Then graph. $z = -5 + 12i$



EX #4: Find $|z|$ and θ. Then graph. $z = -7i$



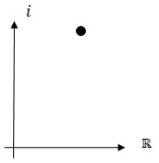
Polar Form of a Complex Number

Rectangular form: $z = a + bi$

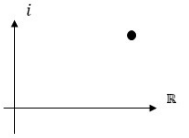
$cos\,\theta =$	$sin\,\theta =$

Substitute: $z = a + bi$

RM:



$\sqrt{3} + i$ in polar form.



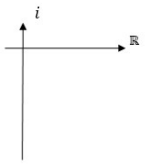
EX #6: Write the complex number $z = \sqrt{6} - i\sqrt{2}$ in polar form.

$a =$

$b =$

$r =$

$\theta =$



Trigonometric Form

EX #7: Change polar form to rectangular form: $z = 5\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$

EX #8: Write the complex number in standard form: $z = 3(\cos 240° + i \sin 240°)$

EX #9: Multiply: $2[(\cos 36° + i \sin 36°)]\,[3(\cos 11$

Product and Quotient of Con

Let $z_1 = r_1(\cos\theta_1 + i\,\sin\theta_1)$ and z_2

PRODUCT: multiply moduli, add arguments	
QUOTIENT: divide moduli, subtract arguments	

EX #10: Divide: $\frac{2(\cos 240° + i \sin 240°)}{6(\cos 32° + i \sin 32°)}$

DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is a positive integer, then . . .

$z^n = [r(\cos\theta + i\sin\theta)]^n$

$z^n = r^n[\cos(n \cdot \theta) + i\sin(n \cdot \theta)]$

EX #12: Find $(\sqrt{3} + i)^4$

EX #13: Evaluate $(1 + i)^5$

Finding n^{th} Roots of Complex Numbers

If a complex number z satisfies the equation $z^n = w$, we say that z is a **complex n^{th} root of w**. We can use DeMoivre's Theorem to find these distinct n^{th} *roots*.

Let $w = r(cos\,\theta + i\,sin\,\theta)$ be a complex number in polar form. If $w \neq 0$, w has n distinct roots given by the formula below, where $k = 0, 1, 2, \ldots, n - 1$

$$z^k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right]$$

EX #14: Find the complex cube roots of $8(\cos 120° + i \sin 120°)$

NOTES

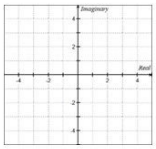
6.4 **DeMoivre's Theorem Homework**

Name _____

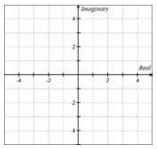
Date _____ Period _____

Problems 1 – 4, plot each complex number in the complex plane and write it in polar form. Express the argument in degrees.

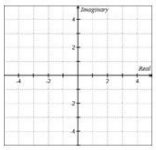
1. $2+\sqrt{3}i$



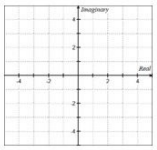
2. $3-4i$



3. $\sqrt{3}+i$



4. $-4i$



Problems 5 - 8, write each complex number in rectangular form.

5. $5(\cos 120° + i \sin 120°)$

6. $2(\cos 225° + i \sin 225°)$

7. $3\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)$

8. $0.4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

Problems 9 – 12, find the product of $z \bullet w$. Leave your answer in trigonometric form.

9. $z = 6(\cos 20° + i \sin 20°)$
$w = 3(\cos 45° + i \sin 45°)$

10. $z = \cos 60° + i \sin 60°$
$w = \cos 75° + i \sin 75°$

11. $z = 2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
$w = 4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

12. $z = 5\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$
$w = 3\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)$

Problems 13 – 16, find the trigonometric form of the quotient.

13. $\frac{3(\cos 50°+i\sin 50°)}{9(\cos 20°+i\sin 20°)}$

14. $\frac{8(\cos 220°+i\sin 220°)}{2(\cos 110°+i\sin 110°)}$

15. $\frac{2(\cos 3\pi+i\sin 3\pi)}{8\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}$

16. $\frac{5\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}{15\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}$

Problems 17 – 20, write each expression in the standard form $a + bi$.

17. $[2(\cos 60° + i \sin 60°)]^3$

18. $[4(\cos 40° + i \sin 40°)]^3$

19. $\left[2\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)\right]^5$

20. $\left[\sqrt{2}\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)\right]^4$

Problems 21 – 24, find all the complex roots. Leave your answers in polar form with the argument in degrees.

21. The complex cube roots of $1 + i$

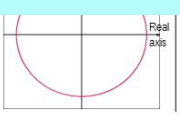
22. The complex cube roots of -8.

23. The complex fourth roots of $-16i$.

24. The complex fifth roots of i.

Problem 25 - 26, solve.

25. Find the sixth roots of unity and plot them.



26. The formula $E = I \bullet Z$, where E represents voltage, I represents current, and Z represents impedance (a measure of opposition to a sinusoidal electric current) is used in electrical engineering. Find E if $I = 12 + 2i$ and $Z = 3 + 5i$.

Practice Sets

Lesson Warm-Ups

3. Find the product of the complex numbers:
$z_1 = 3[\cos(55°) + i\sin(55°)]$
$z_2 = 2[\cos(170°) + i\sin(170°)]$

4. Find $[2(\cos(15°) + i\sin(15°)]^2$

Name _____ Date _____ Period _____

Lesson 6.4: DeMoivre's and nth Root Theorem **Warm-Up # 2**

1. Find the trigonometric form of the complex number $z = \sqrt{2} + i$, where the argument satisfies $0 \le \theta \le 2\pi$

2. Write the complex number
$8\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

3. Find the product of the complex numbers:
$z_1 = 4(\cos(60°) + i\sin(60°))$
$z_2 = 5(\cos(180°) + i\sin(180°))$

4. Find $[2(\cos(100°) + i\sin(100°)]^3$

Form A

_____

_Period _____

of the complex
rgument satisfies

2. Write in standard form $a + bi$
$4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

3. Find the product of the complex numbers.
$z_1 = 3(\cos 50° + i\sin 50°)$
$z_2 = 5(\cos 100° + i\sin 100°)$

6.4 The Complex Plane **Form B**

Name _____

Date _____Period _____

1. Find the trigonometric form of the complex number $-1 - \sqrt{3}i$ where the argument satisfies $0 \le \theta < 2\pi$

2. Write in standard form $a + bi$
$8\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

3. Find the product of the complex numbers.
$z_1 = 2(\cos 35° + i\sin 35°)$
$z_2 = 6(\cos 100° + i\sin 100°)$

Daily Lesson Quiz