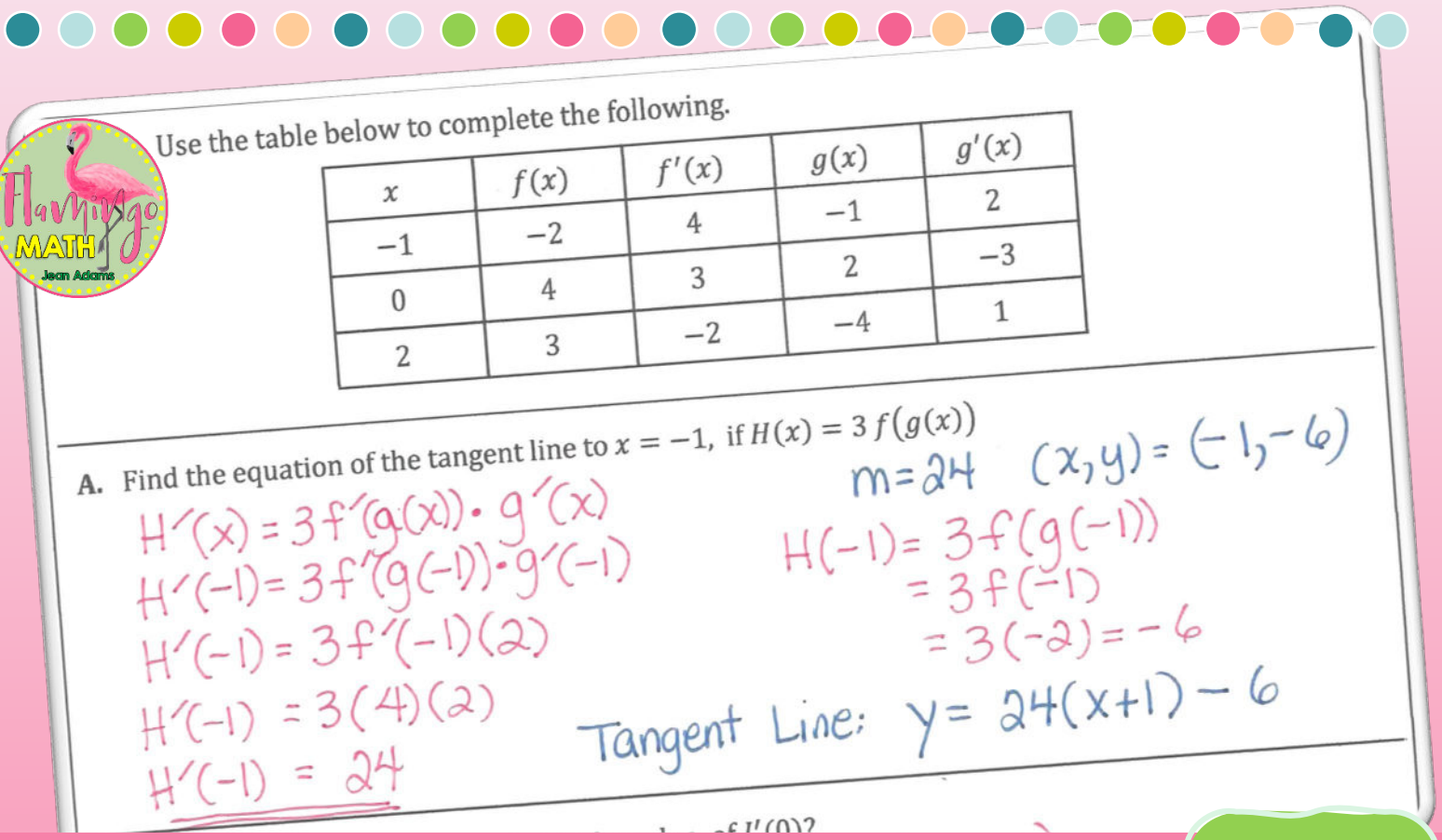
AP® Calculus

The Chain Rule



Use the table below to complete the following.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-2	4	-1	2
0	4	3	2	-3
2	3	-2	-4	1

A. Find the equation of the tangent line to $x = -1$, if $H(x) = 3f(g(x))$

$H'(x) = 3f'(g(x)) \cdot g'(x)$

$H'(-1) = 3f'(g(-1)) \cdot g'(-1)$

$H'(-1) = 3f'(-1)(2)$

$H'(-1) = 3(4)(2)$

$H'(-1) = 24$

$m = 24 \quad (x, y) = (-1, -6)$

$H(-1) = 3f(g(-1))$

$= 3f(-1)$

$= 3(-2) = -6$

Tangent Line: $y = 24(x+1) - 6$

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Lesson I: The Chain Rule

Topic 3.1: The Chain Rule

The chain rule is used to differentiate composite functions such as $y = \sqrt{x^3 + 2}$ or $y = \sin(x^2)$. Remember that you create a composite function by "plugging" one function into another function. The composition of f and g is denoted by $(f \circ g)(x) = f(g(x))$. We can find the derivative of each separate function by using the rules we've learned so far. But, we need the Chain Rule to find the derivative of two functions "plugged into one another."

The Chain Rule:

If f and g are differentiable, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable and $f'(g(x)) \cdot g'(x)$

Leibniz notation will allow us to use "u-substitution." Let $y = f(u)$ and $u = g(x)$, notice that y is a function of u and u is a function of x. So, it follows that,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

This looks more complicated than it really is. Basically, the Chain Rule says to:

MULTIPLY THE DERIVATIVE OF THE INSIDE FUNCTION BY THE DERIVATIVE OF THE OUTSIDE FUNCTION.

EX #1: Find the derivative of $f(x) = (5x + 3)^2$ with and without the chain rule.

A. Without the chain rule

B. With the chain rule

EX #2: Find $f'(x)$ given $f(x) = (4 - x^2)^3$

$f'(x) =$ (**D**erivative of **O**utside **F**unction) × (**D**erivative of **I**nside **F**unction)

General Power and Exponential Rules:

If $g(x)$ is differentiable, then

- ❑ $\frac{d}{dx}\, g(x)^n = n\,(g(x))^{n-1}\, g'(x)$ (for any number n)
- ❑ $\frac{d}{dx}\, e^{g(x)} = g'(x)e^{g(x)}$

EX #3: For each function below, find the derivative.

A. $f(x) = \sqrt{(x^2 - 1)^3}$

B. $f(x) = \frac{-7}{(2x-3)^2}$

C. $y = (5 - 4x^2)^{2/3}$

D. $y = e^{3x^2}$

E. $y = \frac{-2}{\sqrt[3]{6x+3}}$

F. $y = \cos^3(e^{5\theta})$

G. $f(x) = \ln(x^2 + x + 1)^2$

H. $f(x) = x^2 e^{-4\ln x}$

I. $g(x) = \cos^2(4x) - \sin^2(4x)$

J. $y = x^4 \sin^2(x)$

K. $H(x) = \ln(\ln x)$

L. $f(x) = \sin(e^{\pi x})$

More Derivatives Using Tables

EX #4: Let $f(x)$ be a continuous and differentiable function on the interval $0 \le x \le 1$, and let $g(x) = f(4x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. Find the value of $g'(0.2)$.

x	0.1	0.2	0.4	0.5	0.6	0.7	0.8
$f'(x)$	0.75	0.823	0.964	1.033	1.081	1.104	1.125

EX #5: Use the table below to complete the following.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-2	4	-1	2
0	4	3	2	-3
2	3	-2	-4	1

A. Find the equation of the tangent line to $x = -1$, if $H(x) = 3\,f(g(x))$

NOTES

3.1 **The Chain Rule Homework**

Name _____

Date _____ Period _____

Problems 1 – 11, find the derivative of the function.

1. $y = (3x - 2)^4$

2. $f(x) = 5(2 - x^2)^3$

3. $y = (6 + 3x)^{2/3}$

4. $f(x) = \sqrt{x + 3}$

5. $g(x) = \sqrt[3]{x^2 - 9}$

6. $y = \frac{-2}{(x^2 - 3x - 4)^2}$

7. $f(x) = \left(\frac{3}{3-x}\right)^2$

9. $y = \frac{1}{2}x^2\sqrt{4 - x^2}$

10. $f(x) = \frac{x}{\sqrt{x^2+1}}$

11. $s(t) = \left(\frac{t-3}{t^2+2}\right)^2$

each trigonometric function using the Chain Rule.

13. $y = \tan(\ln x)$

14. $f(\theta) = \csc(\theta^2)$

15. $y = \cos\sqrt{x}$

16. $g(\theta) = 4\sin^2(\pi\theta)$

17. $y = \sin(\cos 3x^2)$

Problems 18 – 21, find the equation of the tangent line at the given point.

18. $y = \sqrt{x^2 + 3x + 6}$ at $(2,\ 4)$

19. $y = (x^2 + x - 2)^2$ at $(-1, 4)$

20. $y = \sin(2x)$ at $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

21. $y =$

Problems 22 – 31, find the derivative.

22. $h(x) = e^x \sin^2(x)$

24. $g(x) = \ln(x^3 - 2x)$

26. $y = x\cos(\ln x)$

28. $g(x) = \frac{\pi}{x^2} - \cos x \tan x$

29. $h(x) = \cos\left(\frac{1}{x}\right)$

Problems 32 – 35, use the numerical values in the table below to evaluate the derivatives at the given values of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	6	3	1	-2
3	2	-4	2π	5

32. $f(g(x))$ at $x = 2$

33. $\sqrt{f(x) \cdot g(x)}$ at $x = 2$

34. If $H(x) = \left(4f(x) + g(x)\right)^{\frac{2}{3}}$, find the value of $H'(3)$.

Practice Sets

3.1 The Chain Rule **Form A**

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = \sqrt{x^2 - 4x + 5}$

2. $g(x) = \cos(e^{2x})$

3. $y = \ln(5x - e^x)^2$

4. $g(\theta) = \sin^2(3\theta)$

5. $y = \cot(x^2 + 3)$

3.1 The Chain Rule **Form B**

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = \sqrt{x^2 + 6x + 7}$

2. $g(x) = 2e^{\cos x}$

3. $y = \ln(3x - \pi e^x)^2$

4.

5.

3.1 The Chain Rule **Form C**

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = \sqrt{x^2 - 7x - 8}$

2. $g(x) = \sin\left(e^{x^2}\right)$

3. $y = \ln(3e^x - 4x)^2$

4. $y = \sin\left(\sqrt{2x}\right)$

3.1 The Chain Rule **Form D**

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = \sqrt{x^2 + 5x + 6}$

2. $g(x) = e^{\sin(3x)}$

3. $y = \ln(3x + e^x)^3$

4. $y = \tan(\ln x^2)$

Daily Lesson Quiz