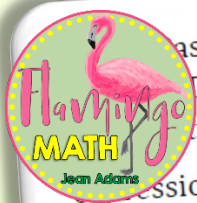


Precalculus Honors

The Binomial Theorem



Pascal (1623-1662), a French mathematician, arranged the combinations in a triangular. This is shown in the table below. The table has 1's down the sides. In order to get to any entry, you add the two nearest entries in the row above it. Now you can understand that the expression $\binom{n}{r}$ is known as a binomial coefficient.

Pascal's Triangle

Pascal's Triangle	Combinations (Binomial Coefficients)	Binomial Expansion
1	${}_0C_0$	$(x + y)^0 = 1$
1 1	${}_1C_0$ ${}_1C_1$	$(x + y)^1 = x + y$
1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$	$(x + y)^2 = x^2 + xy + y^2$
1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

BINOMIAL THEOREM

Completely Editable

9.5

Lesson 9.5: The Binomial Theorem

In this lesson, we will develop the Binomial Theorem, a formula for the expansion of binomials in the form $(x + a)^n$ for any positive integer n . Before we get to that theorem, let's look at how to evaluate the combination of n items taken r at a time.

This is denoted on a calculator as ${}_nC_r$ or, in a textbook, with the symbol $\binom{n}{r}$.

Combinations, n items taken r at a time

If n and r are integers with $0 \leq r \leq n$, the symbol $\binom{n}{r}$ is defined as

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Four Useful Formulas

$$\binom{n}{0} = 1$$

$$\binom{n}{n} = 1$$

$$\binom{n}{1} = n$$

$$\binom{n}{n-1} = n$$

EX #1: Evaluate the following, show your work that leads to your answer:

A. $\binom{5}{1}$

B. $\binom{6}{4}$

C. $\binom{7}{6}$

D. $\binom{15}{12}$

EX #2: Use a graphing utility to evaluate the expression $\binom{26}{8}$

EX #4: Use the Binomial Theorem to expand each binomial.

A. $(3x - 2)^3$

B. $(2m + 5n)^4$

Finding a Coefficient in a Binomial Expansion

EX #5: Find the coefficient of the x^7 term in the expansion of $(3x - 2)^{10}$.

To find the term containing x^k in the expansion of $(x + a)^n$ is:

$$\binom{n}{n-k} a^{n-k} \cdot x^k$$

EX #6: Find the fifth term in the expansion of $(x + 2)^8$

Blaise Pascal (1623-1662), a French mathematician, arranged the combinations in a triangular display. This is shown in the table below. The table has 1's down the sides. In order to get to any other entry, you add the two nearest entries in the row above it. Now you can understand that the expression $\binom{n}{r}$ is known as a binomial coefficient.

Pascal's Triangle		
Pascal's Triangle	Combinations (Binomial Coefficients)	Binomial Expansion
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1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

BINOMIAL THEOREM

For any whole number n ,

$$(x + y)^n = {}_nC_0x^n y^0 + {}_nC_1x^{n-1}y^1 + {}_nC_2x^{n-2}y^2 + \dots + {}_nC_{n-1}x^1y^{n-1} + {}_nC_nx^0y^n$$

Expanding Binomials Using a TI-84

To find a binomial coefficient like ${}_3C_2$ on the TI-84: press MATH, arrow over to PROB, choose 3: ${}_nC_r$, press 5, arrow right, press 2 click ENTER.



NOTES

9.5

The Binomial Theorem Homework

Name _____

Date _____ Period _____

Problems 1-6, evaluate each expression.

1. $\binom{6}{3}$

2. $\binom{8}{6}$

3. $\binom{15}{15}$

4. $\binom{45}{0}$

5. $\binom{30}{29}$

6. $\binom{26}{17}$

Problems 7-12, expand each expression using the Binomial Theorem.

7. $(x + 1)^4$

8. $(x - 1)^5$

9. $(x + 2)^4$

10. $(x - 2)^6$

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11. $(2x + 1)^4$

12. $(3x - 2)^5$

Problems 13-18, use the Binomial Theorem to find the indicated coefficient or term.

13. The coefficient of x^6 in the expansion of $(x + 4)^9$

14. The coefficient of x^3 in the expansion of $(x - 4)^9$

15. The coefficient of x^2 in the expansion of $(2x + 1)^8$

16. Find the third term in the expansion of $(x - 3)^7$

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Practice Sets

Lesson Warm-Ups

2. Expand the expression $(3x - 4)^3$

3. Find the fifth term in the expansion of $(x + 3)^7$

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Name _____ Date _____ Period _____

Lesson 9.5 The Binomial Theorem

Warm-Up #1

1. Evaluate $\binom{10}{4}$

2. Expand the expression $(2x - 3)^3$

3. Find the fourth term in the expansion of $(x + 3)^7$

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Form A

9.5 Binomial Theorem

Form B

Name _____

Date _____ Period _____

1. Evaluate $\binom{7}{2}$

2. Expand the binomial $(2x - 4)^3$

3. Find the coefficient of x^2 in the expansion of $(3x + 5)^5$

4. Find the third term in the expansion of $(x + 3)^7$

2. Expand the binomial $(2x + 5)^3$

3. Find the coefficient of x^3 in the expansion of $(3x - 5)^5$

4. Find the fourth term in the expansion of $(x + 2)^7$

Daily Lesson Quiz

Grade: _____

Grade: _____