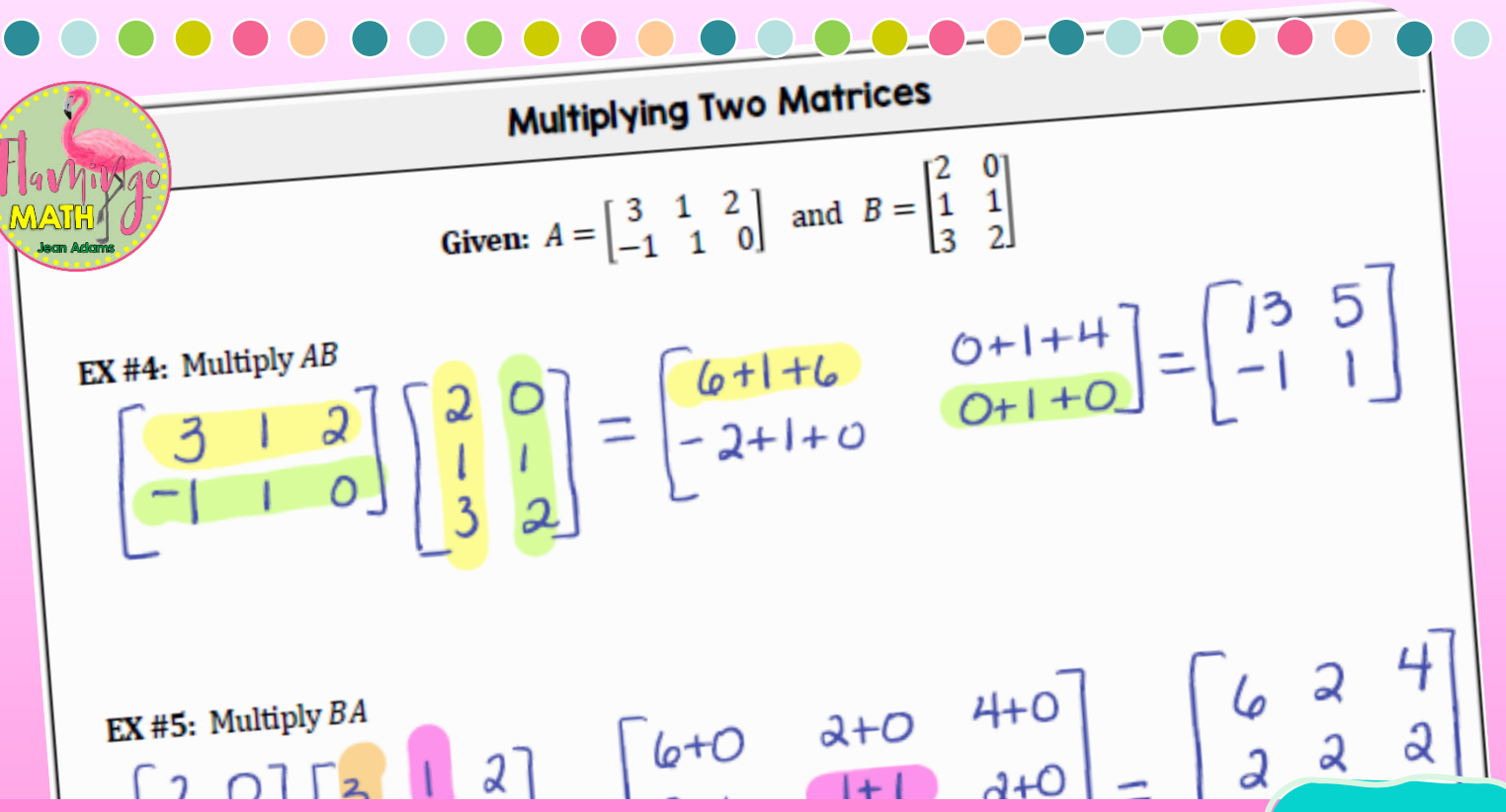
Precalculus Honors

The Algebra of Matrices



Completely Editable 7.4

Lesson 7.4: The Algebra of Matrices

A **matrix** is a rectangular array of numbers written within brackets; usually named by capital letters. In the example at right, $A_{2\times3}$ read this as A is a 2 by 3 matrix.

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix}$$

Any $m \times n$ matrix with the same number of rows as columns is known as a **square matrix**. The **identity matrix, *I***, is a square matrix where the diagonal entries are 1's and all other entries are 0's.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sum and Differences of Two Matrices

Let A and B be matrices of order $m \times n$:

1. $A + B = [a_{ij} + b_{ij}]$
2. $A - B = [a_{ij} - b_{ij}]$

* Matrices must be the same size.

EX #1: Add and subtract, given: $A = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & 1 \\ 3 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 & 0 \\ 4 & 6 & 9 \\ -5 & 1 & -2 \end{bmatrix}$

A. Find $A + B$

B. Find $A - B$

Operations Using Matrices

Let A, B, and C be matrices of order $m \times n$:
* Matrices must be the same size.

Commutative Property	
Associative Property	
Zero Matrix Property	
Scalar Multiplication	

Scalar Multiplication and Matrix Subtraction

EX #2: Find the following, **given:** $A = \begin{bmatrix} 5 & 1 & 3 \\ 6 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 & 4 \\ -3 & 1 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 12 & 0 \\ -4 & 8 \end{bmatrix}$

A. $2A - 3B$

B. $\frac{1}{4}C$

The Product of Two Matrices

Row by Column matrix multiplication of RC is defined as a single number.

$$RC = [r_1 \quad r_2 \quad \cdots \quad r_n] \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = _____$$

Let A denote an $\boldsymbol{m \times r}$ matrix and let B denote an $\boldsymbol{r \times n}$ matrix.
The product of AB is an $\boldsymbol{m \times n}$ matrix whose element in row I, column j is the product of the i-*th* row of A, and the j-*th* column of B.

***** The columns of *A* must equal the rows of *B*. *****

EX #3: Multiply RC, given: $R = [3 \quad -4 \quad 5]$ and $C = \begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix}$

Multiplying Two Matrices

Given: $A = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$

EX #4: Multiply AB

EX #5: Multiply BA

Applications

EX #6: A citrus grower has oranges and grapefruit trees in their groves. They ship the citrus to three different packing houses. The table below shows how many bushels of each fruit are shipped to each packing house. Profits for oranges are $8.25 per bushel, and the profit for grapefruit is $12.50. Find the profits for each packing house.

	Hall's Packing	Florida Citrus	Indian River Fruit

NOTES

7.4 **The Algebra of Matrices Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, find, if possible, (a) $A + B$, (b) $A - B$, (c) $2A$, and (d) $3A - 2B$

1. $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$

2. $A = \begin{bmatrix} 6 & 3 \\ -1 & -2 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 9 \\ 1 & -3 \\ 1 & 6 \end{bmatrix}$

3. $A = \begin{bmatrix} -2 & 1 & 0 \\ -3 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 9 & -3 \\ 4 & -2 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & -5 \\ -4 & 7 & -1 \end{bmatrix}$

Problems 5 – 6, solve for X when $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 4 & 0 \\ -2 & -1 \end{bmatrix}$

5. $X = 3A - 2B$

6. $2X + 3A = B$

Problems 7 – 11, find AB, if possible.

7. $A = \begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}$; $B = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$

8. $A = \begin{bmatrix} -4 & 5 \\ -1 & 6 \\ 0 & 3 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 2 \\ 8 & 0 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \\ 3 & -1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & -2 \\ -1 & 1 & 0 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & -2 \\ -1 & 1 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 & -2 \\ 2 & -3 \\ 3 & -1 \end{bmatrix}$

11. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Problems 12 – 16, use the matrix capabilities of a graphing utility to evaluate each expression.

12. Multiply. $\begin{bmatrix} 8 & 5 & -4 \\ -3 & 6 & 1 \\ 9 & -7 & -2 \end{bmatrix} \begin{bmatrix} 3 & -3 & 2 \\ 8 & 2 & 5 \\ -4 & -3 & -8 \end{bmatrix}$

13. $-3\left(\begin{bmatrix} 5 & -1 & 6 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -3 \\ 1 & 3 \\ 4 & -2 \end{bmatrix}\right)$

14. $\begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & -1 & -1 \\ -2 & -1 & 0 \end{bmatrix}$

15. $\begin{bmatrix} -0.5 & 0.75 & 0.25 \\ 1 & 0 & -1.5 \\ 0 & -1 & 0.5 \end{bmatrix} \begin{bmatrix} -12 & -5 & -9 \\ -4 & -2 & -4 \\ -8 & -4 & -6 \end{bmatrix}$

Applications

17. A manufacturer produces three models of Bluetooth earbuds and shipped to two distribution centers. The number of units of model m that are shipped to distribution center n is represented by a_{mn} in the matrix. The price for each unit is represented by matrix B. Compute BA and state what each entry of the product represents.

$A = \begin{bmatrix} 3{,}800 & 4{,}200 \\ 5{,}500 & 12{,}500 \\ 8{,}000 & 6{,}400 \end{bmatrix}$

$B = [\$42.50 \quad \$55.50 \quad \$78.50]$

18. The costs of equipment for the track team is shown in the table at right. Each jersey costs $25, shoes cost $125, and warm-ups cost $165. Use matrices to find the total cost of equipment for each team.

Equipment	Boy's Team	Girl's Team
Jersey	38	32
Track shoes	18	15
Team Warm-Up	20	17

19. The matrix below shows the inventory in a t-shirt shop. The shop owner needs to know

Small	Medium	Large

T-shirts	$12
Hoodies	$16

Practice Sets

Lesson Warm-Ups

1. Find $3A - 2B$

2. Multiply BA

3. Solve for X. $2X - B = A$

Grade:

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1. Find $3A - 2B$

2. Multiply BA

3. Solve for

Grade:

Name _____ Date _____ Period _____

Lesson 7.4 Matrix Algebra **Warm-Up # 1**

1. Given $A = \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -3 \\ -3 & -2 \end{bmatrix}$

Find $3A - 2B$

2. Multiply BA

3. Find A^{-1}

Name _____ Date _____ Period _____

Lesson 7.4 Matrix Algebra **Warm-Up # 2**

1. Given $A = \begin{bmatrix} 4 & 1 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 3 \\ -4 & -2 \end{bmatrix}$

Find $3A - 2B$

2. Multiply BA

Daily Lesson Quiz