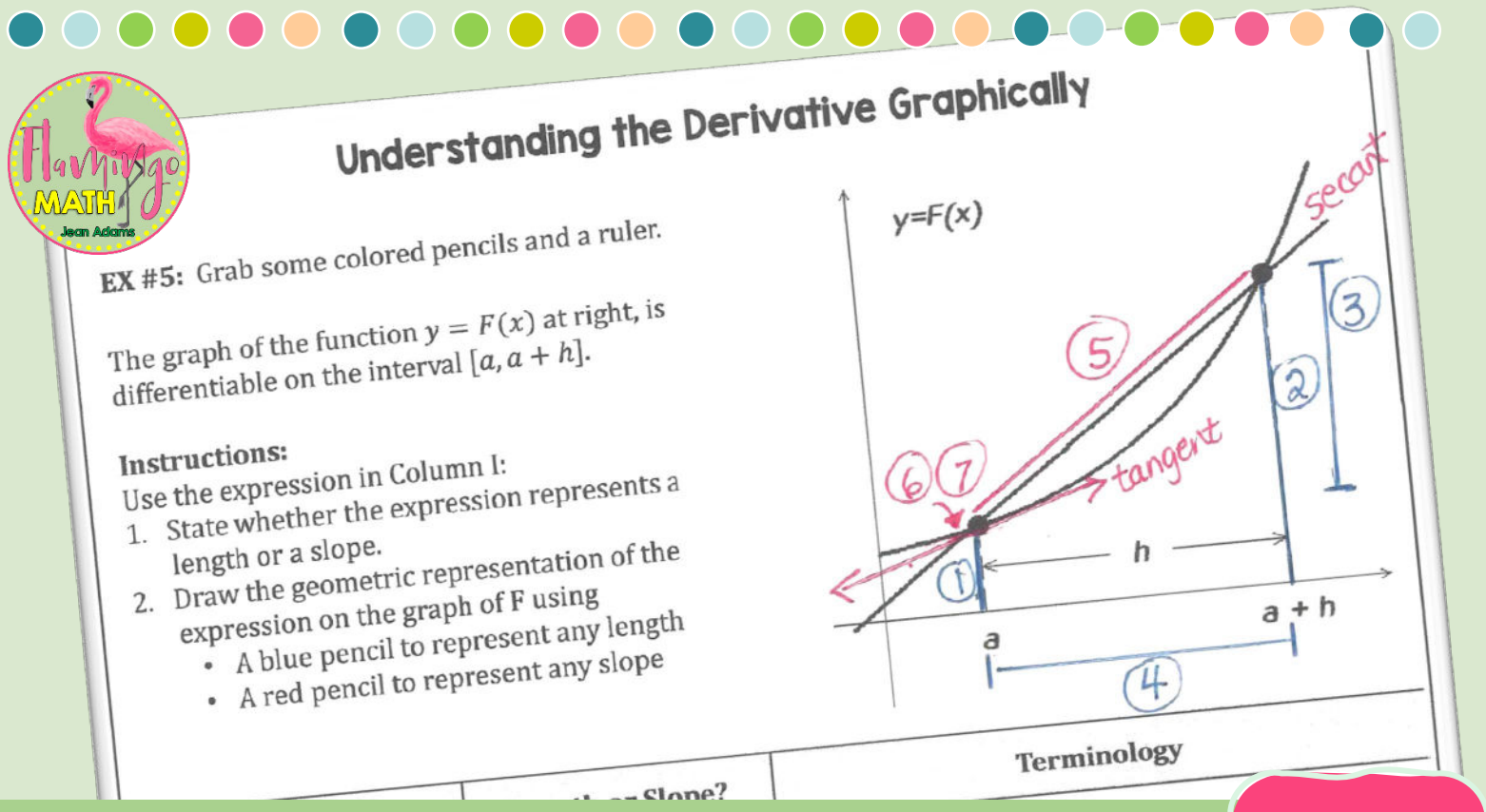
AP® Calculus

Tangent Lines and the Derivative



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Lesson 2: Tangent Lines and the Derivative

Topic 2.2: Defining the Derivative of a Function and Using Derivative Notation

There are two main concepts in calculus, known as the ***tangent line problem*** (derivatives) and the ***area problem*** (integrals). Both of these big ideas are related to limits.
In the previous lesson, we applied the difference quotient to a function $y = f(x)$, and found a general "slope formula." This calculated the slope of a tangent line at any point on the curve, provided the limit exists at that point. This general "slope formula" is also a function and is called the **derivative function** of $f(x)$ or $f'(x)$ [read as "f prime of x"]. A function that has a derivative at a point is said to be **differentiable** at that point.

Definition of the Derivative

The derivative of the function f with respect to x is the function f' whose value at x is

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

ALTERNATE FORMS OF THE DERIVATIVE:

The **numeric value of the derivative** at a point $x = a$ is also found by:

$$f'(a) = m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \textbf{or by} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

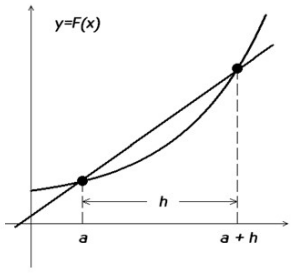
provided the limit exists.
For all *a* for which the limit exists, $f'(x)$ is a function of *a*.

EX #3: Use the alternate form of the derivative to find $f'(a)$ for $f(x) = \sqrt{3x+1}$ at $a = 1$.

Understanding the Derivative Graphically

EX #5: Grab some colored pencils and a ruler.

The graph of the function $y = F(x)$ at right, is differentiable on the interval $[a, a+h]$.



Instructions:
Use the expression in Column I:
1. State whether the expression represents a length or a slope.
2. Draw the geometric representation of the expression on the graph of F using
   - A blue pencil to represent any length
   - A red pencil to represent any slope

Notation	Length or Slope?	Terminology
1. $f(a)$		
2. $f(a+h)$		
3. $f(a+h) - f(a)$		
4. h		
7. $f'(a)$		

The Limit Definition of the Derivative

Graphically: The derivative of a function at a point P is the slope of a tangent line to the graph of $y = f(x)$ at P.

Numerically: The derivative at a point is the *limit of slopes of the secant lines* or the *limit of the difference quotient.*

Verbally: A function f is said to be "differentiable" at a point $(a, f(a))$ if and only if it has a defined slope at $x = a$. If a function does not have a tangent at a specified point, then the function cannot have a slope there either.

EX #1: Use the limit process of the derivative to find the general formula, $\frac{dy}{dx}$ for the derivative of $f(x) = 2x^2 - 3x$.

Topic 2.3: Estimating Derivatives of a Function at a Point

EX #4: Find the slope of the tangent line to the hyperbola $y = \frac{2}{x}$ at $x = 3$.

A. Using $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

B. Using $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Understanding the Derivative Symbolically

Oftentimes you will be asked to extend your knowledge base for operations in calculus where you need to recognize functions, procedures, processes, theorems, and properties. You might also need to "decompose or deconstruct" mathematical operations symbolically. Here is a sample of such instances.

EX #6: Each of the following expressions represents the derivative of some function $y = f(x)$ for some value, $x = c$. Identify the function $y = f(x)$ and the value for $x = c$.

A. $\lim_{h \to 0} \frac{\sqrt{h+4} - 2}{h}$

B. $\lim_{h \to 0} \frac{e^{\cos(\pi + h)} - e^{-1}}{h}$

C. $\lim_{h \to 0} \frac{\tan 2(2+h) - \tan(4)}{h}$

D. $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{6+h}{6}\right)$

NOTES

2.2 **Tangent Lines and the Derivative Homework**

Name _____

Date _____ Period _____

Problems 1– 6, Find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for each of the functions given below. Write the equation of the tangent line and the normal line to the graph of $f(x)$ at the given value of x.

1. $f(x) = 7 - 3x - x^2$

2. $g(x) = \sqrt{2-x}$

3. Find the equation of the tangent line to the graph of $f(x) = 7 - 3x - x^2$ at $x = 1$

4. Find the equation of the tangent line to the

5. Find the equation of the normal line to the graph of $f(x) = 7 - 3x - x^2$ at $x = 1$

6.

Problems 7 – 12, given the function $g(x) = \sqrt{x-2}$

7. Find $g'(x)$ by using the limit process.

8. Find the slope of the tangent line drawn to the graph of $g(x)$ at $x = 4$

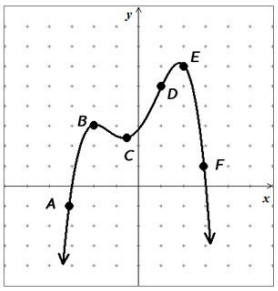
9. Find the slope of the tangent line drawn to the graph of $g(x)$ at $x = 6$

10. Write the equation of the tangent line to the graph of $g(x)$ at $x = 6$.

11. Write the equation of the normal line to the graph of $g(x)$ at $x = 6$.

$\frac{f(x)-f(a)}{x-a}$, where $a = 4$.

13. Complete the table below. Estimate the value of $f'(x)$ at the indicated x-values by drawing a tangent line and estimating its slope. Describe the behavior at the given point. Write the equation of the tangent line.



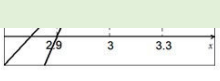
Point of tangency	Behavior at this point? increasing, decreasing, relative maximum or relative minimum	Estimated Slope of Tangent Line	Equation of the tangent line at given point:
A			
B			
C			
D			
E			
F			

Problems 14 – 18, finding a numeric derivative from a
estimated by finding the slope of the secant line
at either side of the point.

x	−3	−2	−1	0	1
$g(x)$	4	3.6	2	−3	−1

x-value	Estimate of the derivative	Is the funct increasing decreasing h
14. $x = -2$		
15. $x = 0$		
16. $x = 1$		
17. $x = 4$		
18. $x = 5$		

19. Given $y = g(x)$ at right, where $g(3) = 18$ and



Problems 20 – 23, Each of the following expressions represents the derivative of some function $y = f(x)$ for some value, $x = c$. Identify the function $y = f(x)$ and the value for $x = c$.

20. $\lim_{h\to 0}\frac{7-2(2+h)-3}{h}$

21. $\lim_{x\to 9}\frac{81-x^2}{x-9}$

22. $\lim_{h\to 0}\frac{(-1+h)^3+1}{h}$

23. $\lim_{x\to -7}\frac{\sqrt{2-x}-3}{x+7}$

24. If $f'(5) = -4$ and $f(5) = 8$, write the equation of the normal line to $y = f(x)$ at $x = 5$.

Practice Sets

2.2 Tangent Lines and the Derivative
Form A

Name _____

Date _____ Period _____

1. Find the derivative of $f(x) = x^2 - 3$ by the limit process.

2. Write the equation for the tangent line to the graph of $f(x) = x^2 - 3$ where $x=2$

3. If $f'(-3) = 2$ and $f(-3) = 5$, write the equation of the normal line to $y = f(x)$ at $x = -3$.

2.2 Tangent Lines and the Derivative
Form B

Name _____

Date _____ Period _____

1. Find the derivative of $f(x) = -x^2 + 4$ by the limit process.

2. Write the equation for the tangent line to the graph of $f(x) = -x^2 + 4$ where $x = -3$.

3.

2.2 Tangent Lines and the Derivative
Form C

Name _____

Date _____ Period _____

1. Find the derivative of $f(x) = -x^2 + 3$ by the limit process.

2. Write the equation for the tangent line to the graph of $f(x) = -x^2 + 3$ where $x = 3$.

3. If $f'(-1) = 3$ and $f(-1) = 4$, write the equation of the normal line to $y = f(x)$ at $x = -1$.

2.2 Tangent Lines and the Derivative
Form D

Name _____

Date _____ Period _____

1. Find the derivative of $f(x) = x^2 - 4$ by the limit process.

2. Write the equation for the tangent line to the graph of $f(x) = x^2 - 4$ where $x = -2$.

3. If $f'(5) = -4$ and $f(5) = -9$, write the equation of the normal line to $y = f(x)$ at $x = 5$.

Daily Lesson Quiz