

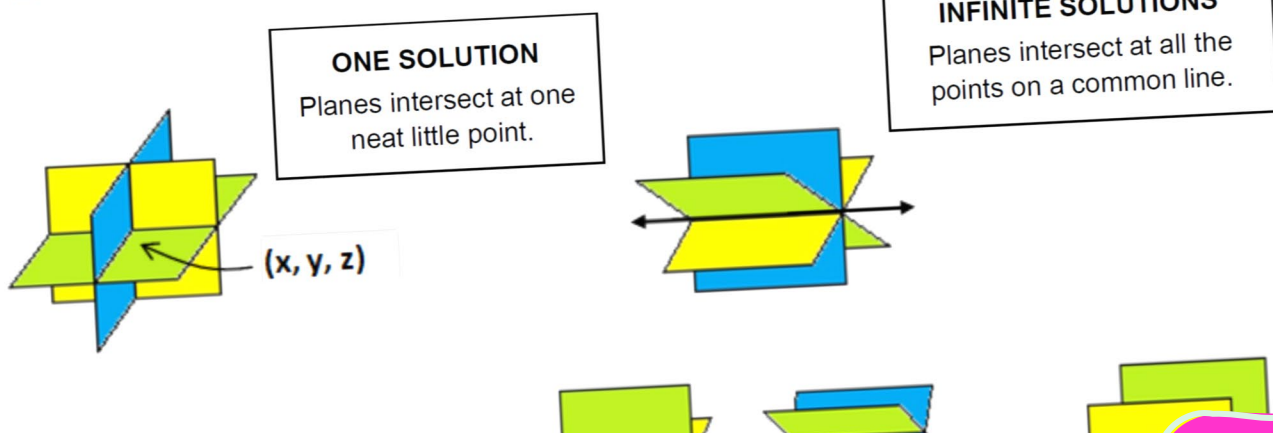
Algebra 2 Honors

Systems with Three Variables

Lesson 5: Systems with Three Variables

Here's a visual picture of three planes in three dimensions

Remember that a 3×3 system is really three planes that are living in 3-D space... They're actually locked into specific locations (not floating around) and we want to find where they intersect. Let's look at the possibilities.



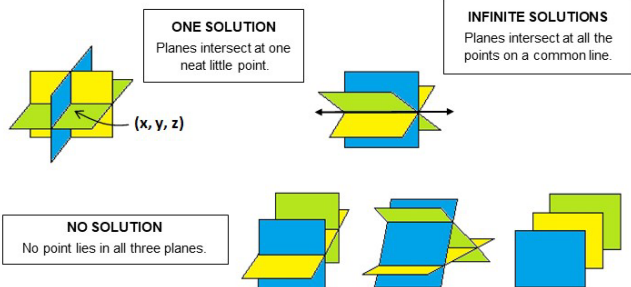
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3.5

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Finding a Procedure to Follow:

One method for solving three equations with three unknowns is stated below.

1. Number the equations in the system 1, 2, and 3.
2. Choose any variable to eliminate.
3. Pick any two of the equations from the original system and use the elimination method learned earlier. Call the result equation #4.
4. Go back to the original system and pick two different equations from the system. Eliminate the **same** variable by using the elimination method. Call the result equation #5.
5. Use equations #4 and #5 together to eliminate a second variable by the elimination method. This will give you one of your unknowns.
6. Take your solution for step 5 and substitute back into equation #4 or #5 to find a second unknown.
7. Find your third unknown.
8. Write your solution as an ordered triple (x, y, z)
9. Check your solution in the original system.

Solving a System Using Elimination

EX #1:
$$\begin{cases} x - y + z = -1 \\ x + y + 3z = -3 \\ 2x - y + 2z = 0 \end{cases}$$

Solving an Equivalent System

EX #2:
$$\begin{cases} x - 2y + 3z = 12 \\ 2x - y - 2z = 5 \\ 2x + 2y - z = 4 \end{cases}$$

Solving a System Using Substitution

EX #3: Solve,
$$\begin{cases} x - 2y + z = -4 \\ -4x + y - 2z = 1 \\ 2x + 2y - z = 10 \end{cases}$$

Solving Real-World Problems

EX #4: You manage a clothing store and budget \$5400 to restock 200 shirts. You can buy T-shirts for \$12 each, polo shirts for \$24 each, and rugby shirts for \$36 each. Suppose you want to have the same number of T-shirts as polo shirts. How many of each shirt should you buy?

EX #5: A company placed \$500,000 in three different accounts. It placed one account in short-term notes paying 5.5% per year, three times as much in government bonds paying 7% per year, and the rest in utility bonds paying 4.5% each. The income after one year was \$31,000. Determine how much money was placed in each account.

DEFINE:

Let x = the amount of money invested in short term notes
Let y = the amount invested in government bonds
Let z = the amount invested in utility bonds.

NOTES

Solving Systems with Three Variables Practice

Problems 1–4, solve each system.

1.
$$\begin{cases} x + y + z = -1 \\ 3x - y + 2z = -7 \\ -x + 2y - z = 7 \end{cases}$$

2.
$$\begin{cases} x + y + z = 9 \\ 2x - y + 2z = 3 \\ 3x + 2y - z = -2 \end{cases}$$

3.
$$\begin{cases} 2x - y - z = 12 \\ -x - 2y - z = 1 \\ 3x - y - z = 16 \end{cases}$$

4.
$$\begin{cases} 6x - y + 2z = 4 \\ 2x + 3y - 3z = -5 \\ 4x + 2y + 5z = -9 \end{cases}$$

Problems 5–6, solve each system by substitution.

5.
$$\begin{cases} x + y - z = 4 \\ 3x - y + z = 0 \\ 5x + z = 3 \end{cases}$$

6.
$$\begin{cases} 4x + y + 3z = 13 \\ -3x - 2y + z = 5 \\ 2x + 5y + 4z = -4 \end{cases}$$

Problems 7–12, solve each system by any method.

7.
$$\begin{cases} 4y + 3z = 3x - 2 \\ x + z - 2y = -4 \\ x - 2z = 3y + 1 \end{cases}$$

8.
$$\begin{cases} x + y - z = 3 \\ 3x + 4y - z = 1 \\ 6x + 8y - 2z = 7 \end{cases}$$

9.
$$\begin{cases} 8x + 2y - z = 6 \\ -2x + 5y - 3z = -1 \\ 4y + z = -3 \end{cases}$$

10.
$$\begin{cases} x - y + 2z = 7 \\ x + y + 2z = 7 \\ 2x + 4z = 14 \end{cases}$$

11.
$$\begin{cases} x + y = -1 \\ y + z = 4 \\ x + z = 1 \end{cases}$$

12.
$$\begin{cases} 4x - y + z = 1 \\ -x + y - z = -1 \\ 2x - z - 5 = y \end{cases}$$

Problems 13–16, write and solve a system of equations for each problem.

13. The sum of three numbers is twelve. The first number is equal to twice the second number minus 6. The second number is two more than the sum of the first and third numbers.

14. Collin is training for a triathlon which includes biking, swimming, and running. Last week he biked 5 times as far as he ran. He ran 4 times farther than he swam. At the end of the week Collin had a total of 200 miles in training. How many miles did he train for each event?

15. The sum of three numbers is 7. The sum of three times the first number, twice the second number, and the third number is 28. The difference between the second number and half the third number is 8. Find the numbers.

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 3.5: Solving Systems with Three Variables Warm-Up # 1	
1. Solve the system. $\begin{cases} -x - 5y - 5z = 2 \\ 4x - 5y + 4z = 19 \\ x + 5y - z = -20 \end{cases}$	2. Solve the system. $\begin{cases} 4x + 4y + z = 24 \\ 2x - 4y + z = 0 \\ 5x - 4y - 5z = 12 \end{cases}$

Name _____ Date _____ Period _____	
Lesson 3.5: Solving Systems with Three Variables Warm-Up # 2	
1. Solve the system. $\begin{cases} -x - 5y + z = 17 \\ -5x - 5y + 5z = 5 \\ 2x + 5y - 3z = -10 \end{cases}$	2. Solve the system. $\begin{cases} x - 6y + 4z = -12 \\ x + y - 4z = 12 \\ 2x + 2y + 5z = -15 \end{cases}$

3.5 Systems with Three Variables Form A	3.5 Systems with Three Variables Form B
Name _____	Name _____
Date _____ Period _____	Date _____ Period _____
1. Solve the system: $\begin{cases} x + y - z = -4 \\ x + y + z = 2 \\ y = -2 \end{cases}$	1. Solve the system: $\begin{cases} x + y - z = 4 \\ x - y + z = 6 \\ z = 3 \end{cases}$
2. Solve the system: $\begin{cases} x + y - z = -2 \\ x - y = -2 \\ 2x - y + 3z = 3 \end{cases}$	2. Solve the system: $\begin{cases} x + y - z = -2 \\ x - y = 5 \\ 2x - y + 3z = 10 \end{cases}$
Grade: _____	Grade: _____

Daily Lesson Quiz