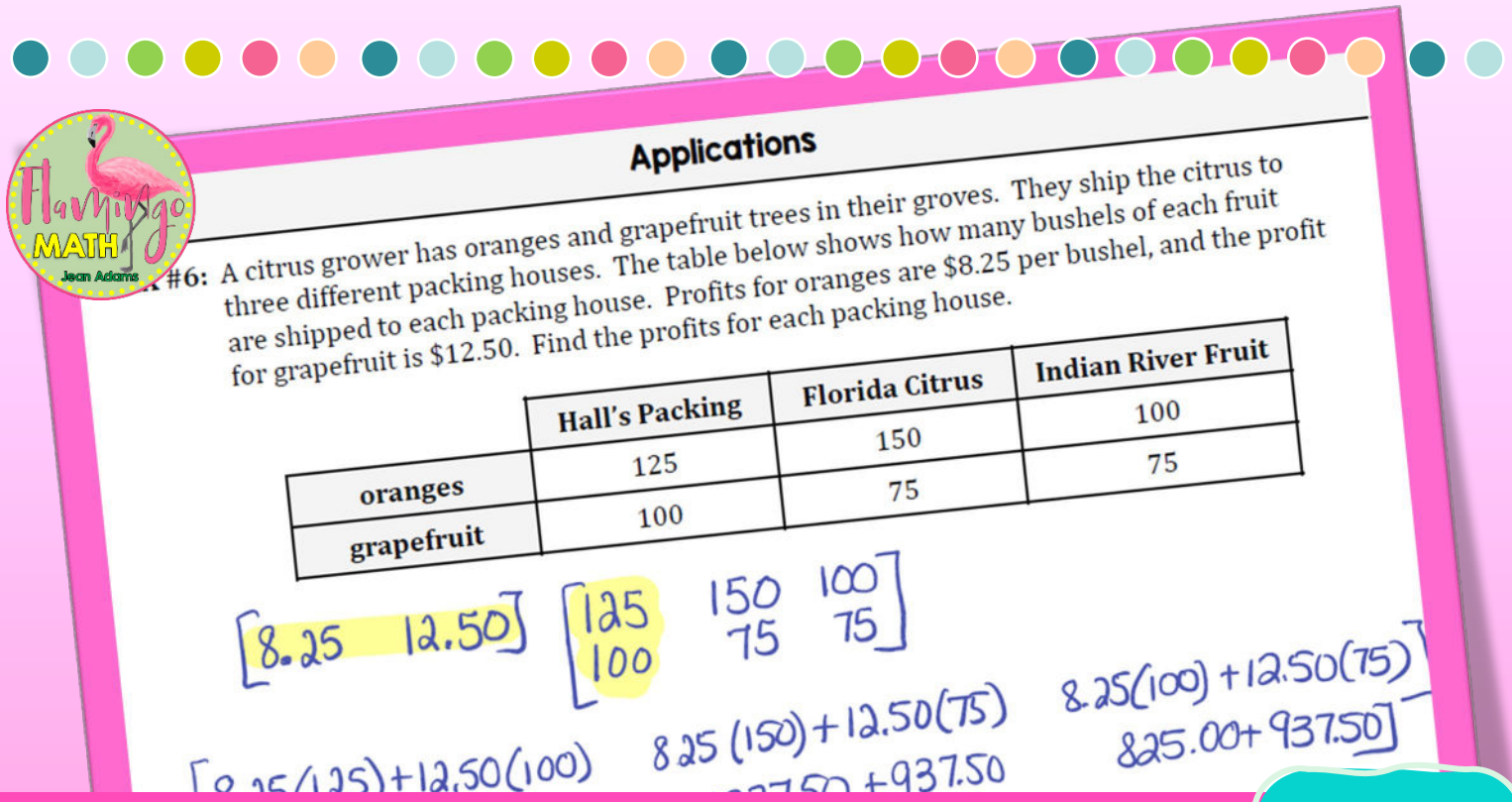
Precalculus Honors

Systems of Equations and Matrices



Guided Notes

7

Completely Editable

PRECALCULUS CURRICULUM

Scope and Sequence

Unit 7: Systems of Equations and Matrices

Lesson	Skills & Learning Objectives
[7.1] Solving Systems of Equations	❑ Solve systems of equations in two variables by elimination method. ❑ Solve systems of three equations with three unknowns, including real-world applications.
[7.2] Multivariable Linear Systems	❑ Use back-substitution to solve linear systems in row-echelon form. ❑ Use Gaussian elimination to solve systems of linear equations. ❑ Solve non-square systems of linear equations ❑ Use systems of linear equations in three or more variables to solve real-world applications.
[7.3] Matrices and Systems of Equations	❑ Write matrices and identify their orders. ❑ Perform elementary row operations on matrices. ❑ Use matrices and Gaussian elimination to solve systems of linear equations ❑ Use matrices and Gauss-Jordan elimination to solve systems of linear equations.
[7.4] The Algebra of Matrices	❑ Determine whether two matrices are equal. ❑ Add and subtract matrices and multiply matrices by scalars. ❑ Multiple two matrices. ❑ Use matrix operations to model and solve real-world problems.
[7.5] Inverses of Matrices and Matrix Equations	❑ Verify that two matrices are inverses of each other. ❑ Use Gauss-Jordan elimination to find the inverses of matrices. ❑ Use a formula to find the inverses of matrices. ❑ Use inverse matrices to solve systems of linear equations.

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PRECALCULUS CURRICULUM

Scope and Sequence

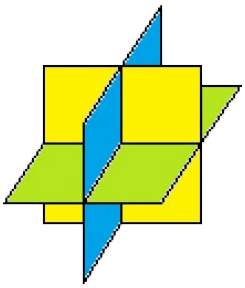
Unit 7: Systems of Equations and Matrices

Lesson	Skills & Learning Objectives
[7.6] The Determinant of a Square Matrix	❑ Find the determinants of matrices. ❑ Find minors and cofactors of square matrices. ❑ Find the determinants of square matrices.
[7.7] Partial Fractions	❑ Recognize partial fraction decompositions of rational expressions. ❑ Find partial fraction decompositions of rational expressions.
[7.8] Systems of Nonlinear Equations	❑ Solve systems of nonlinear equations by graphing
[7.9] Linear Programming	❑ Sketch the graphs of inequalities in two variables ❑ Use linear programming to maximize and minimize real-world applications

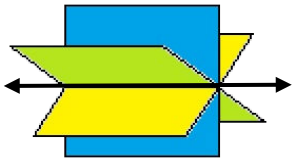
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Graphical Interpretation of Three-Variable Systems

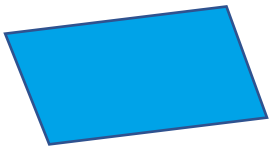
In our first lesson we categorized solutions to linear equations in two variables. The solutions for linear equations in more than two variables must also fall into one of the three categories of exactly one solution, infinitely many solutions, and no solution. The graph of an equation in three variables consist of all points (x, y, z) that are solutions of the equation.



Solution:
One point

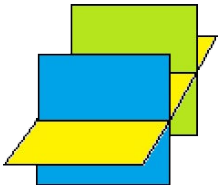


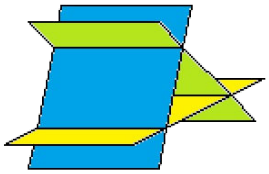
Solution:
One line

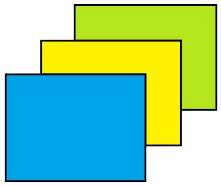


Solution:
One plane

No Solutions:







Identify an Inconsistent System of Equations

EX #3: Solve. $\begin{cases} x - y + 2z = 2 \\ x + 2y - z = 5 \\ 2x - y + 3z = 1 \end{cases}$

Lesson 7.4: The Algebra of Matrices

A **matrix** is a rectangular array of numbers written within brackets; usually named by capital letters. In the example at right, $A_{2\times3}$ read this as A is a 2 by 3 matrix.

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix}$$

Any $m \times n$ matrix with the same number of rows as columns is known as a **square matrix**. The **identity matrix, *I***, is a square matrix where the diagonal entries are 1's and all other entries are 0's.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sum and Differences of Two Matrices

Let A and B be matrices of order $m \times n$:

1. $A + B = [a_{ij} + b_{ij}]$
2. $A - B = [a_{ij} - b_{ij}]$

* Matrices must be the same size.

EX #1: Add and subtract, given: $A = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & 1 \\ 3 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 & 0 \\ 4 & 6 & 9 \\ -5 & 1 & -2 \end{bmatrix}$

A. Find $A + B$

B. Find $A - B$

Operations Using Matrices

Let A, B, and C be matrices of order $m \times n$:
* Matrices must be the same size.

Commutative Property	
Associative Property	
Zero Matrix Property	
Scalar Multiplication	

Lesson 7.5: Inverses of Matrices and Matrix Equations

We have explored **Gaussian elimination** in a previous lesson. This method is used to transform a system of equations into triangular form. Named after German mathematician Carl Friedrich Gauss (1777-1855).

$$\begin{aligned} x + y - z &= 1 \\ -y + z &= 5 \\ z &= 3 \end{aligned}$$

When using row operations to obtain triangulation of the matrix is called **reduced row echelon form** when the solutions are obtained.
In this lesson we will explore solving systems by using inverse functions.

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

The Inverse of a Matrix

Consider the real number equation $ax = b$. To solve this equation for x, multiply each side of the equation by the multiplicative inverse of a, that is, a^{-1} $(a \neq 0)$. This general idea is similar in matrix operations.

Definition of the Inverse of a Square Matrix

Let A be a square $n \times n$ matrix. If there exists an n × n matrix, A^{-1} , read "A inverse," where $AA^{-1} = A^{-1}A = I$, then A^{-1} is called the inverse of matrix A.

It should be noted that it is not always true that $AB = BA$. If A and B are both square matrices and $AB = I$, it can be shown that $BA = I$.

EX #1: Show that B is the inverse of A by showing $AB = I$ and $BA = I$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

Finding Inverse Matrices

A matrix A is called **invertible** (or **nonsingular**) if the matrix has an inverse, otherwise A is called **singular**. A nonsquare matrix cannot have an inverse. Not all square matrices have inverses, but if a matrix does have an inverse, that inverse is unique.

Also consider, if matrix A is of order $m \times n$ and matrix B is of order $n \times m$ where $m \neq n$, the products AB and BA are different orders and cannot be equal to each other.

Lesson 7.7: Partial Fractions

In this lesson, we will learn to write a rational expression as the sum of two or more simpler rational expressions. In this example, each fraction on the right side of the equation is a **partial fraction**. Together, they make up the **partial fraction decomposition** of the left side:

$$\frac{6x+2}{x^2+3x-10} = \frac{4}{x+5} + \frac{2}{x-2}$$

Decomposition of N(x)/D(x) into Partial Fractions

1. ***Divide if Improper.*** If the degree of $N(x) \geq$ degree of $D(x)$, divide the denominator into the numerator to obtain proper form.

$$\frac{N(x)}{D(x)} = (polynomial) + \frac{R(x)}{D(x)}$$

2. ***Factor denominator:*** Completely factor the denominator into factors of the form $(px+q)^m$ and $(ax^2+bx+c)^n$ where (ax^2+bx+c) is irreducible over the real numbers.

3. ***Linear factors:*** For *each* factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of *m* fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \ldots + \frac{A_m}{(px+q)^m}$$

4. ***Quadratic factors:*** For *each* factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of *n* fractions.

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \ldots + \frac{B_nx+C_n}{(ax^2+bx+c)^m}$$

The Basic Guidelines for Decompositions

1. Use long division to obtain proper form when the degree of the numerator degree is greater than the degree of the denominator.

2. Factor the denominator as a product of linear factors or irreducible quadratic factors.

3. Write a partial fraction with a constant numerator for each linear factor of the denominator.

4. Clear the denominator.

5. Equate coefficients to write a linear system.

6. Solve the system to find the partial fraction decomposition.

Note: If you go on to take a calculus course, this procedure will be helpful in the course for an operation called antidifferentiation.

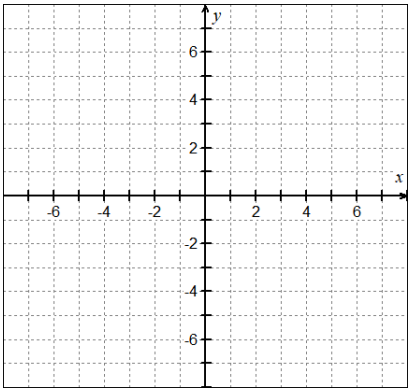
Lesson 7.8: Systems of Nonlinear Equations

Let's revisit systems of equations in which the equations are not linear. To solve a system of nonlinear equations, we can use graphing, substitution or elimination methods that we used in the earlier lessons for this unit.

Substitution and Elimination Methods

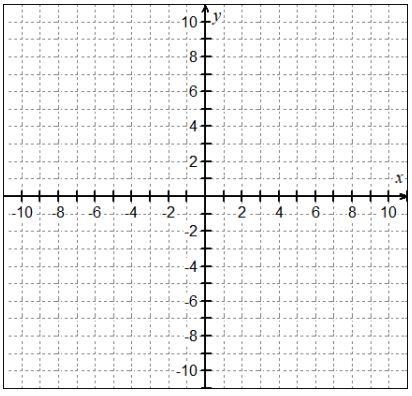
EX #1: Solve by substitution method.

$$\begin{cases} x^2 + y^2 = 25 \\ y = -2x - 5 \end{cases}$$



EX #2: Solve by elimination method.

$$\begin{cases} 6x^2 + 5y = 9 \\ 3x^2 + 2y = 9 \end{cases}$$



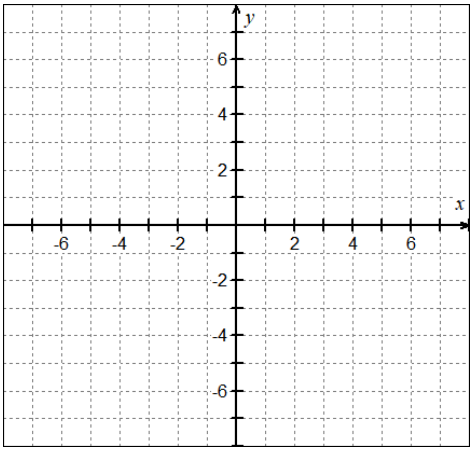
Lesson 7.9: Systems of Inequalities & Linear Programming

In this lesson we will explore systems of inequalities and applications of **optimization.** The strategy of finding minimum and maximum values of a quantity, given a set of constraints, is known as **linear programming.** A linear programming problem consists of a linear **objective function** and a system of linear inequalities called **constraints.**

Graphing Quadratic Inequalities

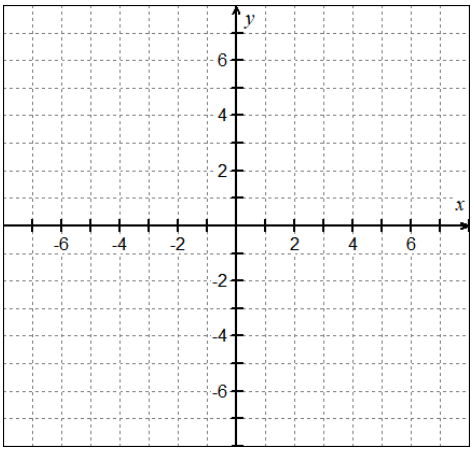
EX #1: Sketch the graph of the solution of the system.

$$\begin{cases} x - y^2 \geq 0 \\ x - y < 2 \end{cases}$$



EX #2: Sketch the graph of the solution of the system.

$$\begin{cases} y \geq x^2 - 4x + 3 \\ y < -x^2 + 2x + 3 \end{cases}$$



Feasible Regions

The objective function gives the quantity that is to be maximized or minimized. The constraints determine the set of **feasible solutions.** A **feasible region** is the shaded polygonal area created by the intersection of the graphs of inequalities. It is the location where every constraint is satisfied.

Any region that can be contained within a defined region or space by the vertices of a feasible region is said to be **bounded**.

