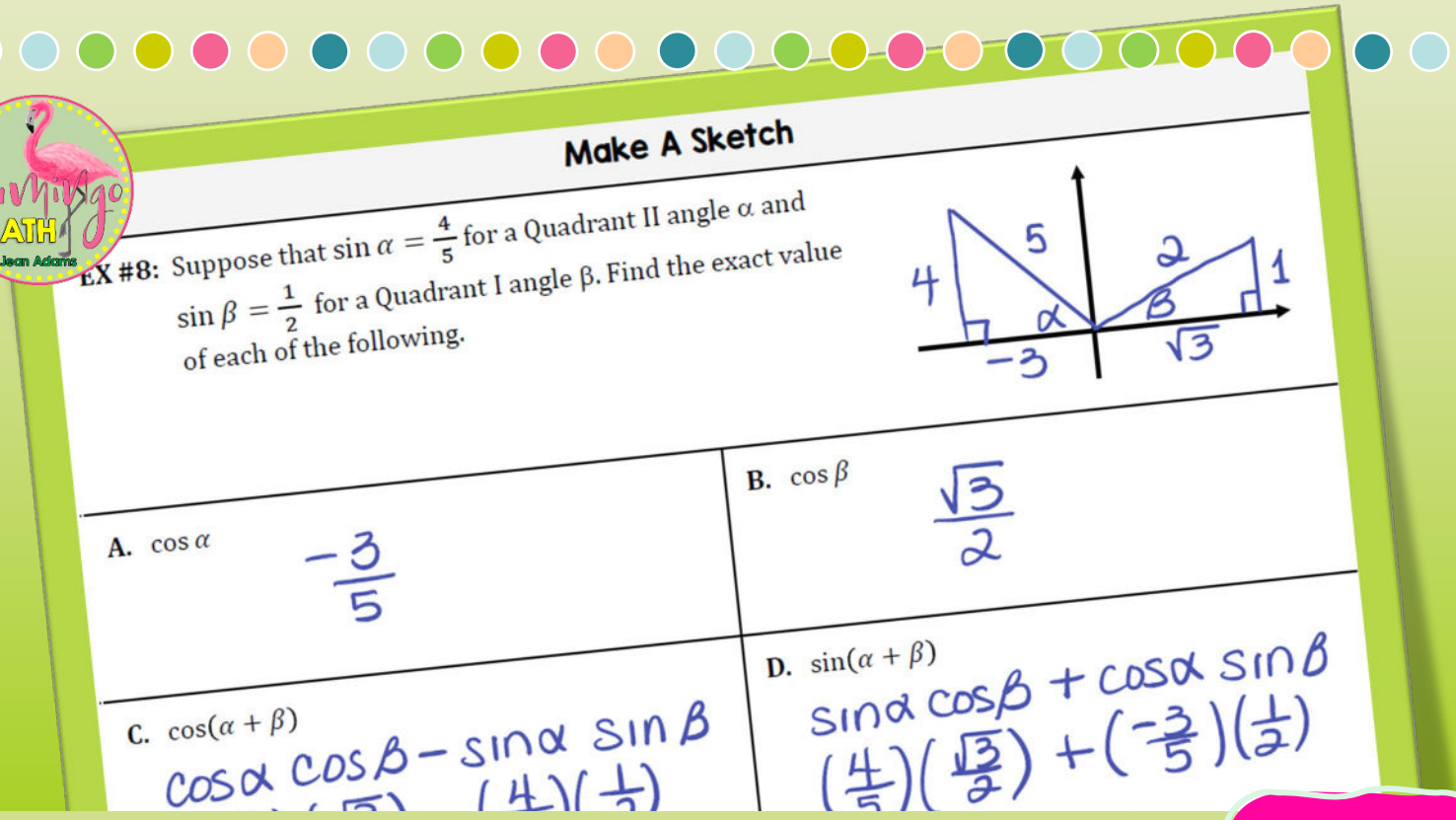
Precalculus Honors

Sum & Difference Identities



Completely Editable

5.3

Lesson 3: Sum and Difference Identities

In this section we use formulas that involve sum and differences to verify identities . . .

Six Formulas:

Sums	Differences
$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$	$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$	$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

What patterns do you see that could help you memorize these formulas?

Sums and Differences of an Angle

EX #1: Find the exact value of $\cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ$

EX #2: Write the following expressions as the sine or cosine of an angle.

A. $\sin 22^\circ \cos 13^\circ + \cos 22^\circ \sin 13^\circ$

B. $\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$

EX #3: To find the exact value for $\cos 15^\circ$ use the fact that $\cos 15^\circ = \cos(45^\circ - 30^\circ)$ and follow the difference formula for cosines.

Verify an Identity

EX #4: Verify the identity:

$$\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta}=1+\tan\alpha\tan\beta$$

EX #5: You try: $\sin\left(\frac{\pi}{2}-x\right)=\cos x$

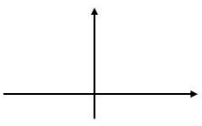
Sine and Cosine Formulas

EX #6: Find the exact value of $\sin\frac{5\pi}{12}$ using the fact that $\frac{5\pi}{12}=\frac{\pi}{6}+\frac{\pi}{4}$

EX. #7: Find the exact value of $\cos\frac{7\pi}{12}$ using the fact that $\frac{7\pi}{12}=\frac{\pi}{4}+\frac{\pi}{3}$

Make A Sketch

EX #8: Suppose that $\sin\alpha=\frac{4}{5}$ for a Quadrant II angle α and $\sin\beta=\frac{1}{2}$ for a Quadrant I angle β. Find the exact value of each of the following.



A. $\cos\alpha$

B. $\cos\beta$

Reduction Formulas

If one of the angles in a sum or difference is a quadrantal angle, then the sum or difference identity yields a single-termed expression. Since the result is a less complicated expression the resulting identity is called a ***reduction formula****.*

EX #9: Prove the reduction formula:

$\sin(x+\pi)=-\sin x$

$\cos\left(x+\frac{3\pi}{2}\right)=\sin x$

Tangent Formulas

EX. #10: Find the exact value of $\tan\frac{5\pi}{12}$ if $\alpha=\frac{2\pi}{3}$ and $\beta=\frac{\pi}{4}$

NOTES

5.3 **Sum and Difference Identities Homework**

Name _____

Date _____ Period _____

Problems 1 – 6, Use sum or difference identity to find an exact value for each expression.

1. $\cos 15°$
2. $\sin 75°$
3. $\tan 105°$
4. $\sin \frac{\pi}{12}$
5. $\cos \frac{7\pi}{12}$
6. $\tan \frac{11\pi}{12}$

Problems 7 – 14, Find the exact value of each expression.

7. $\cos 87° \cos 33° - \sin 87° \sin 33°$
8. $\sin 20° \cos 80° - \cos 20° \sin 80°$
9. $\sin 22° \cos 8° + \cos 22° \sin 8°$
10. $\cos 40° \cos 10° + \sin 40° \sin 10°$
11. $\sin \frac{\pi}{12} \cos \frac{5\pi}{12} + \sin \frac{5\pi}{12} \cos \frac{\pi}{12}$
12. $\cos \frac{7\pi}{18} \cos \frac{\pi}{18} + \sin \frac{7\pi}{18} \sin \frac{\pi}{18}$
13. $\frac{\tan 40° - \tan 10°}{1 + \tan 40° \tan 10°}$
14. $\frac{\tan 25° + \tan 20°}{1 - \tan 25° \tan 20°}$

Problems 15 – 16, Use the given conditions to find the exact value of each of the following.

A) $\sin(\alpha + \beta)$ **B)** $\cos(\alpha + \beta)$ **C)** $\sin(\alpha - \beta)$ **D)** $\tan(\alpha - \beta)$

15. $\sin \alpha = \frac{5}{13}; \frac{\pi}{2} < \alpha < \pi$; $\tan \beta = \sqrt{3}; 0 < \beta < \frac{\pi}{2}$
16. $\cos \alpha = \frac{1}{2}; 0 < \alpha < \frac{\pi}{2}$; $\sin \beta = \frac{1}{3}; 0 < \beta < \frac{\pi}{2}$

Problems 17 – 18, Find the exact value of each under the given conditions.

17. If $\sin \theta = \frac{1}{3}$, θ in Quadrant II, find $\sin\left((\theta + \frac{\pi}{6}\right)$
18. $\cos \theta = \frac{1}{4}$, θ in Quadrant IV, find $\cos\left(\theta + \frac{\pi}{3}\right)$

Problems 19 – 26, Prove each identity.

19. $\sin(\pi - \theta) = \sin \theta$
20. $\sin(\pi + \theta) = -\sin \theta$
23. $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$
24. $\frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} = \tan \alpha + \tan \beta$
25. $\sin(\alpha - \beta) \sin(\alpha + \beta) = \sin^2 \alpha - \sin^2 \beta$

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____

Lesson 5.3: Sum and Difference Formulas **Warm-Up # 1**

1. Use identities to find the exact value:

$$\cos\left(\frac{5\pi}{3}\right)\cos\left(\frac{4\pi}{3}\right)+\sin\left(\frac{5\pi}{3}\right)\sin\left(\frac{4\pi}{3}\right)$$

2. Use identities to find the exact value:

$$\sin(60°)\cos(150°)+\cos(60°)\sin(150°)$$

3. Use identities to find the exact value:

$$\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{5\pi}{4}\right)+\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{5\pi}{4}\right)$$

4. Find the exact value:

$$\frac{\tan(145°)-\tan(25°)}{1+\tan(145°)\tan(25°)}$$

Name _____ Date _____

Lesson 5.3: Sum and Difference Formulas

1. Use identities to find the exact value:

$$\cos\left(\frac{\pi}{6}\right)\cos\left(\frac{7\pi}{6}\right)-\sin\left(\frac{\pi}{6}\right)\sin\left(\frac{7\pi}{6}\right)$$

2. Use identities to fin

$$\sin(240°)\cos(105°)$$

3. Use identities to find the exact value:

$$\sin\left(\frac{5\pi}{3}\right)\cos\left(\frac{2\pi}{3}\right)-\cos\left(\frac{5\pi}{3}\right)\sin\left(\frac{2\pi}{3}\right)$$

4. Find the exact value

$$\frac{\tan(175°)+\tan(5}{1-\tan(175°)\tan(}$$

Daily Lesson Quiz

5.3 Sum & Difference Identities **Form A**

Name _____

Date _____ Period _____

1. Use identities to find the exact value:

$$\cos\frac{4\pi}{3}\cos\frac{\pi}{6}+\sin\frac{4\pi}{3}\sin\frac{\pi}{6}$$

2. Use identities to find the exact value:

$$\sin 85°\cos 140°+\cos 85°\sin 140°$$

3. Use identities to find the exact value:

$$\sin\frac{13\pi}{6}\cos\frac{\pi}{2}-\cos\frac{13\pi}{6}\sin\frac{\pi}{2}$$

Grade:

5.3 Sum & Difference Identities **Form B**

Name _____

Date _____ Period _____

1. Use identities to find the exact value:

$$\cos\frac{4\pi}{3}\cos\frac{\pi}{2}+\sin\frac{4\pi}{3}\sin\frac{\pi}{2}$$

2. Use identities to find the exact value:

$$\sin 32°\cos 13°+\cos 32°\sin 13°$$

3. Use identities to find the exact value:

$$\sin\frac{\pi}{2}\cos\frac{\pi}{3}-\cos\frac{\pi}{2}\sin\frac{\pi}{3}$$

Grade: