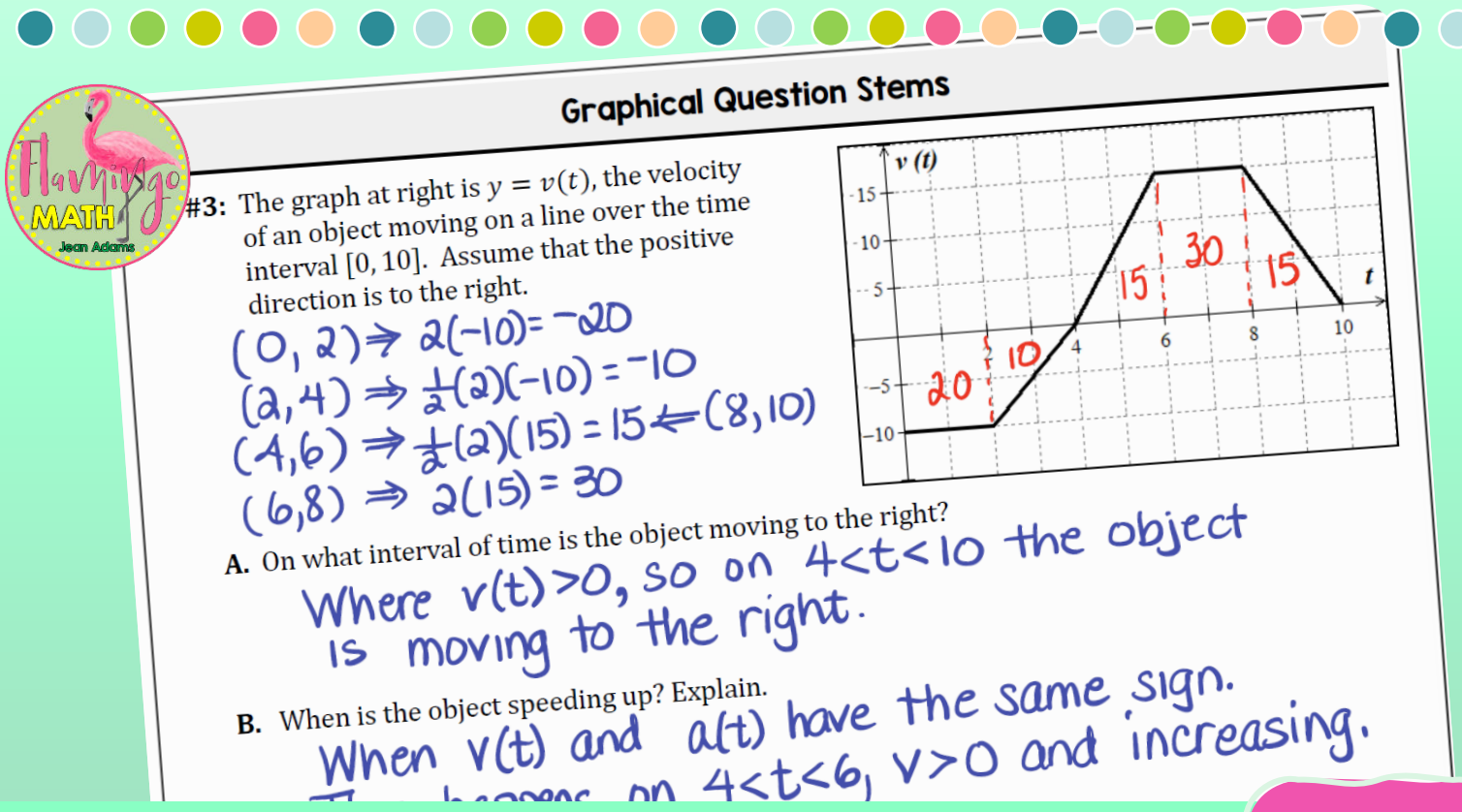
AP® Calculus

Straight Line Motion



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4.2

Lesson 2: Straight Line Motion
Connecting Position, Velocity & Acceleration

Topic 4.2: Straight Line Motion - Connecting Position, Velocity & Acceleration

In Unit 2, Lesson 7, the big ideas of position, velocity and acceleration were introduced. Particle motion problems deal with motion along a straight line. This includes horizontal and vertical motion of a falling object. It's important to note that this motion means the particle is moving along an axis, not along the graph of the function.

Vocabulary Review

The vocabulary terms for this lesson are restated here for your convenience.

Position	$s(t), x(t), y(t)$ tells where the particle is located at a given time, t.
Average Velocity	The average rate of change over $a \le t \le b$ $v_{avg.} = \frac{\Delta s}{\Delta t} = \frac{\text{displacement}}{\text{elapsed time}} = \frac{s(b) - s(a)}{b - a} = m_{\text{secant}}$
Instantaneous Velocity	$s'(t) = v(t)$ is rate of change of position at a given time and direction of movement. $\frac{ds}{dt} = s'(t) = v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$
Speed	The rate at whic
Acceleration	$s''(t) = v'(t) =$ given time. The s''
Displacement over an interval $[a, b]$	$s(b) - s(a)$ is th
Distance Traveled	The sum of the a points.
Free Fall Motion	$s(t) = -\frac{1}{2}gt^2 +$ influence of grav
Direction changes	When velocity ch to positive at a p zero or even be for direction to c
Gravitational Constants	9.8 $\frac{m}{sec^2}$ or 32 $\frac{ft}{sec}$

The focus of this lesson is to make connections and transition between position, velocity and acceleration functions as they relate to the skills and rigor on the AP Calculus Exam®. We will look at algebraic/ numerical problems, table problems, and graphical interpretations related to straight line motion in this lesson.

Algebraic and Numeric Question Stems

EX #1: A particle moves along the x-axis so that at any time $t \ge 0$, the position is given by the function $x(t) = t^3 - \frac{9}{2}t^2 - 12t + 5$

A. At $t = 0$, determine whether the particle is moving to the right or to the left. Explain how you know.

B. At what time(s) does the particle change directions. Justify your answer.

creasing or decreasing?

easing or decreasing. Justify.

1/4? Explain the meaning of your answer in
ld units be explained in context to the particle?

Table Question Stems

EX #2: The table below gives the velocity, in feet per second, of a particle moving along a horizontal axis. The velocity v is a twice-differentiable function of time t.

Time, t (sec)	0	2	3	5	7	10	12	15
Velocity, $v(t)$ (ft/sec)	-4	-1	1	2	3	5	3	6

A. At $t = 0$, determine whether the particle is moving to the right or to the left. Explain how you know.

B. Is there any time(s) during the interval $0 \le t \le 15$ seconds when the particle is at rest? Explain your answer.

C. Use the data from the table to find an approxim
$v'(8)$ in terms of the particle motion. Show the
and indicate the units of measure.

D. Find the average acceleration of the particle fo
number means in terms of the particle's velocit

E. Let $a(t) = v'(t)$. Is there a time $t = c$ guarante
$0 \le t \le 15$? Justify your answer.

Graphical Ques

EX #3: The graph at right is $y = v(t)$, the velocity of an object moving on a line over the time interval $[0, 10]$. Assume that the positive direction is to the right.

A. On what interval of time is the object moving t

B. When is the object speeding up? Explain.

C. How fast is the object moving at time $t = 3$, a

D. What is the average velocity over the time interval $0 < t < 4$ and $4 < t < 10$? Show the computations that lead to your answer.

E. What is the total distance traveled by the object? Show the computations that lead to your answer.

H. On what interval(s) does the object move at a constant speed?

Calculator Question Stems

EX #4: An object moves along the x-axis for time $0 \le t \le 6$. The velocity of the object at time t is given by $v(t) = \ln(t) - 0.15t^3 + (t^2 + 2t)^{4/5}$.

A. For $t = 4$, is the object speeding up or slowing down? Justify.

B. At what time on $0 \le t \le 6$ does the particle change directions? Justify.

EX #5: A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given in meters per second by the function $v(t) = 4 + 2.3\sin(0.85t)$. What is the acceleration of the particle at $t = 4$? What does this tell you about the direction of the particle?

EX #6: An object moves along a line so that at time t, $0 \le t \le \pi$, its position is given by $s(t) = -3\sin(0.25t^2) + 1$. What is the velocity of the object when its acceleration is zero? Describe the behavior of the object.

EX #7: A particle moves along the x-axis so that at time t, $0 \le t \le 15$, the velocity of the particle is given by $v(t) = \tan(\sqrt[3]{t} - 0.2t)$.

NOTES

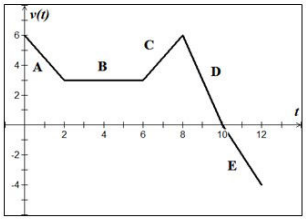
4.2 **Straight Line Motion - Connecting Position, Velocity & Acceleration Homework**

Name _____

Date _____ Period _____

Problems 1 – 2, Use the velocity vs. time graph below to answer the questions.

1. In the velocity graph, at right, let the units for velocity be in feet per second.



	A	B	C	D	E
Acceleration (slope) (+, –, 0)					
Acceleration (yes/no)					
Velocity/Acceleration (+, –)					
Speed up, slow down, constant					

2. Use the graph of the velocity function above to find
lead to your answer, include units.

A. How far did the particle move in the first 10 se

B. What is the average velocity over the 12 second

C When does the particle change directions?

Problems 3 – 8, Evaluate each scenario, in context.

3. If a calculus book is dropped from a height of 100 feet, its height s at time t is given by the position function $s(t) = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity over the time interval [1, 2].

4. Hayden is standing on a ledge that is 32 feet above a mountain stream. At time $t = 0$, he jumps from the ledge into the water below. Hayden's position at any time $t \geq 0$ can be modeled by the function $s(t) = -16t^2 + 16t + 32$, where s is measured in feet, and t is measured in seconds.

A. When does Hayden hit the water?

B. What is Hayden's velocity at impact?

ce of the earth with an initial velocity of 120
the rocket at $t \geq 0$ is modeled by the function

B. When does the rocket hit the ground?

D. At $t = 15$ seconds, is the rocket speeding up or slowing down? Justify.

6. A drone delivery service drops a package when the drone is 120 ft above the ground, rising at 12 ft/sec. The position function is $s(t) = -16t^2 + 12t + 120$.

A. How long will it take for the package to hit the ground?

B. What is the speed of the package at impact?

7. A potato is fired vertically upward from a potato cannon with an initial velocity of 84 ft/sec from a building that is 224-feet high. The position function is given by $s(t) = -16t^2 + 84t + 224$.

A. How long will it take for the potato to reach its maximum height?

B. What is the maximum height?

C. How long will it take the potato to reach its starting height on the way down?

8. A particle is moving along a line so that at any time
particle can be modeled by $s(t) = 5 - 0.5t^2 - 3$ co
what is the velocity of the object?

9. Randy the Roach is back. He is moving along a hori
counter at 2 a. m.) in a smooth and continuous ma
minute, is a differentiable function and has been tr
system. Randy's velocity is shown for $0 \leq t \leq 10$

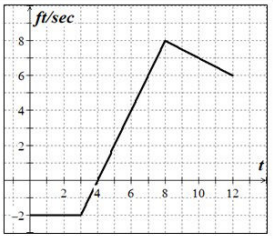
t	0	0.5	1	2	2.5	4
$v(t)$	−1.7	0.5	0.5	.75	−0.5	−0.25

A. At what time(s) is Randy moving to the left? Exp

B. Is there any time(s) during the interval $0 \leq t \leq$
your answer.

C. Use the data from the table to find an approximation for $v'(7)$. Explain the meaning of $v'(7)$ in terms of Randy's movement. Show the computations that lead to your answer, and indicate the units of measure.

10. The graph at right shows the velocity $v(t)$ of a particle, in ft/sec, moving along a horizontal line for time, $0 \leq t \leq 12$ seconds.



A. On what open intervals or at what time $0 < t < 12$ is the particle at rest? Justify.

B. On what open intervals $0 < t < 12$ is the particle moving to the right? Justify.

C. On what open intervals or at what time(s) $0 < t < 12$ is the particle's speed increasing?

D. Does the particle slow down? If so, when? Explain how you know.

E. How far does the particle move during the first four seconds, and in which direction? Show the computations that lead to your answer.

Practice Sets

4.2 Straight Line Motion — Form A

Name _____

Date _____ Period _____

1. For time $t \geq 0$, the velocity of a particle is given by $v(t) = 3t^4 - 3t^3 + 5$. How many times is the particle's acceleration equal to zero?

2. For time $t \geq 0$, a particle moves along the x-axis, its position is given by the function $s(t) = -t^4 - 4t^3 + 18t^2 + 4$. At what time does the particle obtain its maximum velocity?

3. For time $t \geq 0$, a particle moves along the x-axis, its position is given by the function $s(t) = -t^3 + 3t^2 - 6$. What is the velocity when acceleration is zero?

Grade:

4.2 Straight Line Motion — Form B

Name _____

Date _____ Period _____

1. For time $t \geq 0$, the velocity of a particle is given by $v(t) = -t^4 + 8t^2 + 12$. How many times is the particle's acceleration equal to zero?

2. For time $t \geq 0$, a particle moves along the x-axis, its position is given by the function $s(t) = -t^4 + 2t^3 + 36t^2 + 5$. At what time does the particle obtain its maximum velocity?

3. For ti
x-axis
funct
veloc

Grade:

4.2 Straight Line Motion — Form C

Name _____

Date _____ Period _____

1. For time $t \geq 0$, the velocity of a particle is given by $v(t) = 3t^4 - 6t^2 + 8$. How many times is the particle's acceleration equal to zero?

2. For time $t \geq 0$, the position of a particle is given by $s(t) = -t^4 - 2t^3 + 36t^2 + 9$. At what time does the particle obtain its maximum velocity?

3. For time $t \geq 0$, the position of a particle is given by $s(t) = -t^3 + 6t^2 - 12t + 4$. What is the velocity when acceleration is zero?

Grade:

4.2 Straight Line Motion — Form D

Name _____

Date _____ Period _____

1. For time $t \geq 0$, the velocity of a particle is given by $v(t) = t^4 - 3t^3 + 5$. How many times is the particle's acceleration equal to zero?

2. For time $t \geq 0$, the position of a particle is given by $s(t) = -t^4 + 6t^3 + 24t^2 + 15$ At what time does the particle obtain its maximum velocity?

3. For time $t \geq 0$, the position of a particle is given by $s(t) = t^3 - 3t^2 - 9t + 6$. What is the velocity when acceleration is zero?

Grade:

Daily Lesson Quiz