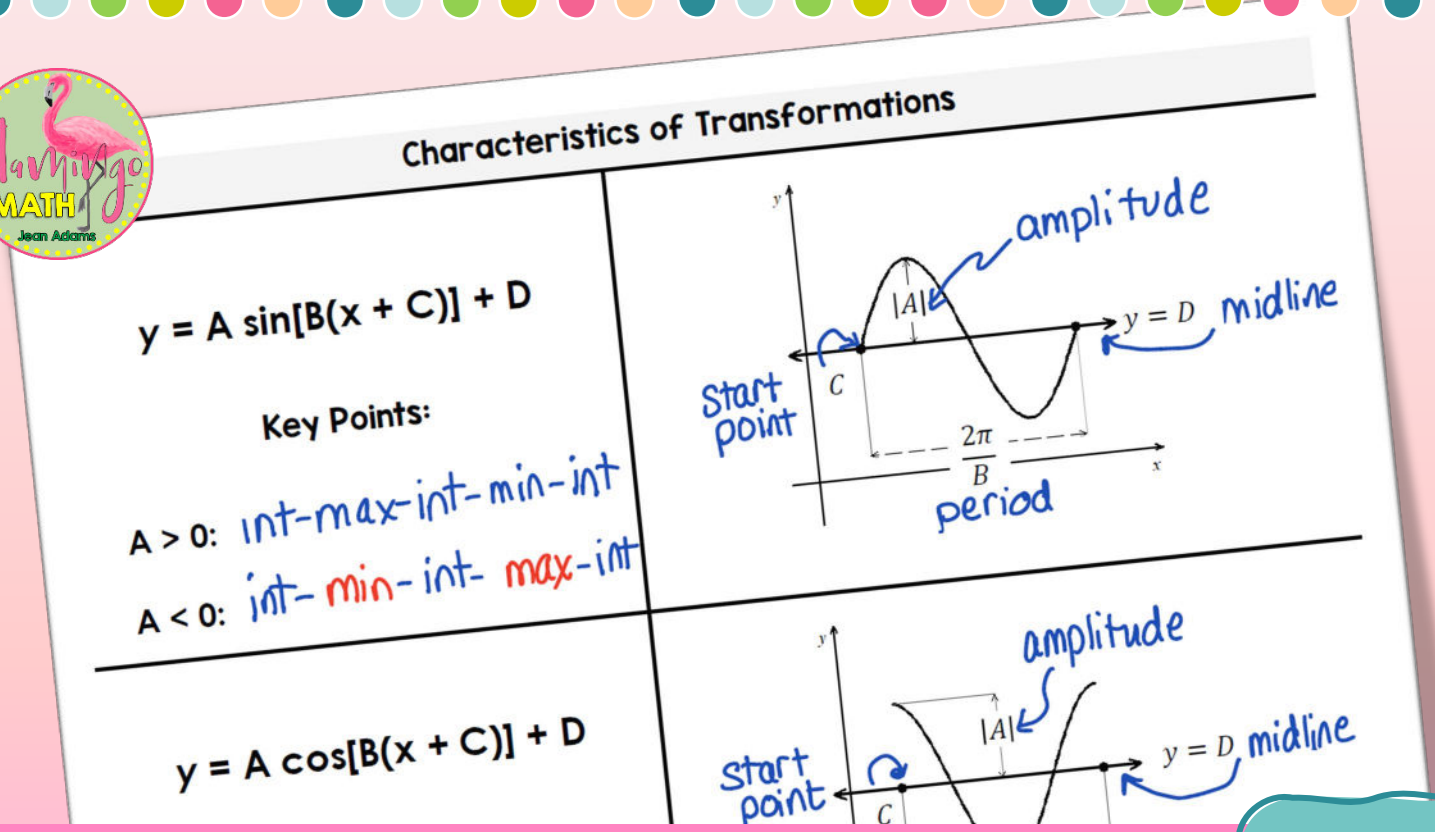
AP® Precalculus

Sinusoidal Function Transformations



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Lesson 6-A: Sinusoidal Function Transformations

Graphing the basic sine and cosine functions in a rectangular coordinate system should now be a comfortable task for you to perform. Let's complete or toolkit by considering more advanced transformations of the parent functions.

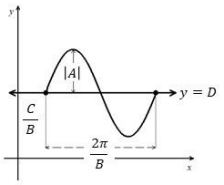
Characteristics of Transformations

$y = A\sin(Bx - C) + D$

Key Points:

A > 0:

A < 0:



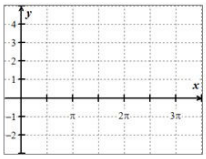
$y = A\cos(Bx - C) + D$



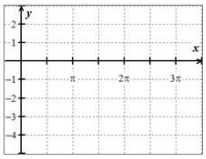
Vertical Shift (Midline)

If you add or subtract a number from the entire equation you can shift a trigonometric graph "up or down." This is a shift in the x-axis, which creates a new "midline" for the graph.

EX #3: Graph $y = \sin x + 1$



EX #4: Graph $y = \cos x - 2$



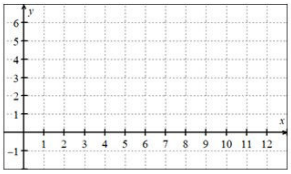
Therefore, if $y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$ then, the vertical shift of the entire graph is equal to D. The midline of the graph is $\boldsymbol{y = D}$.

Summary

EX #5: Given the standard form of a sine or cosine function, complete the summary in the

More Graphing

EX #9: Determine the amplitude, period, and midline of $y = -2\cos\left(\frac{\pi}{3}x\right) + 3$. Then, graph the function from $0 \le x \le 12$.



Writing Functions from Characteristics

EX #10: Write the equation for the given function that has the following characteristics.

A. Sine function

Amplitude: 3 Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{3}$ right Vertical shift: none

B. Cosine function

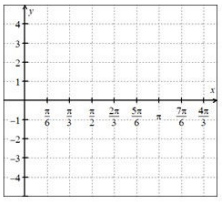
Amplitude: 5 Period: 4π

Phase Shift: $\frac{\pi}{4}$ left Vertical shift: 2 down

Graphing by Key Points y = A sin (Bx - C)

The graph of $y = A\sin(Bx - C)$ is a horizontal shift of the graph of $y = A\sin(Bx)$. The starting point of the cycle is shifted from $x = 0$ to $x = \frac{C}{B}$. We can find an interval containing one cycle by solving the argument for the period of 2π. This will give you the starting point and the ending point of one period.

EX #1: Graph $y = 3\sin\left(3x - \frac{\pi}{2}\right)$



Graphing by Key Points y = A cos (Bx - C)

EX #2: $y = 2\cos\left(\frac{x}{2} + \pi\right)$

Lesson 6-B: Sinusoidal Function Transformations

Consider how you evenly space the five key points. It is possible to summarize this procedure using two different methods.

Finding an Interval for the First Cycle

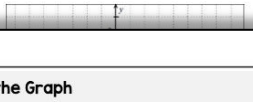
EX #6: We can find the endpoints for the first cycle of a trigonometric function in two ways. Let's explore.

	Method #1	Method #2
Period Begins:		
Period Ends:		

NOTE: *Use the first and last points of one cycle to divide the interval into four equal segments.*

EX #7: Find the amplitude, period and phase shift for $y = 2\cos(2x + 3\pi)$. Graph the function.

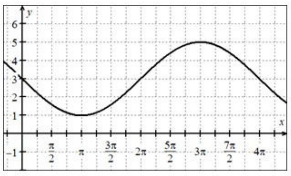
Amplitude:



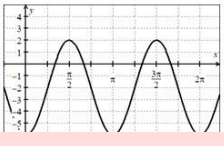
Writing Equations Given the Graph

EX #12: Find an equation for each graph. State the amplitude, period, phase shift, midline. Verify your equation with a graphing utility. State the maximum and minimum points in one period and the points of inflection.

A.



B.



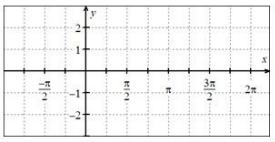
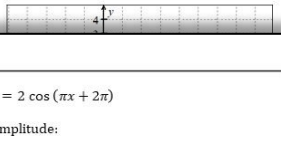
Guided Notes

3.6 **Sinusoidal Function Transformations Practice Set 1**

Name _____

Date _____ Period _____

Problems 1-8, Graph each equation.

Equation	Characteristics	Graph
1. $y = 2\cos\left(x + \frac{\pi}{2}\right)$	Amplitude: Period: Phase Shift:	
2. $y = -3\sin(x - \pi)$	Amplitude: Period: Phase Shift:	
3. $y = 2\cos\left(x - \frac{\pi}{2}\right)$	Amplitude: Period: Phase Shift:	
4. $y = 4\sin[2(x - \pi)]$	Amplitude: Period: Phase Shift:	

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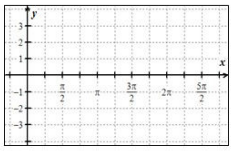
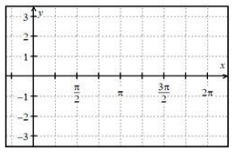
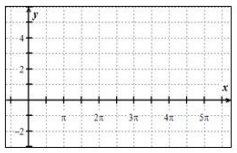
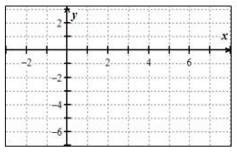
Equation	Characteristics	Graph
5. $y = \sin\left(x - \frac{\pi}{4}\right) + 1$	Amplitude: Period: Phase Shift: Midline:	
6. $y = -\cos\left[2\left(x - \frac{\pi}{2}\right)\right]$	Amplitude: Period: Phase Shift:	
		
		

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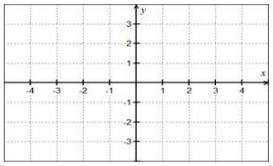
7. $y = 2\cos(\pi x + 2\pi)$

Amplitude:

Period:

Phase shift:

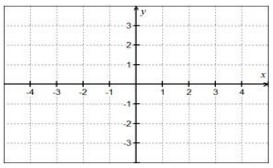


8. $y = -3\sin(\pi x - 4\pi)$

Amplitude:

Period:

Phase shift:

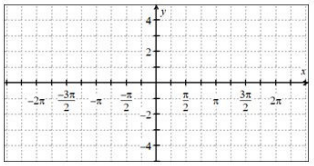


9. $y = -4\cos\left(\frac{1}{2}x - \pi\right)$

Amplitude:

Period:

Phase shift:



10. $y = 2\sin\left(\frac{x}{2} + \frac{3\pi}{2}\right)$

Amplitude:

Period:

Phase

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3.6 **Sinusoidal Function Transformations Practice Set 2**

Problems 1 – 4, Write the equation for the

1. Sine function

Amplitude: 4 Period: π

Phase Shift: $\frac{1}{2}$ Vertical shift: no

3. Cosine function

Amplitude: 5 Period: 3π

Phase Shift: $-\frac{1}{3}$ Vertical shift: 2

Amplitude: 3 Period: $\frac{\pi}{2}$

Phase Shift: -2 Vertical shift: 4

Problems 5–10, Find amplitude, period, and phase shift of each function. Graph the function.

5. $y = 2\cos(3x - \pi)$

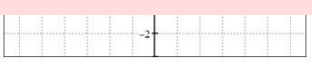
Amplitude:

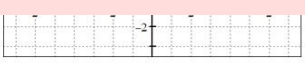
Period:

6. $y = -2\sin\left(2x - \frac{\pi}{2}\right)$

Amplitude:

Period:





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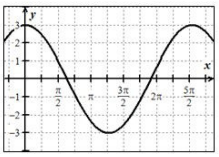
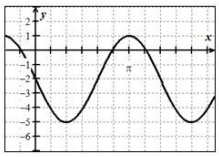
Problems 11-12, Find a, b, c, and d for each graph. Then, write an equation for each graph.

11.

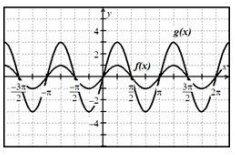


12.



Problems 13- 16, Describe the relationship between the graphs of f and g. Consider amplitudes, periods, and shifts.

13.



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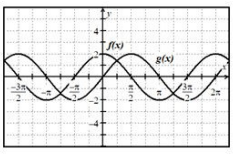

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Practice Sets

3.6 Sinusoidal Function Transformations Set 1 **Form A**

Name _____

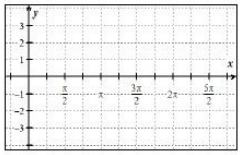
Date _____ Period _____

State the characteristics and draw the graph.

1. $y = \cos\left[2\left(x + \frac{\pi}{4}\right)\right] - 1$

Amplitude: Period:

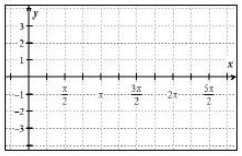
Phase Shift: Midline:



2. $y = -3\sin\left(x - \frac{\pi}{2}\right)$

Amplitude: Period:

Phase Shift: Midline:



3. List the characteristics, but don't draw the graph. $y = -5\sin\left[\frac{\pi}{4}(x - 3)\right] + 2$

Amplitude: Period:

Phase Shift: Midline:

Grade:

3.6 Sinusoidal Function Transformations Set 1 **Form B**

Name _____

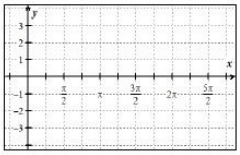
Date _____ Period _____

State the characteristics and draw the graph.

1. $y = \sin\left[4\left(x - \frac{\pi}{4}\right)\right] + 2$

Amplitude: Period:

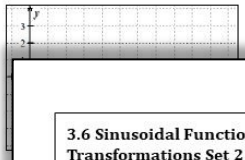
Phase Shift: Midline:



2. $y = -2\cos\left(x + \frac{\pi}{2}\right)$

Amplitude: Period:

Phase Shift: Midline:



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3.6 Sinusoidal Function Transformations Set 2 **Form A**

Name _____

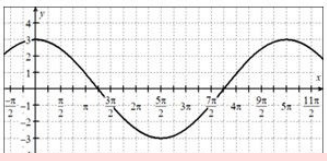
Date _____ Period _____

Write the equation for the given function that has the following characteristics.

1. Sine function
Amplitude: 3 Period: 3π
Phase Shift: $-\frac{2}{3}$ Vertical Shift: 5

2. Cosine function
Amplitude: 1 Period: 4π
Phase Shift: 6 Vertical Shift: -3

3. Find *a*, *b*, *c*, and *d* for the graph. Then write an equation for the graph.



Grade:

3.6 Sinusoidal Function Transformations Set 2 **Form B**

Name _____

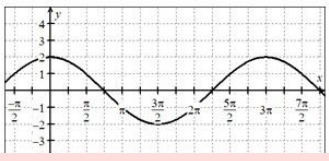
Date _____ Period _____

Write the equation for the given function that has the following characteristics.

1. Cosine function
Amplitude: 2 Period: 5π
Phase Shift: $-\frac{1}{3}$ Vertical Shift: 4

2. Sine function
Amplitude: 1 Period: 6π
Phase Shift: 2 Vertical Shift: -5

3. Find *a*, *b*, *c*, and *d* for the graph. Then write an equation for the graph.



Grade:

Daily Lesson Quiz