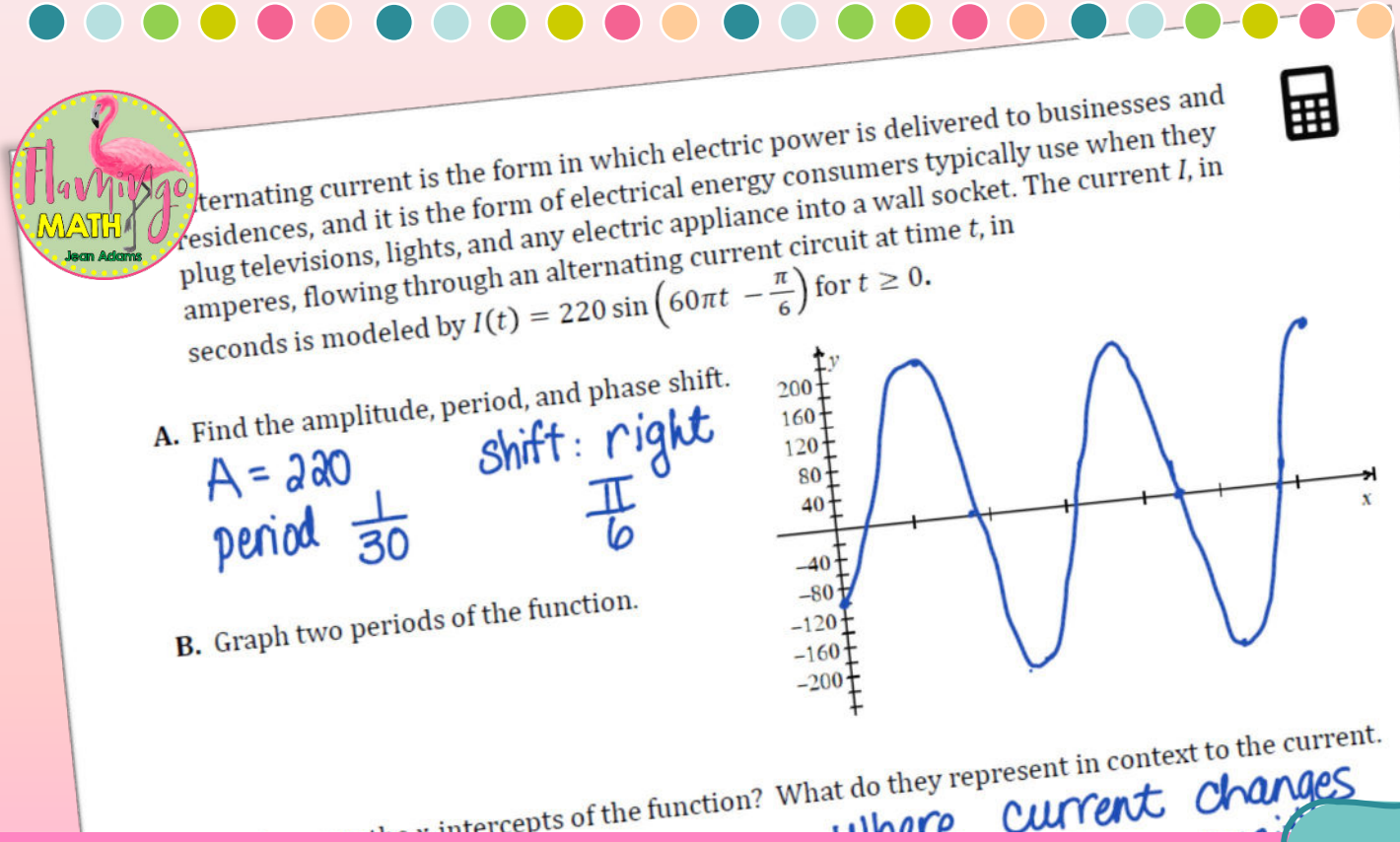
AP® Precalculus

Sinusoidal Function

Context and Data Modeling



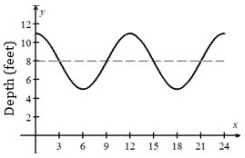
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3.7

Lesson 7-A: Sinusoidal Function Context and Data Modeling

Now we can put together the concepts from the last few lessons to write equations using sine or cosine functions given a verbal description of periodic behavior. Then we can use the equation as a mathematical model to make predictions and interpretations related to real-world phenomena.

EX #1: The figure at right shows the depth of water along a seawall. The depth is 5 feet at low tide and 11 feet at high tide. On a given day, low tide occurs at 6 a.m. and high tide at noon.



Number of Hours after Midnight

A. If d represents the depth of the water t hours after midnight, use a cosine function of the form $d(t) = A\cos(Bt) + D$ to model the depth of the water.

B. Use your model to find the interval(s) of time when the tide is rising.

EX #2: The height above the ground of a yo-yo can be modeled by the equation $h(t) = 1.5\cos(\pi t) + 2.25$, where h represents the height of the yo-yo in feet above the ground and t represents time in seconds since the yo-yo was first dropped from its maximum height.

A. When does the yo-yo return to the maximum height for the first time?

B. Find the maximum and minimum heights reached above the ground by the yo-yo.

Simple Harmonic Motion

An object that moves on a coordinate axis is in **simple harmonic motion** if its distance from the origin, d, at time t is given by either

$d = a\cos(\omega \cdot t)$	$d = a\sin(\omega \cdot t)$

The motion has **amplitude** $|a|$, the maximum displacement of the object from its rest position. The **period** of the motion is, $\frac{2\pi}{\omega}$ where $\omega > 0$. The period gives the time it takes for the motion to go through one complete cycle. The motion has **frequency** $\frac{\omega}{2\pi}$, which is the number of oscillations per unit of time.

EX #3: A point on the tip of a tuning fork vibrates in harmonic motion described by the equation $d = 12\sin(\omega \cdot t)$. Find ω for a tuning fork that has a frequency of 386 vibrations per second.
Answer in π radians

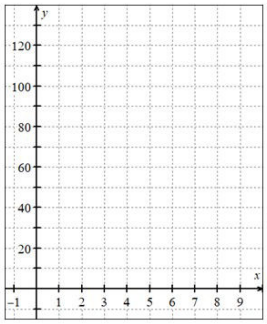


EX #4: A ball on a spring is pulled 6 inches below its rest position and then released. The period for the motion is 4 seconds. Write the equation for the ball's simple harmonic motion.

EX #5: A mass oscillating up and down on the bottom of a spring (assuming perfect elasticity and no friction or air resistance) can be modeled as harmonic motion. If the weight is displaced a maximum of 4 cm, find the modeling equation if it takes 3 seconds to complete one cycle. Assume the object starts at rest.

EX #6: A Ferris wheel is constructed such that a person gets onto the ride at its lowest point, five feet above the ground and the ride reaches its highest point at 125 feet above the ground. The amount of time it takes to complete one full rotation is equal to 8 minutes. The vertical position of the rider can be modeled as a function of time t in minutes since boarding by the equation $y = A\sin(Bt - C) + D$.

A. Sketch a graph of the rider's height with respect to time, for one cycle.



B. Find the values of A, B, C and D.

C. Write an equation to model the function.

D. At what time is the rider at 60 feet above the ground?

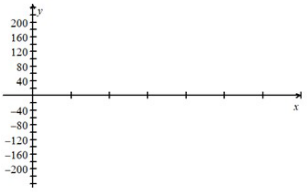
Lesson 7-B: Sinusoidal Function Context and Data Modeling

Let's continue our investigation of real-world applications related to sinusoidal behavior.

EX #7: Alternating current is the form in which electric power is delivered to businesses and residences, and it is the form of electrical energy consumers typically use when they plug televisions, lights, and any electric appliance into a wall socket. The current I, in amperes, flowing through an alternating current circuit at time t, in seconds is modeled by $I(t) = 220\sin\left(60\pi t - \frac{\pi}{6}\right)$ for $t \geq 0$.

A. Find the amplitude, period, and phase shift.

B. Graph two periods of the function.



C. Where are the x-intercepts of the function? What do they represent in context to the current.

EX #8: Cheryl opened a bakery and found that her monthly profit followed an up-and-down pattern during the first year. She discovered that her profit could be modeled by $P(t) = 2t - 7\sin\left(\frac{\pi}{3}t\right)$, where P is measured in hundreds of dollars and t was

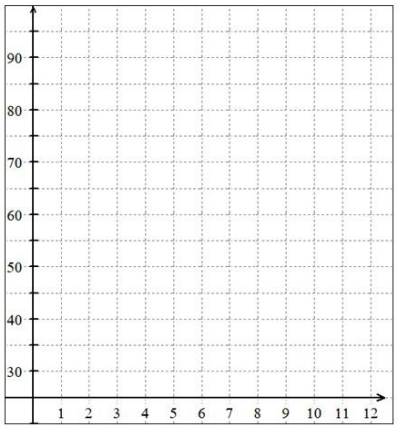
Guided Notes

Finding Sinusoidal Functions from Data

EX #10: The table below shows the average temperature per month for Atlanta, Georgia. Information was found at the World Climate website (www.worldclimate.com).

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
°F	51.6	55.9	63.7	72.7	80.1	86.2	88.3	87.4	82.0	72.7	62.6	54.0

A: Plot the data below, and notice that they approximate sinusoidal behavior.



B. Why does it make sense that average monthly temperatures can be modeled by a periodic function?

EX #9: Hurricanes are categorized using the Saffir-Simpson Hurricane Scale. The table below represents the number of major hurricanes in the Atlantic Basin that are rated as major storms of category 3,4, or 5 for each decade from 1921 to 2020.

A. Use a graphing utility to create a scatter plot of the data for $t = 1$ to $t = 10$ for the decades shown in the table below.

Decade, x	**Hurricanes**	**Decade, x**	**Hurricanes**
1921-1930	5	1971-1980	4
1931-1940	8	1981-1990	5
1941-1950	10	1991-2000	5
1951-1960	8	2001-2010	11
1961-1970	6	2011-2020	9

Source: U.S. National Oceanic and Atmospheric Administration

B. Find a sinusoidal regression function that best models the data.

C. What is the maximum and minimum number of storms based on your model?

D. In what decade will the next maximum number of storms occur based on your model?

C. Let's model this graph with a sine function in the form of $y = A\sin(Bx) + D$. Use the data plot to estimate the amplitude, period, vertical translations.

D. Use these values in the data to find a sinusoidal regression equation of the form $y = A\sin(Bx - C) + D$.

E. Explain the meaning of the points of inflection in context to the data.

F. Explain the meaning of the maximum and minimum values in the regression model as

Guided Notes

3.7 **Sinusoidal Function Context and Data Modeling Practice Set 1**

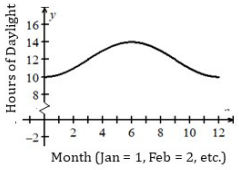
Name _____

Date _____ Period _____

Problems 1- 10, solve each problem using sinusoidal models.

1. The figure at right shows the number of hours of daylight in St. Augustine, Florida. Hours of daylight are at a minimum of 10 hours in December. Hours of daylight are at a maximum of 14 hours in June.

Let x represent the month of the year, where Jan. =1, Feb. = 2, and Dec. = 12. If y represents the number of hours of daylight in month x, use a sine function in the form $y = A\sin(Bx - C) + D$ to model the hours of daylight.



Month (Jan = 1, Feb = 2, etc.)

Explain the meaning of the points of inflection in context to the scenario.

2. The London Eye Ferris wheel has a diameter o
rotation every 30 minutes. Passengers board f
Write an equation that models the height abov

3. A person's blood pressure can be modeled by
represents the blood pressure at time t, meas
period and frequency.

4. The voltage V, produced in an alternating current generator can be modeled by $V(t) = 220\sin(120\,\pi t)$.

A. Find the amplitude and period.

B. If a resistance of $R = 20$ ohms is present, what is the current I? Use Ohm's Law $V = IR$.

C. Fine the amplitude and period of the current I.

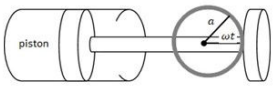
5. A buoy in the Gulf of Mexico sends a signal beacon to a Coast Guard station. The behavior of the buoy can be modeled by the function $h = a\sin(bt) + 4$, where h is measured in feet above sea level. During a recent tropical storm, the height varies from 1 foot to 8 feet with a 4.5 second interval between one 8-foot height to the next. Find the values of the constants a and b.

es of sunlight on the summer solstice, but only
inter solstice.

using the format:

mber of hours of sunlight on April 1, the 91st day

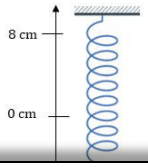
7. A piston operates by a wheel rotating at a constant rate, ω radians per second. The back-and-forth motion of the piston is an example of *simple harmonic motion*. In a mechanical piston, like shown at right, a wheel with a 6-cm radius turns with an angular velocity of 6π radians/second.



A. What is the frequency of the piston?

B. What is the distance from the starting position $t = 0$ exactly 2.4 seconds after starting?

8. A weight oscillating up and down on the bottom of a spring (assuming no air resistance or friction) can be modeled as harmonic motion.



A. If the weight is displaced a maximum of 8 cm, and it takes 2 seconds to complete one cycle, what is the frequency of the oscillating weight?

B. Find the modeling equation of the action if the spri
the origin when $t = 0$.

9. A point on the tip of a tuning fork vibrates in harmonic
by the equation $d = 16\sin(\omega t)$.

Find ω for a tuning fork that has a frequency of 560 vibrations per second.

Free Resp
Calculat

10. A weight is attached to the end of a long sprin
the distance from the floor varies sinusoidall
stopwatch reads 0.4 second, the weight first r
the floor. The next low point, 48 centimeters

A. Write an equation expressing the distance fr
the stopwatch reads.

B. Predict the distance from the floor when the s

C. What was the distance from the floor when you started the stopwatch?

D. Predict the first positive value of time at which the weight is 55 centimeters above the floor.

3.7 **Sinusoidal Function Context and Data Modeling Practice Set 2**

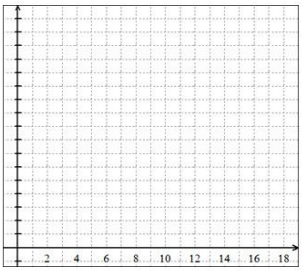
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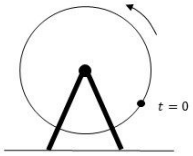
Date _____ Period _____

Problems 1- 8, solve each problem using sinusoidal models.

1. The Ferris Wheel at the Florida State Fair in Tampa is a popular ride that soars to a maximum height of 45 feet and a low point of 3 feet. When the last seat is filled and the Ferris Wheel begins to revolve, your seat is at the position marked in the figure below at $t=0$. Your stopwatch records that it takes you 4 seconds to reach the top, and that the wheel makes a complete revolution once every 12 seconds. The wheels diameter is 42 feet.

A. Graph the sinusoid.





B. Write an equation that models the behavior of the Ferris Wheel ride.

D. What is the value of t the second time that you are 15 feet above the ground?

Practice Sets

2. During a 90-day period, daily rainfall can be modeled by sinusoidal functions. If the rainfall fluctuates between a low of 2 inches on day 8 and 12 inches on day 53, during what period is daily rainfall more than 10 inches.

3. The possible hours of daylight in a given day is a function of the day of the year. In Atlanta, Georgia, the minimum hours of daylight (occurring on the Winter solstice) is 9.9 hours, and the maximum hours of daylight (during Summer Solstice) is equal to 14.4 hours.

A. If the hours of daylight can be modeled using a sinusoidal equation, what is the equations amplitude?

B. Where does the midline occur and what does this represent in context to the daylight hours in Atlanta, Georgia?

4. Outside temperatures over the course of a day can be modeled as a sinusoidal function. Suppose the high temperature of 105°F occurs at 5PM and the average temperature for the day is 85°F. Find the temperature, to the nearest degree, at 9:00 a.m.

5. Chase wanted to lose weight and noticed that his weight loss program W, over a two-year period, followed a curve that could be modeled by the function $W(t) = 250 - 1.5t + 9.82\sin\left(\frac{\pi}{4}t\right)$, where t was measured in months after January 1 and W was measured in pounds.

A. What did Chase weight initially? What was his weight at the end of two years?

B. What was his maximum weight during the two-year period?

C. What was his minimum weight during the two-year period?

6. The length of time between consecutive high tides is 12 hours and 25 minutes according to NOAA. On Friday, April 7, 2023, high tide occurred at 9:30 a.m. (9.5 hours) and low tide occurred at 3:27 p.m. (15.45 hours). Water heights are measured as the amount above or below the mean lower low water. The height of the water at high tide was 6.85 feet, and the height of the water at low tide was −0.35 foot.

A. Approximately when will the next high tide occur?

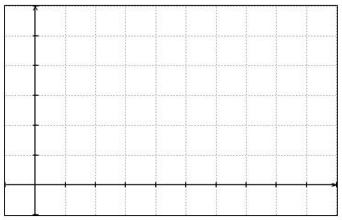
B. Find a sinusoidal function of the form $y = A\sin(Bx - C) + D$ that models the data.

C. Use your function model to predict the height of the water at noon on April 7th, 2023.

7. The depth of the water at the end of the Cocoa Beach Pier varies with the tide on any given day. Suppose the high tide occurs at 5:15 a.m. and the depth of the water is 15 feet, then low tide occurs at 11:27 a.m. with a depth of 9.5 feet.



A. Sketch a graph showing how the depth of the water depends on the time since midnight.

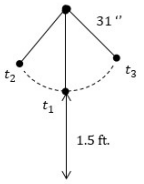


B. Write a trigonometric equation as a sine function that models the depth of the water t hours after midnight.

C. What is the depth of the water at noon?

8. The pendulum on a grandfather clock moves in a sinusoidal fashion. The length of the pendulum is 31 inches at its lowest point. The pendulum is 1.5 feet from the floor.

The pendulum begins to move at t_1 so that the interior angle between t_2 and t_3 is 90°. It takes 0.875 second for the pendulum to swing from t_2 to t_3.



A. What is the minimum height of the pendulum in inches?

B. What is the maximum height of the pendulum?

C. Write the sinusoidal equation that models the movement of the pendulum as a cosine function $y = A(Bx - C) + D$.

Practice Sets

3.7 Sinusoidal Function Context and Data Modeling Set 1 **Form A**

Name _____

Date _____ Period _____

A weight is attached to the end of a long spring that is bouncing up and down. As it bounces the distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point of 64 centimeters above the floor. The next low point, 42 centimeters above the floor, occurs at 2.3 seconds.

1. Write an equation expressing the distance from the floor in terms of the number of seconds the stopwatch reads.

2. Predict the distance from the floor when the stopwatch reads 5.3 seconds.

3. Predict the first positive value of time at which the weight is 50 centimeters above the floor.

Grade:

3.7 Sinusoidal Function Context and Data Modeling Set 1 **Form B**

Name _____

Date _____ Period _____

A weight is attached to the end of a long spring that is bouncing up and down. As it bounces the distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.6 seconds, the weight first reaches a high point of 71 centimeters above the floor. The next low point, 47 centimeters above the floor, occurs at 3.6 seconds.

1. Write an equation expressing the distance from the floor in terms of the number of seconds the stopwatch reads.

2. Predict the distance from the floor when the stopwatch reads 7.6 seconds.

3. Predict th
which the
the floor.

Grade:

3.7 Sinusoidal Function Context and Data Modeling Set 2 **Form A**

Name _____

Date _____ Period _____

The Texas Star is the Ferris Wheel at the State Fair of Texas. It has a maximum height of 212 feet and a low point of 8 feet. When the last seat is filled and the Ferris Wheel begins to move, you start your stopwatch. You reach the top when your stopwatch reads 15 seconds. The wheel makes a complete rotation every 40 seconds. The wheels diameter is 204 feet.

1. Write a sinusoidal equation to model the behavior of the Ferris Wheel ride.

2. Use your equation to find the height above the ground at your starting location ($t = 0$).

3. What is the value of t the second time you were at 200 feet?

Grade:

3.7 Sinusoidal Function Context and Data Modeling Set 2 **Form B**

Name _____

Date _____ Period _____

The Texas Star is the Ferris Wheel at the State Fair of Texas. It has a maximum height of 212 feet and a low point of 8 feet. When the last seat is filled and the Ferris Wheel begins to move, you start your stopwatch. You reach the top when your stopwatch reads 10 seconds. The wheel makes a complete rotation every 40 seconds. The wheels diameter is 204 feet.

1. Write a sinusoidal equation to model the behavior of the Ferris Wheel ride.

2. Use your equation to find the height above the ground at your starting location ($t = 0$).

3. What is the value of t the second time you were at 200 feet?

Grade:

Daily Lesson Quiz