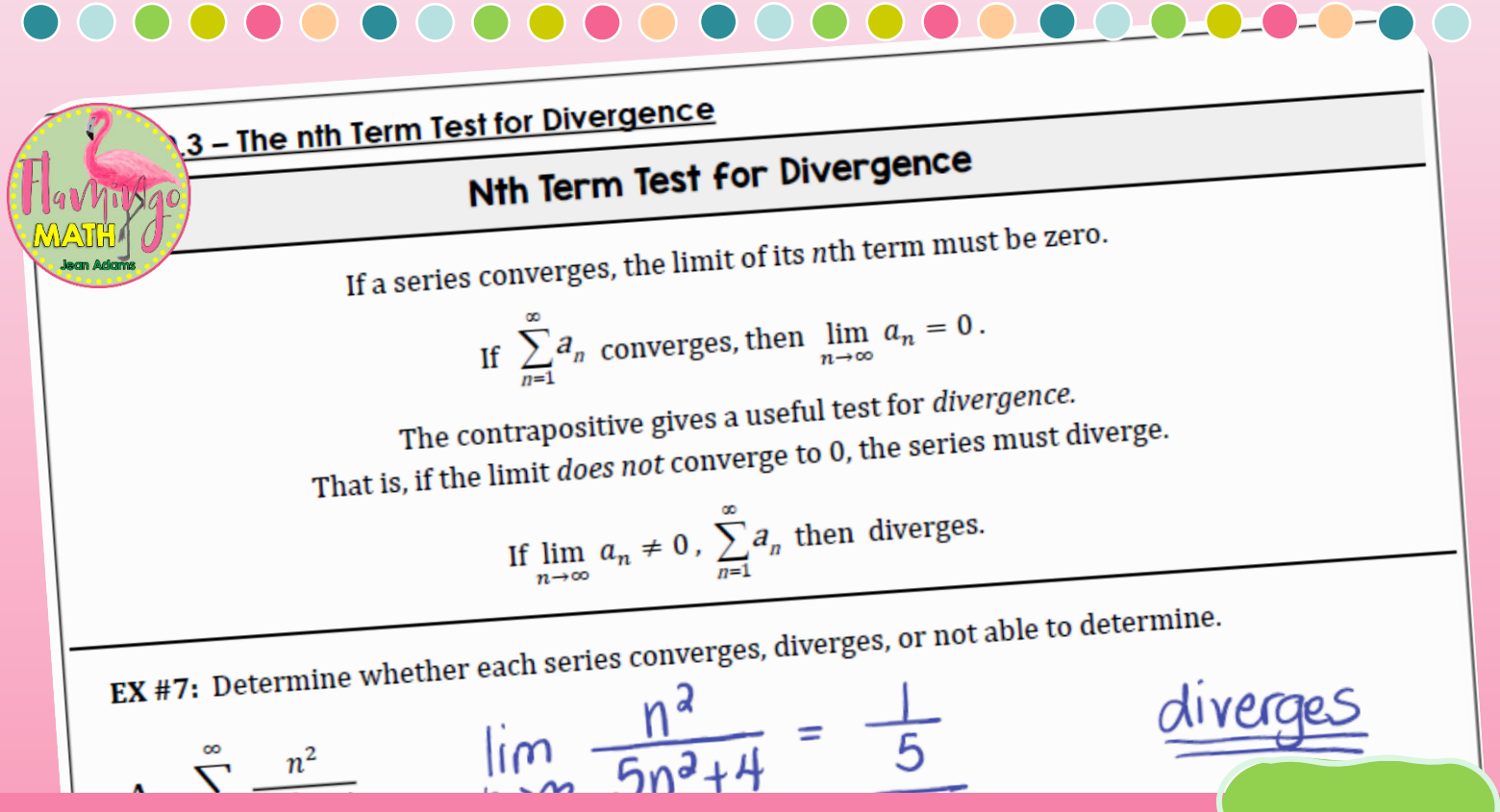
AP® Calculus BC

Series & Convergence



Completely Editable

10.1

Lesson 10.1: Series and Convergence

Topic 10.1-Defining Convergent and Divergent Infinite Series

An **infinite series** is defined as the summation of a sequence with an infinite number of terms. That is, if $\{a_n\}$ is an infinite sequence, then

Infinite Series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

Finding Partial Sums

We will find that when a sequence of partial sums converges, then the series will also converge. This lesson begins with a look at **geometric series** and **telescoping series**. The next lesson will explore **p-series** and **harmonic series**.

Infinite sums are called **series**. We want to investigate limits of sequences of numbers.

Series	Sequence	The value of a series is the limit of a sequence
$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$	$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots, \frac{2^n - 1}{2^n}, \ldots$	$\sum_{n=1}^{\infty} \frac{1}{2^n} = \lim_{n \to \infty} \frac{2^n - 1}{2^n}$

EX #1: Write the indicated partial sums S_n for the series $\sum_{k=1}^{n} \frac{1}{2^{k-1}}$

A. S_1 **B.** S_2

C. S_3 **D.** S_n

Definitions of Convergent and Dive

For the infinite series $\sum_{n=1}^{\infty} a_n$ the ***n*th partial sum** is give

If the sequence of partial sums $\{S_n\}$ converges to S, th

The limit S is called the **sum of the**

If $\{S_n\}$ diverges, then the series **div**

Topic 10.2 – Working with Geometric Series

Geometric Series

A geometric series is $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots, a \neq 0$ or $\sum_{n=1}^{\infty} ar^{n-1}$

Convergence of a Geometric Series

A geometric series with ratio r diverges if $|r| \geq 1$.
If $0 < |r| < 1$, then the series converges to the sum:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Partial Sum of Geometric Series

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r} = a_1 \left[\frac{1 - r^n}{1 - r}\right]$$

EX #2: Consider the infinite geometric series $8 + \frac{8}{5} + \frac{8}{25} + \frac{8}{125} + \cdots$

A. Find a_1 **B.** What is r?

d the 4th partial sum, S_4

d $\lim_{n \to \infty} S_n$

explain why.

$\sum_{=0}^{\infty} \frac{4^n - 7^n}{5^n}$

Properties of Infinite Series

Given two convergent series $\sum a_n$ and $\sum b_n$ where A, B, and c are real numbers.
If $\sum a_n = A$ and $\sum b_n = B$, then the following series converge to the given sums.

$\sum_{n=1}^{\infty} ca_n = cA$	$\sum_{n=1}^{\infty} (a_n + b_n) = A + B$	$\sum_{n=1}^{\infty} (a_n - b_n) = A - B$

Telescoping Series

Let's investigate a fun series where the terms collapse like an old-fashioned spyglass or telescope. This type of series is not used very often, but they are fun to consider.

EX #4: Use partial fractions and to find the sum of the series.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1(2)} + \frac{1}{2(3)} + \cdots$$

Topic 10.3 – The nth Term Test for Divergence

Nth Term Test for Divergence

If a series converges, the limit of its *n*th term must be zero.

If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$.

The contrapositive gives a useful test for *divergence*.
That is, if the limit *does not* converge to 0, the series must diverge.

If $\lim_{n \to \infty} a_n \neq 0$, $\sum_{n=1}^{\infty} a_n$ then diverges.

EX #7: Determine whether each series converges, diverges, or not able to determine.

A. $\sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 4}$

B. $\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$

Formulas and Term Values from nth Term Partial Sums

EX #8: The nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = \frac{n+2}{n+5}$. Write a rule for a_n.

EX #9: The *n*th partial sum of a series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = \frac{n^2+2}{n+1}$. Find a_6.

NOTES

10.1 **Series and Convergence Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, Find the sequence of partial sums, S_1, S_2, S_3, S_4, S_5.

1. $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{9}+\frac{1}{11}+\cdots$

2. $2+1+\frac{4}{5}+\frac{5}{7}+\frac{6}{9}+\cdots$

3. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$

4. $\sum_{n=1}^{\infty}\frac{3}{2^n}$

Problems 5 – 10, Determine whether the following series converge or diverge. Justify your answer.

5. $\sum_{n=1}^{\infty}\frac{3n+2}{2n-5}$

6. $\sum_{n=1}^{\infty}\frac{n!}{2n!+1}$

7. $\sum_{n=1}^{\infty}\ln\left(\frac{n}{2n+5}\right)$

8. $\sum_{n=1}^{\infty}\frac{4^{n+1}}{5^n}$

9. $\frac{1}{3}+\frac{1}{7}+\frac{1}{11}+\frac{1}{15}+\cdots$

10. $\sum_{n=1}^{\infty}\frac{1}{e^{2n}}$

Problems 11 – 14, Find a formula or term values for the nth partial sums given below.

11. Find a rule for a_n given:

$S_n=\frac{7n}{n+3}$

12. Find a rule for a_n given:

$S_n=\frac{n+1}{n+5}$

13. Find a_7 given:

$S_n=\frac{2n+3}{3n}$

14. Find a_5 given:

$S_n=\frac{n^2+1}{n+2}$

Problems 15 – 16, Use partial fractions to find the sum of the convergent telescoping series.

15. $\sum_{n=1}^{\infty}\frac{-2}{(n+1)(n+2)}$

16. $\sum_{n=1}^{\infty}\frac{2}{n^2+4n+3}$

Practice Sets

10.1 Series and Convergence **Form A**

Name _____

Date _____ Period _____

Determine if the sequence converges. If so, find the limit. If the sequence diverges, explain why.

1. $a_n = \left\{\frac{(-1)^n (n)}{n+1}\right\}$

2. $a_n = \frac{8 + 5n - 3n^2}{4n^2 - 1}$

3. $\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \ldots$

$\sum_{n=1}^{\infty}$ (5)

10.1 Series and Convergence **Form B**

Name _____

Date _____ Period _____

Determine if the sequence converges. If so, find the limit. If the sequence diverges, explain why.

1. $a_n = \frac{10n^2 + 4n + 5}{6 - 2n^2}$

2. $a_n = \left\{\frac{(-1)^n (n-1)}{n}\right\}$

3. $\frac{1}{2}, \frac{3}{3}, \frac{5}{4}, \frac{7}{5}, \frac{9}{6}, \ldots$

$\sum_{n=1}^{\infty}$ (3)

Daily Lesson Quiz