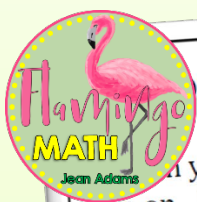


Precalculus Honors

Sequences & Summation Notation



Unit 1: Sequences and Summation Notation

When you think of a “sequence of events” you likely would consider what happened first, second, and so on. In mathematics, the word *sequence* is also used in the same way, some list of numbers has a first element, a second element, a third element, and so on. One famous example, known as the **Fibonacci sequence** is found throughout nature: 1, 1, 2, 3, 5, 8, 13, 21, 34, ... Can you discover the rule for the next term?

DEFINITION OF A SEQUENCE

An **infinite sequence** is a function whose domain is the set of positive integers. The function values are the **terms** of the sequence.

$$a_1, a_2, a_3, a_4, \dots, a_n, \dots$$

If the domain of the function consists of the first n positive integers only, the sequence is a **finite sequence**.

Writing Terms of a Sequence

EX #1: Write the first six terms of the following sequences. Use the graphs shown below to verify.

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9.1

Lesson 9.1: Sequences and Summation Notation

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Writing Terms of a Sequence

EX #1: Write the first six terms of the following sequences. Use the graphs shown below to verify.

A. $a_n = \frac{n-1}{n+1}$

B. $b_n = \left\{(-1)^n \left(\frac{2}{n}\right)\right\}$

Evaluating Fractions with Factorial Notation

EX #5: Evaluate each factorial expression:

A. $\frac{12!}{2!10!}$

B. $\frac{(n+2)!}{n!}$

Summation Notation

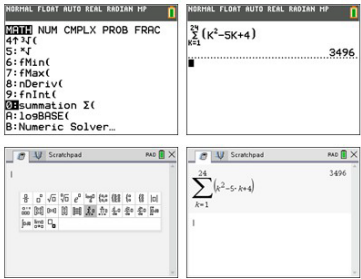
The sum of the first n terms of a sequence is represented by the **summation notation**

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + a_4 + \dots + a_n$$

Using Technology to Sum a Sequence

EX #9: Use the formulas to find the sum, then use a graphing utility to verify the sum of the sequence.

$$\sum_{k=1}^{24} (k^2 - 5k + 4)$$



Annuity and Amortization Formulas

In earlier courses, you have used simple interest and compound interest formulas. Many times, money is invested in small amounts on periodic intervals. An **annuity** is a sequence of equal periodic deposits made in daily, monthly, quarterly, or annually. The **amount of an annuity** is the sum of all deposits made plus all interest paid. An **ordinary annuity** is an account where the deposits are made at the same time the interest is credited.

When equal periodic payments are made to pay off a loan, we say that the loan is **amortized**, such as home mortgages. **Recursive sequences** are also used to compute loans of this type for any given time.

AMORTIZATION FORMULA

If B dollars are borrowed at an interest rate $r\%$ (expressed as a decimal) per year compounded monthly, the balance A_n due after n monthly payments of P dollars is given by the recursive sequence

$$A_0 = B, A_n = \left(1 + \frac{r}{12}\right) A_{n-1} - P, n \geq 1$$

Using a Recursive Formula

In our first example we were given an **explicit** rule. The n th term formulas expressed the term as a function of n . Sequences can be defined using **recursive formulas**. In a recursive rule, each term makes a reference to a previous term.

EX #2: Find the first five terms in the recursive forms for $n > 1$.

A. $a_1 = 7, a_n = 2a_{n-1} + 3$

B. $a_1 = -5, a_n = a_{n-1} + 10$

EX #3: Look for simple addition or subtraction patterns to relate consecutive terms. Write a recursive rule for the sequence.

A. 1, 4, 7, 10, 13, ...

B. 4, -1, -6, -11, -16, ...

Factorial Notation

If n is a positive integer, the notation $n!$ (read " n factorial") is the product of all positive integers from n down through 1. By definition (zero factorial) $0! = 1$

Properties of Sums

Property	Example
1. $\sum_{i=1}^n c a_i = c \cdot \sum_{i=1}^n a_i$ c any real number	
2. $\sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$	
3. $\sum_{i=1}^n (a_i - b_i) = \sum_{i=1}^n a_i - \sum_{i=1}^n b_i$	

Real-World Applications

EX #10: On January 1, 2015, Abby's parents decided to put \$75 at the end of each month into the Florida Pre-Paid Education Program.

A. Write a recursive formula that represents the balance at the end of each month if the rate-of-return is assumed to be 6.5% per year compounded monthly.

B. How long will it take for the value of the account to be more than \$5000?

C. What will be the value of the account in 16 years when Abby goes to college?

EX #11: Alex and Tina borrowed \$175,000 at 3.6% per year compounded monthly for a 30-year mortgage. Their monthly payment is \$796.

A. Write a recursive formula for the balance remaining after each monthly payment has been made.

B. Use a graphing utility to find the balance at the end of 15 years.

E. How much interest will be paid at this payment schedule? How much is saved?

NOTES

9.1

Sequences & Summation
Notation Homework

Name _____

Date _____ Period _____

Problems 1 – 4, write the first five terms of each sequence.

1. $a_n = \frac{n}{n+2}$

2. $b_n = \frac{n}{e^n}$

3. $s_n = \frac{n^2}{2^n}$

4. $d_n = \frac{(-1)^n}{(n+1)(n+2)}$

Problems 5 – 8, find the n th term of a sequence $\{a_n\}$ from the given pattern.

5. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

6. $3, -6, 9, -12, 15, \dots$

7. $1, -1, 1, -1, 1, -1, \dots$

8. $\frac{1}{1 \cdot 2}, \frac{1}{2 \cdot 3}, \frac{1}{3 \cdot 4}, \frac{1}{4 \cdot 5}, \dots$

Problems 9 – 12, write the first five terms of the recursively-defined sequence.

9. $a_1 = 3; a_n = 5 - a_{n-1}$

10. $a_1 = 2; a_n = \sqrt{2 + a_{n-1}}$

11. $a_1 = K; a_n = a_{n-1} + d$

12. $a_1 = 243; a_n = \frac{6n-1}{3}$

Problems 13 – 16, Evaluate each factorial expression.

13. $\frac{14!}{8!11!}$

14. $\frac{3!7!}{4!}$

15. $a_n = \frac{(n+1)!}{n!}$

16. $a_n = \frac{(2n+1)!}{(2n)!}$

Problems 17 – 20, write out each sum.

17. $\sum_{k=1}^n (2k + 3)$

18. $\sum_{k=0}^n \left(\frac{3}{2}\right)^k$

19. $\sum_{k=0}^n \frac{1}{2^{k+1}}$

20. $\sum_{k=2}^n (-1)^k \ln k$

Problems 21 – 24, write each sum using summation notation.

21. $1 + 3 + 5 + 7 + \dots + [2(15) - 1]$

22. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + (-1)^8 \left(\frac{1}{2^8}\right)$

23. $1^3 + 2^3 + 3^3 + \dots + 9^3$

24. $0 + \frac{1}{e} + \frac{2}{e^2} + \frac{3}{e^3} + \dots + \frac{n}{e^n}$

Problems 25 – 28, find the sum of each sequence.

25. $\sum_{k=1}^{30} 3$

26. $\sum_{k=1}^{50} k$

Problems 29-30, solve.



29. Max bought a car by taking out a loan for \$23,500 at 2.5% interest per month. His normal payment is \$417 per month, but he wants to pay an extra \$100 toward the balance each month. His balance each month is modeled by the recursively defined sequence

$$B_0 = \$23,500, B_n = \left(1 + \frac{0.025}{12}\right) B_{n-1} - 517.$$

A. Using a graphing utility, determine when Max's balance will be below \$12,000. How many payments of \$517 have been made?

B. When will Max pay off the balance? What is the total of all the payments?

C. What was the total interest Max paid on the loan?

30. Andrea deposited \$500 at the end of each quarter beginning January 1, 2010, into a Mutual Fund Account.

A. Write a recursive formula that represents Andrea's balance at the end of each quarter if the rate of return is assumed to be 8.5% per year compounded quarterly.

Practice Sets

Lesson Warm-Ups

3. Evaluate the factorial: $\frac{12!}{5! \cdot 8!}$	4. Evaluate the series: $\sum_{k=1}^6 (k^2 - 3k)$

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Name _____ Date _____ Period _____	
Lesson 9.1: Sequences and Summation Notation Warm-Up # 2	
1. In the recursively defined sequence: $a_1 = 5$; $a_n = a_{n-1} - 3$ for $n \geq 2$. Find the first and fourth terms of the sequence.	2. Write an explicit rule for the n^{th} term of the arithmetic sequence. 9, 5, 1, -3, ...
3. Evaluate the factorial: $\frac{15!}{3! \cdot 12!}$	4. Evaluate the series: $\sum_{k=1}^6 (k^2 + 4k)$

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Form A	9.1 Sequences & Summation Notation	Form B
Name _____	Name _____	Name _____
Period _____	Date _____ Period _____	Date _____ Period _____
sequence, find ence. 3 for $n \geq 2$.	1. In the recursively defined sequence, find the first 4 terms of the sequence. $a_1 = 5$ and $a_n = a_{n-1} - 4$ for $n \geq 2$.	
	2. Write a recursive rule for the sequence. 9, 4, -1, -6, ...	
	3. Evaluate the factorial expression: $\frac{16!}{12!}$	
	4. Evaluate the series	

Daily Lesson Quiz

Grade: _____	Grade: _____
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