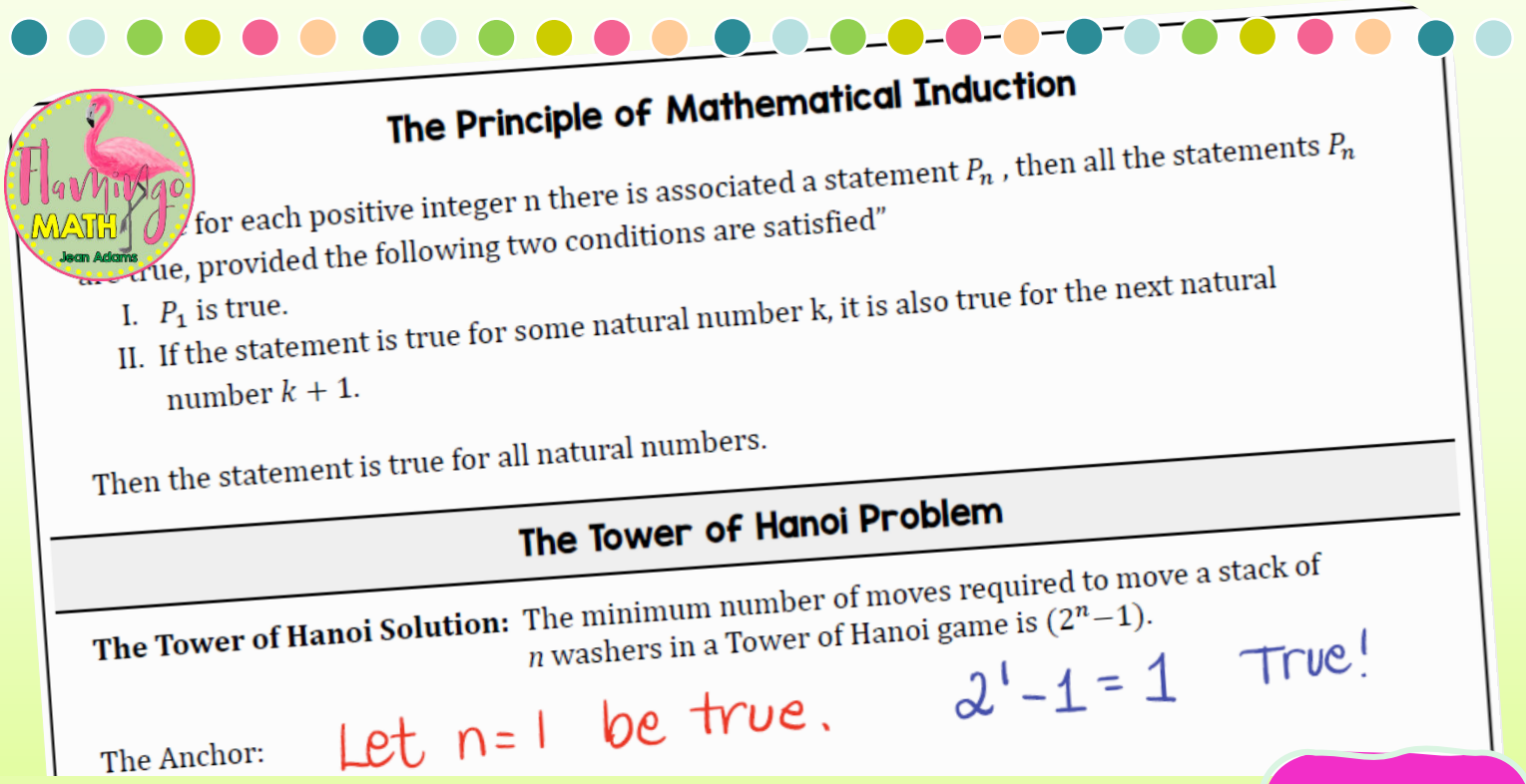
Precalculus Honors

Sequences & Series
Discrete Mathematics



The Principle of Mathematical Induction

for each positive integer n there is associated a statement P_n, then all the statements P_n true, provided the following two conditions are satisfied"

I. P_1 is true.

II. If the statement is true for some natural number k, it is also true for the next natural number $k + 1$.

Then the statement is true for all natural numbers.

The Tower of Hanoi Problem

The Tower of Hanoi Solution: The minimum number of moves required to move a stack of n washers in a Tower of Hanoi game is $(2^n - 1)$.

The Anchor: Let n=1 be true. $2^1 - 1 = 1$ True!

Guided Notes 9

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PRECALCULUS CURRICULUM

Scope and Sequence

Unit 9: Sequences and Series

Lesson	Skills & Learning Objectives
[9.1] Sequences and Summation Notation	❑ Write the terms of a recursively-defined sequence ❑ Use factorial notation ❑ Use summation notation to write sums ❑ Find the sum of a sequence algebraically and using technology ❑ Solve annuity, amortization, and real-world applications
[9.2] Arithmetic Sequences and Partial Sums	❑ Recognize, write and find the *n*th terms of arithmetic sequences ❑ Find *n*th term partial sums of arithmetic sequences ❑ Use arithmetic sequences and series to model real-world problems ❑ Find limits of converging sequences
[9.3] Geometric Sequences and Series	❑ Recognize, write and find the *n*th terms of geometric sequences ❑ Find *n*th term partial sums of geometric sequences ❑ Find sums of infinite geometric series ❑ Use geometric sequences to model real-world problems
[9.4] Mathematical Induction	❑ Use mathematical induction to prove statements involving a positive integer *n*. ❑ Find the sums of powers of integers ❑ Find finite differences of sequences
[9.5] Binomial Theorem	❑ Evaluate $\binom{n}{r}$ ❑ Use the Binomial Theorem

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Lesson 9.1: Sequences and Summation Notation

When you think of a "sequence of events" you likely would consider what happened first, second, and so on. In mathematics, the word *sequence* is also used in the same way, some list of numbers has a first element, a second element, a third element, and so on. One famous example, known as the **Fibonacci sequence** is found throughout nature: 1, 1, 2, 3, 5, 8, 13, 21, 34, … Can you discover the rule for the next term?

DEFINITION OF A SEQUENCE

An **infinite sequence** is a function whose domain is the set of positive integers. The function values are the **terms** of the sequence.

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

If the domain of the function consists of the first n positive integers only, the sequence is a **finite sequence.**

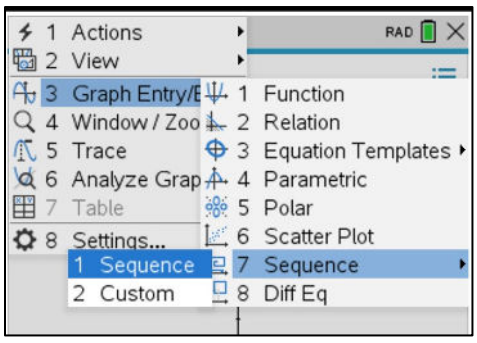
Writing Terms of a Sequence

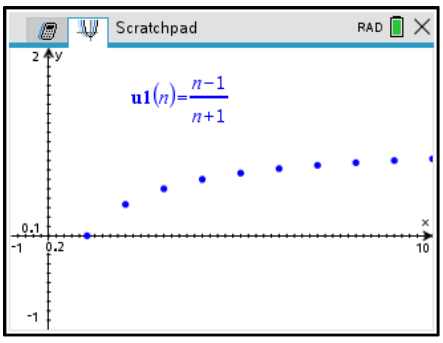
EX #1: Write the first six terms of the following sequences. Use the graphs shown below to verify.

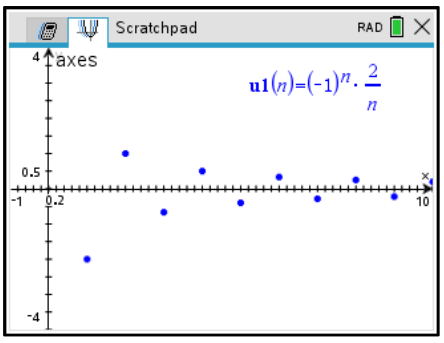
A. $a_n = \frac{n-1}{n+1}$

B. $b_n = \left\{(-1)^n \left(\frac{2}{n}\right)\right\}$

Using Technology to Graph a Sequence







9.2 Arithmetic Sequences and Series

Arithmetic Sequences

Sequence – an ordered set of numbers. The dots (ellipsis) mean there are more numbers. A sequence that does not end is called **infinite**.

EX. 3, 5, 7, 9, . . .

Some sequences have a rule to describe the n^{th} term. The rule for the sequence above is

$$\boldsymbol{a_n = 2n + 1}.$$

Nth term of an Arithmetic Sequence

In an arithmetic sequence with first term a_1 and common difference d, the nth term a_n, is given by the following.

$$a_n = a_1 + (n-1)d$$

EX #1: Find the ninth term of the arithmetic sequence whose first term is 8 and has a common difference of -5.

Finding General Terms

EX #2: The numbers in the sequences below follow a general pattern. Can you write the general rule?

A. $1, 1.25, 1.5, 1.75, \ldots$

B. $-1, -4, -7, -10, -13, \ldots$

9.3 Geometric Sequences and Series

Geometric Sequences

A **geometric sequence** is any sequence with a constant ratio between consecutive terms. With a starting value "*a*" and common ratio "*r*" is in the form $a, ar, ar^2, ar^3, \ldots ar^n$.

EX. $\mathbf{4, 8, 16, 32, 64, \ldots 4(2)^n}$

General Form and Common Ratios

The *n*th term (general term) of a geometric sequence with first term a_1 and common ratio r is

$$a_n = a_1 r^{n-1}$$

Geometric sequences have common ratios between consecutive terms.

Common ratio: $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}$

Each term in a geometric sequence can be obtained recursively from its previous term by multiplying by *r*:

Formula:	
General Form:	$a_n = a_1 r^{n-1}$
Recursion Formula:	$a_1 = a$ $a_n = a_{n-1} \cdot r$ for $n > 1$

Identifying Geometric Sequences

EX #1: Determine whether each sequence is geometric. If so, find the common ratio, *r*.

A. $3, 6, 12, 24, \ldots$

B. $3, -9, 27, -81, \ldots$

C. $1, 5, 9, 13, 17, \ldots$

D. $48, -12, 3, \ldots$

Lesson 9.4: Mathematical Induction

In this last section we will learn how to use a principle of proof called *mathematical induction* to prove statements about the positive integers. **Induction** is reasoning from *particular examples* to draw conclusions about general principles, while **deduction** is reasoning from the *general principles* to draw conclusions about specific cases.

Mathematical Induction is a way of proving that if a hypothesis is true for one natural number it is true for the next consecutive natural number and therefore for all numbers.

The Principle of Mathematical Induction

Suppose for each positive integer n there is associated a statement P_n , then all the statements P_n are true, provided the following two conditions are satisfied"

I. P_1 is true.

II. If the statement is true for some natural number k, it is also true for the next natural number $k + 1$.

Then the statement is true for all natural numbers.

The Tower of Hanoi Problem

The Tower of Hanoi Solution: The minimum number of moves required to move a stack of n washers in a Tower of Hanoi game is $(2^n - 1)$.

The Anchor:

The Inductive Hypothesis:

The Inductive Step:

Lesson 9.5: The Binomial Theorem

In this lesson, we will develop the Binomial Theorem, a formula for the expansion of binomials in the form $(x + a)^n$ for any positive integer n. Before we get to that theorem, let's look at how to evaluate the combination of *n* items taken *r* at a time.

This is denoted on a calculator as ${}_nC_r$ or, in a textbook, with the symbol $\binom{n}{r}$.

Combinations, *n* items taken *r* at a time

If n and r are integers with $0 \leq n \leq r$, the symbol $\binom{n}{r}$ is defined as

$$nCr = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Four Useful Formulas

$$\binom{n}{0} = 1 \qquad \binom{n}{n} = 1 \qquad \binom{n}{1} = n \qquad \binom{n}{n-1} = n$$

EX. #1: Evaluate the following, show your work that leads to your answer.

A. $\binom{5}{1}$

B. $\binom{6}{4}$

C. $\binom{7}{6}$

D. $\binom{15}{12}$

EX #2: Use a graphing utility to evaluate the expression $\binom{26}{8}$