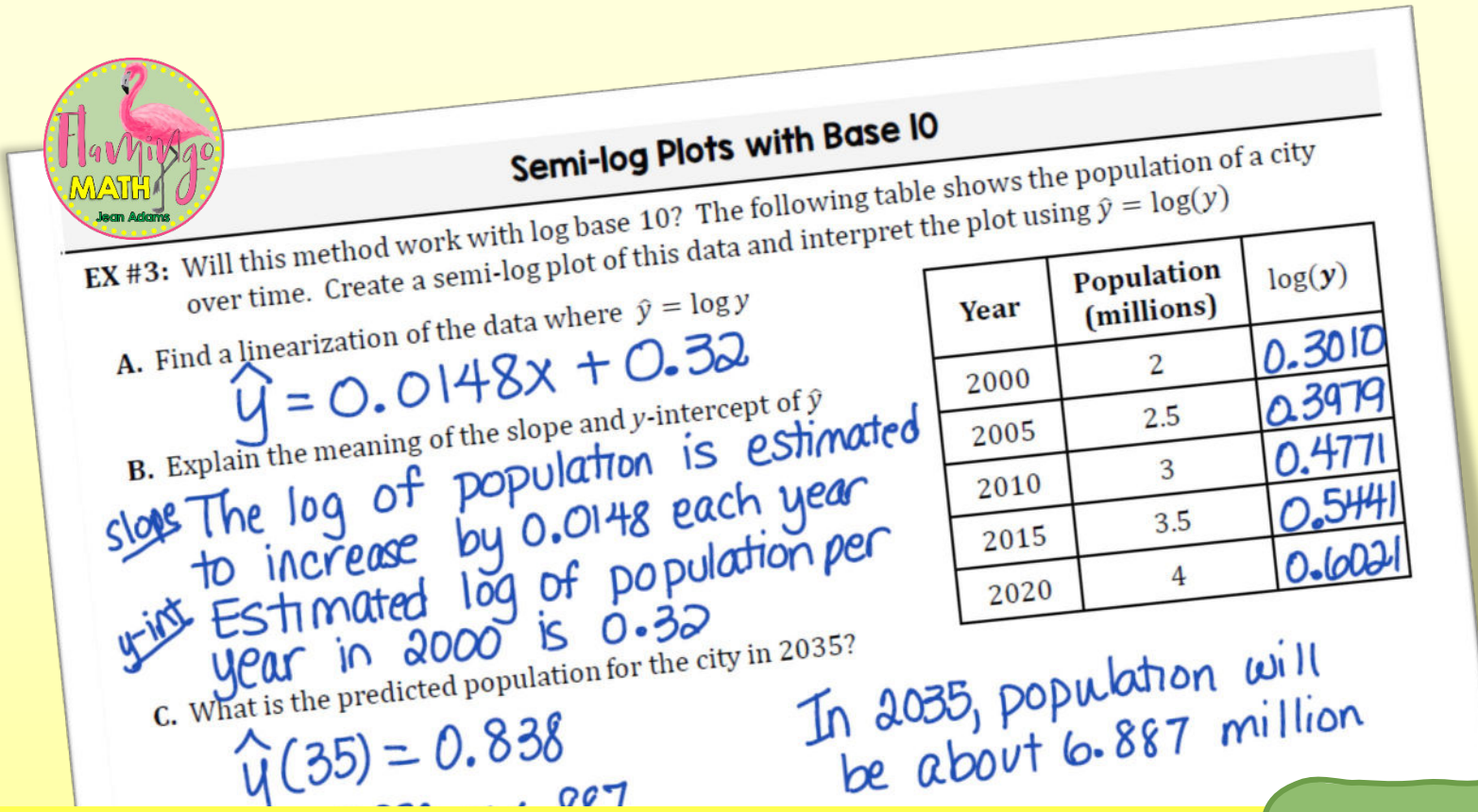
AP® Precalculus

Semi-log Plots



Semi-log Plots with Base 10

EX #3: Will this method work with log base 10? The following table shows the population of a city over time. Create a semi-log plot of this data and interpret the plot using $\hat{y} = \log(y)$

Year	Population (millions)	log(y)
2000	2	0.3010
2005	2.5	0.3979
2010	3	0.4771
2015	3.5	0.5441
2020	4	0.6021

A. Find a linearization of the data where $\hat{y} = \log y$

$\hat{y} = 0.0148x + 0.32$

B. Explain the meaning of the slope and y-intercept of $\hat{y}$

slope: The log of population is estimated to increase by 0.0148 each year

y-int: Estimated log of population per year in 2000 is 0.32

C. What is the predicted population for the city in 2035?

$\hat{y}(35) = 0.838$

In 2035, population will be about 6.887 million

Completely Editable 2.15

Lesson I5-A: Semi-Log Plots

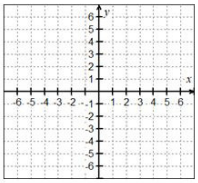
Semi-log plots are a useful tool for visualizing data that covers a wide range of values. A ***semi-log plot*** is a graph that combines a logarithmic scale for one axis with a linear scale for the other axis. It is used to plot data that covers a wide range of values, where a linear plot would make it difficult to see the details. The semi-log plot will create a graph that reveals the details of the data while still showing the overall trend. In this lesson, we will learn how to create and interpret semi-log plots.

Plotting Functions on Semi-Log Graph Paper

EX #1: Let's consider graphing the function $f(x) = x$ on two different types of graph paper.

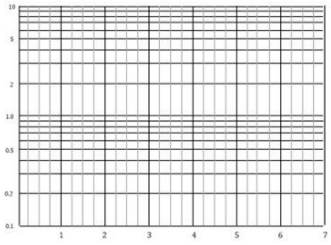
A. On a linear axes, graph $y = x$ for table below.

x	−2	−1	0	1	2	3	4	5	6
$f(x)$	−2	−1	0	1	2	3	4	5	6



B. Use the semi-logarithmic axes, at right, to plot the graph of $y = x$.

Notice that the vertical axis is a logarithmic scale (base 10) so there are no negative values for y since we can't find the log of a negative number. The horizontal axis is a linear scale.



C. What do you notice about the graph of the linear function on this semi-log plot?

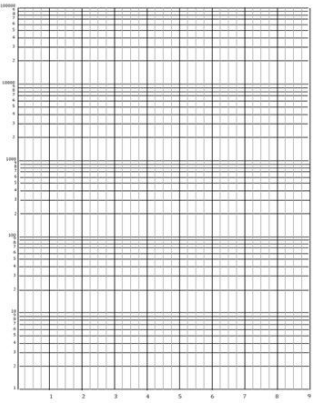
Mathematically, we treat a logarithmic axis as a transformation of the variables. For our course, we will plot data pairs (x, y) in a semi-log plot where we are working with the transformed data pairs $(x, \log y)$ or $(x, \ln y)$. Think about the log y as a variable. The logarithmic scale is a great tool to compress a large range of data into a small space and be able to detect small variations in the values of the data points in a set.

Plotting Functions on Semi-Log Graph Paper

EX #2: Consider the function $f(x) = 9 \cdot 3^x$. The table below shows values of $f(x)$ at selected values of x. To graph this function, we would need a very long y-axis. However, if we use a semi-log plot, where the y-axis is scaled to powers of 10, we can graph many of the function's values.

A. Plot the points for the function, using the table below.

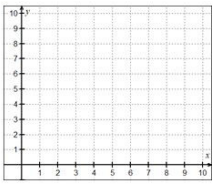
x	$f(x)$
0	9
1	27
2	81
3	243
4	729
5	2187
6	6561
7	19683
8	59049



B. What do you notice about the graph of the exponential function on this semi-log plot?

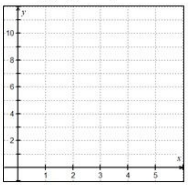
C. Write $\log_3(f(x))$ in the form $\log_3(f(x)) = mx + b$. State the values of m and b.

D. Graph this relation with $\log_3(f(x))$ as the vertical axis and x as the horizontal axis.



Plotting Functions on Semi-Log Graph Paper

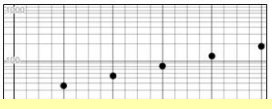
EX #3: Given $g(x) = 8 \cdot 2^{2x}$. Find the values of m and b such that $y = (\log_2 b)x + \log_2 a$. Graph $y = (\log_2 b)x + \log_2 a$ on the graph provided. Label the axes appropriately.



EX #4: Given $h(x) = 5e^{7x/3}$. Find the values of m and b such that $y = (\ln b)x + \ln(a)$

EX #5: The semi-log plot, at right, corresponds to the data table below.

x	1	2	3	4	5
$g(x)$	36	54	81	122	182



A. Write an equation for the linear model in the form $g(x) = (\log_n b)x + \log_n a$

Selecting a Regression Model

EX #6: Consider the data in the tables below. Determine whether the data can best be modeled by a linear, logarithmic, or exponential regression equation. Explain your choice. Support your answer with tables and graphs.

A.

x	1	2	3	4
y	5	8	11	14

B.

x	1	2	3	4
y	6	12	24	48

C.

x	1	2	3	4
y	4	5.4	6.2	6.8

D.

x	1	2	3	4
y	13	6.5	3.25	1.625

In slope-intercept form, $y = mx + b$, the slope of the linear function is $\log_n b$ and the y-intercept is $\log_n a$.

Note: The base n corresponds to the base used for the scaling of the vertical axis.

Guided Notes

Lesson 15-B: Semi-Log Plots

In the first lesson on semi-log plots, we knew the base of the exponential function. But, if we have real world data and do not know the function or the base, we will take the logarithm of the output values and then we can graph using a logarithmically scaled vertical axis. So, whether we have the exponential function of raw data, in either case, we can linearize the data by transforming the y-values to a logarithmic scale.

Using Raw Data

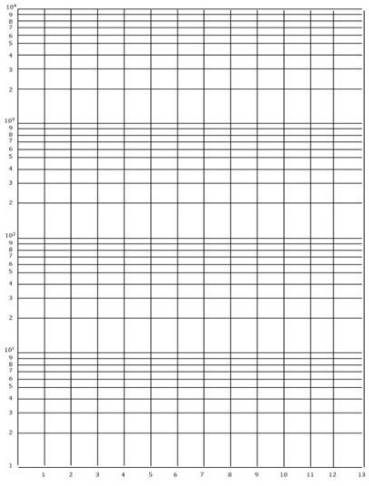
EX #7: The table below gives values for a function $f(x)$ at selected values of t.

x	0	2	4	5	6	7	8	10	12	13
$f(x)$	0.3	1.2	5.8	12.6	28.8	65.4	156.2	820.3	4352.6	9942.5

A. Graph the data on the semi-log plot below. Label your axes.

B. What does this graph say about the data?

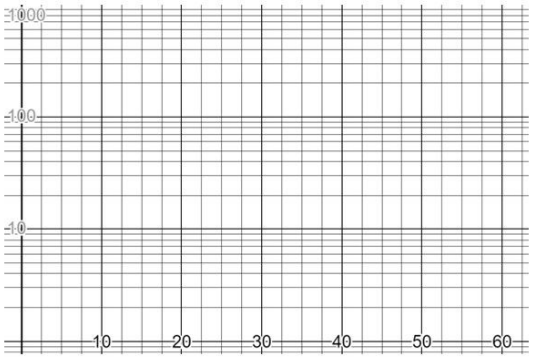
C. Approximate the values of $f(3)$ and $f(9)$ using the graph.



Graphing Data on Semi-log Paper

EX #8: Graph the following data on the semi-log paper.

x	7	12	20	31	42	54
y	3	16	39	53	88	125



Based on the graph, what does this suggest about the data set?

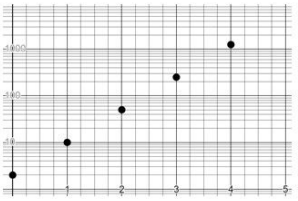
EX #9: The function g is graphed on the semi-log plot at right where the vertical axis has been logarithmically scaled. Which of the following functions could be a model for g?

A. $g(t) = 2\left(\frac{1}{5}\right)^t$

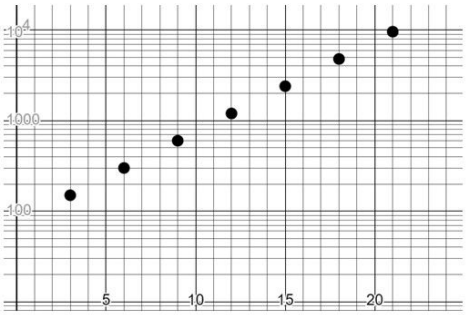
B. $g(t) = 2 + 5t$

C. $g(t) = 2(5)^t$

D. $g(t) = 2 + \log_5(t)$

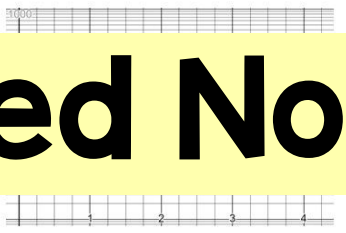


EX 10: The graph below is data collected from a bacteria growth B, in a culture that was recorded over a period of three weeks. The semi-log plot shows the number of bacteria present, beginning on day 3 through day 21. Write the function for $B(t)$.



EX #11: Use the table of values to graph the points on a semi-log plot. Does the graph indicate that an exponential model would be appropriate for the data? Explain why or why not.

x	$h(x)$

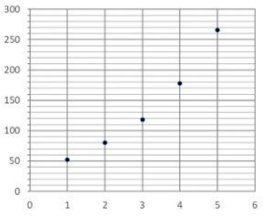


Choosing an Appropriate Graph

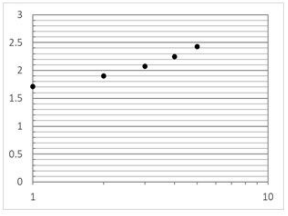
EX #12: The table below gives selected values for the function k. Which of the following graphs could represent these data in a semi-log plot, where the vertical axis is logarithmically scaled?

x	1	2	3	4	5
$k(x)$	52	80	118	178	266

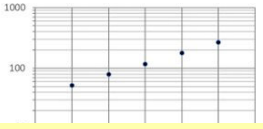
A.



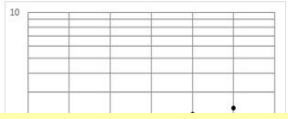
B.



C.



D.



Guided Notes

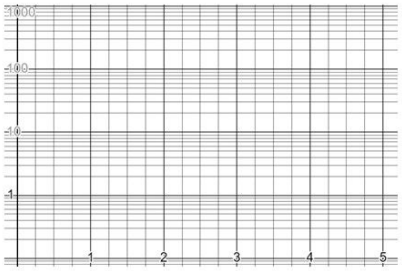
2.15 **Semi-Log Plots Practice Set 1**

Name _____

Date _____ Period _____

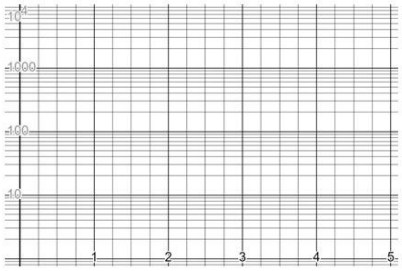
Problems 1-5, Plot the following points on the same coordinate plane below.

1. $A = (0, 0.4)$
2. $B = (1, 60)$
3. $C = (2.5, 300)$
4. $D = (3.25, 140)$
5. $E = (4, 500)$



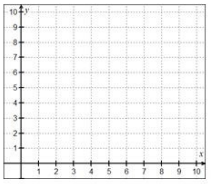
2. Given the exponential function $g(x) = 4(5)^x$, complete the table of values. Graph the points on the semi-log plot. Write the corresponding linear model for the semi-log plot.

x	0	1	2	3	4
$g(x)$					

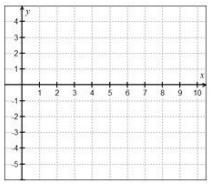


Problems 3-4, Given the exponential functions, Find the values of m and b such that $\log_b(f(x)) = mx + b$. Graph $\log_b(f(x)) = mx + b$ on the graph provided. Label the axes appropriately.

3. Given $g(x) = 16 \cdot 4^x$.



4. Given $g(x) = 27 \cdot 3^{-2x}$.



Problems 5- 8, Consider the data in the tables below. Determine whether the data can best be modeled by a linear, logarithmic, or exponential regression equation. Explain your choice. Support your answer with tables and graphs.

5.

x	1	2	3	4
y	3	5.8	7.4	8.5

6.

x	1	2	3	4
y	14	7	3.5	1.75

7.

x	1	2	3	4
y	3	8	13	18

8.

x	1	2	3	4
y	12	36	108	324

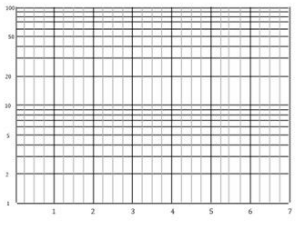
Problems 9-10, For each table of values given below,

A. Plot the points for the function on the semi-log graph.

B. What function would model the set of data values based on your graph? Explain your reasoning.

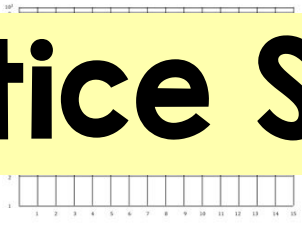
9.

x	1	2	3	4	5	6
y	2	6	10	16	22	30



10.

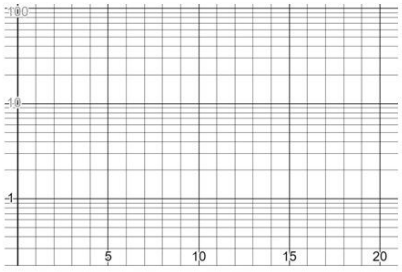
x	1	4	8	11	15



11. Given $h(t) = 84(0.75)^t$

A. Complete the table, plot the data $(t, h(t))$ on the grid provided.

t	$h(t) = 84(0.75)^t$
0	
2	
4	
10	
20	



B. Find a linear regression model $y = (\ln b)x + \ln(a)$ for the graph.

12. Write an equation for the linear model in the form $y = (\log_n b)x + \log_n a$. List the slope and y-intercept of each exponential equation when graphed on a semi-log plot.

A. $f(x) = 3.2(7)^x$

B. $g(x) = 8\left(\frac{2}{3}\right)^x$

Practice Sets

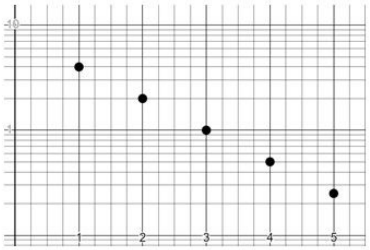
2.15 **Semi-Log Plots**
Practice Set 2

Name _____

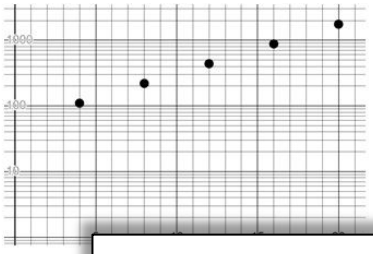
Date _____ Period _____

Problems 1-3, Assume exponential behavior for the data given. Each function is graphed on the semi-log plot where the vertical axis has been logarithmically scaled.

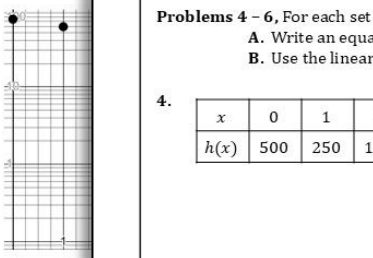
1. Find an equation for the function f in the form $y = ab^x$



2. A bacteria colony is measured every four days for a 20-day period. The data is shown, at right, on the semi-log plot. Let the function N give the total number of bacteria for t days. Write the function $N(t)$.



3. The function g shown at right can be modeled by which of the following functions?



A. $g(x) = 75 - \left(\frac{4}{5}\right)^x$

B. $g(x) = 75\left(\frac{5}{4}\right)^x$

C. $g(x) = 75\left(\frac{4}{5}\right)^x$

D. $g(x) = 75 - 80x$

Problems 4 – 6, For each set of data,

A. Write an equation for the linear model of the form $y = (\log_n b)x + \log_n a$

B. Use the linear model in part **A.** to find the exponential model $y = ab^x$ for the data.

4.

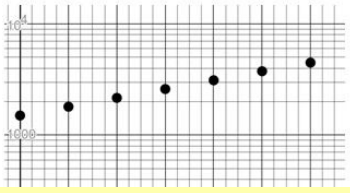
x	0	1	2	3	4
$h(x)$	500	250	125	62.5	31.25

5.

x	10	20	30	40	50
$k(x)$	36	108	324	972	2916

6.

x	0	1	2	3	4	5	6
$g(x)$	1500	1800	2160	2595	3110	3750	4480



Practice Sets

2.15 Semi-log Plots Set 1 **Form A**

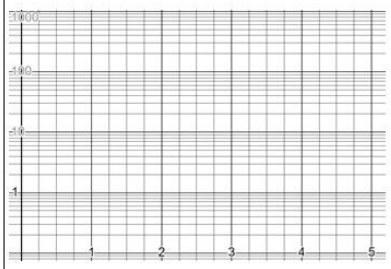
Name _____

Date _____Period _____

1. Given the exponential function $f(x) = 5(3)^x$, complete the table below.

x	0	1	2	3	4
$f(x)$					

2. Graph the points on the semi-log plot.



3. Use $f(x)$ to write the corresponding linear model for the semi-log plot.

Grade:

2.15 Semi-log Plots Set 1 **Form B**

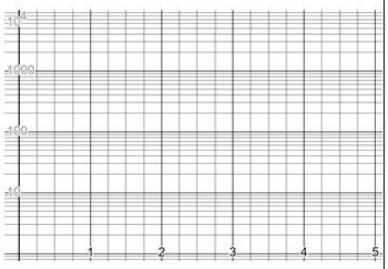
Name _____

Date _____Period _____

1. Given the exponential function $g(x) = 4(5)^x$, complete the table below.

x	0	1	2	3	4
$g(x)$					

2. Graph the points on the semi-log plot.



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2.15 Semi-log Plots Set 2 **Form A**

Name _____

Date _____Period _____

x	1	2	3	4	5
$g(x)$	30	70	150	340	750

1. Use the table above to write an equation for the linear model in the form $g(x) = (\log_n b)x + \log_n a$

2. Use the linear model from (1.) to write the equation of the exponential model $y = ab^x$ for this data.

3. Use your model from (2.) to find the y-value when $x = 6$.

Grade:

2.15 Semi-log Plots Set 2 **Form B**

Name _____

Date _____Period _____

x	1	2	3	4	5
$h(x)$	42	70	120	190	325

1. Use the table above to write an equation for the linear model in the form $h(x) = (\log_n b)x + \log_n a$

2. Use the linear model from (1.) to write the equation of the exponential model $y = ab^x$ for this data.

3. Use your model from (2.) to find the y-value when $x = 6$.

Grade:

Daily Lesson Quiz