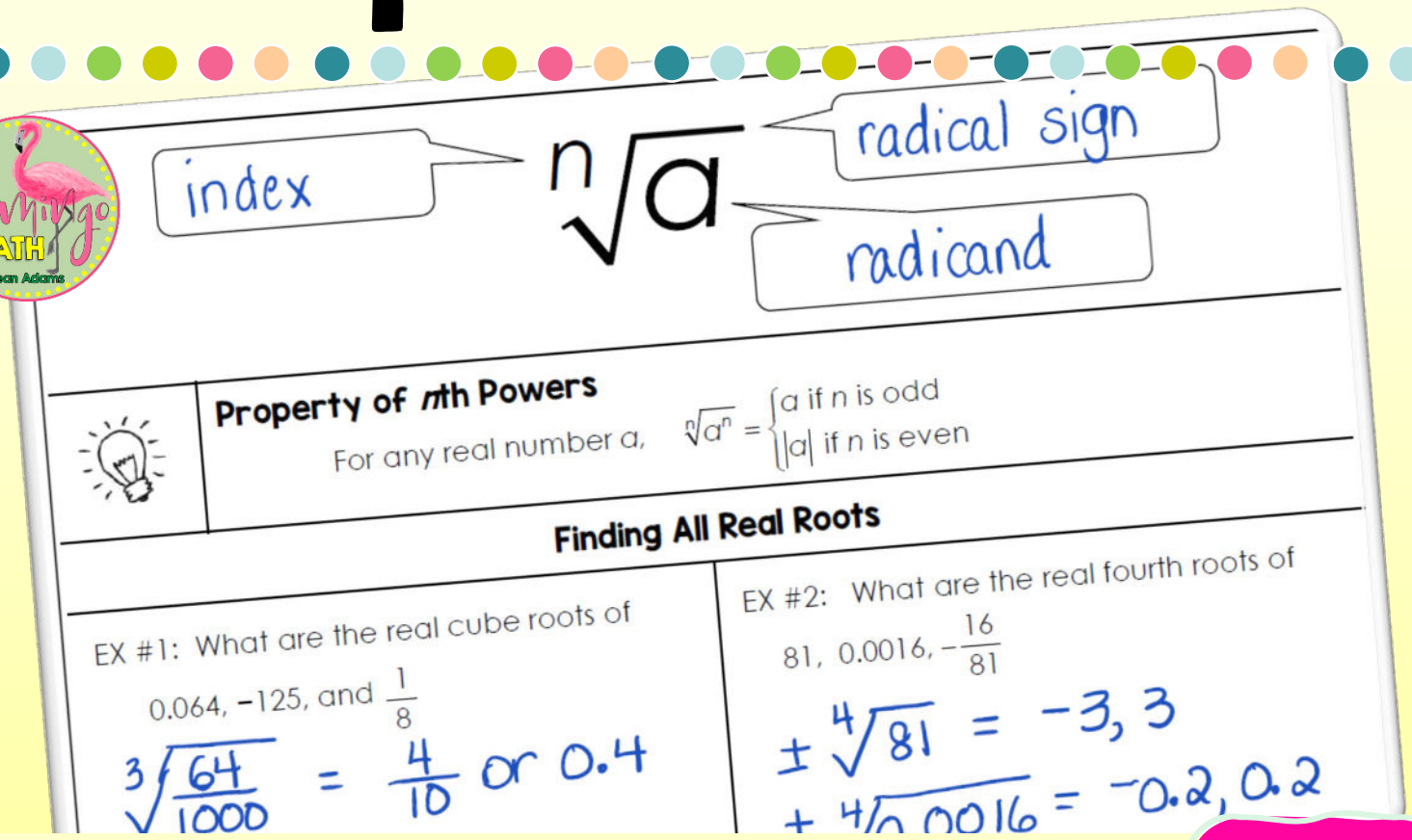
Algebra 2 Honors

# Roots & Radical Expressions




## Completely Editable

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## Lesson 6.I: Roots and Radical Expressions

**Principal root** – When a number has two real roots, the positive root is called the **principal root**. A radical sign indicates the principal root. The principal root of a negative number $a$ is $\sqrt{-a} = i\sqrt{a}$

### *n*th Root Definition

For any real numbers $a$ and $b$, and any positive integer $n$, if $a^n = b$, then $a$ is an ***nth root*** of b.

**Example:**

| If n is odd . . . | If n is even . . . | |
|---|---|---|
| There is one real $n^{th}$ root of b, denoted in radical form as $\sqrt[n]{b}$ | And b is positive, there are two real nth roots of b. | And b is negative, there are no real nth roots of b. |

**The only *n*th root of zero is _____.**

$\sqrt[n]{a}$

### Property of *n*th Powers

For any real number $a$, $\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ |a| \text{ if } n \text{ is even} \end{cases}$

### Finding All Real Roots

EX #1: What are the real cube roots of $0.064, -125$, and $\frac{1}{8}$

EX #2: What are the real fourth roots of $81, 0.0016, -\frac{16}{81}$

EX #3: What is each real-number root, if possible?

A. $\sqrt[3]{-27}$

B. $\sqrt[4]{-81}$

C. $\sqrt{(-7)^2}$

D. $\sqrt{-49}$

### Simplifying Radical Expressions

EX. #4: Find the simplified form of each radical expression.

A. $\sqrt{81x^4}$

B. $\sqrt[3]{a^{12}b^{15}}$

C. $\sqrt[4]{x^{12}y^{16}}$

### Using A Radical Expression

EX #5: One method for adjusting student test scores to relate the score to an AP test is to use the formula $A = 10\sqrt{R}$, where A is the adjusted score, and $R$ is the raw score. If the raw scores on one test range from 34 to 90, what is the range of the adjusted scores?

### Sometimes, Always, Never

EX #6: Is each equation always, sometimes, or never true? Explain your answer.

A. $\sqrt{x^4} = x^2$

B. $\sqrt{x^2} = x^3$

C. $\sqrt[3]{x^3} = x^2$

D. $\sqrt[3]{x^3} = |x|$

### Application

EX #7: A gourmet cheese company wants to ship cheese wedges that weight from 11 to 12 ounces in cartons that will hold 3 layers of cheese wedges in a 3 by 4 pattern. The height of each wedge can vary between 2.25 and 2.5 inches in height. The weight of each cheese wedge is related to its length by the formula $w = L^3 / 5$, where $L$ is the length in inches and $w$ is the weight in ounces. If you assume the dimensions of the carton are in whole inches, what size cartons should the company use for shipping the wedges?

### Application

**EX #8:** Johannes Kepler developed a formula to find the distance of a planet from the Sun in millions of miles, based on the number of Earth-days that it takes for the planet to orbit the Sun. The formula given is $d = \sqrt[3]{6t^2}$. If the length of a year on Mars is 687 Earth days, how far from the Sun is Mars?

NOTES

Name _____ Date_____

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## Roots and Radical Expressions Practice

Problems 1 – 6, find all the real square roots of each number.

| | |
|---|---|
| 1. 169 | 2. $\frac{9}{25}$ |
| 3. $-0.0025$ | 4. 400 |
| 5. 10,000 | 6. 0.0625 |

Problems 7 – 10, find all the real cube roots of each number.

| | |
|---|---|
| 7. $-343$ | 8. $\frac{27}{64}$ |
| 9. $-0.125$ | 10. $\frac{1000}{216}$ |

Problems 11 – 14, find all the real fourth roots of each number.

| | |
|---|---|
| 11. 81 | 12. $-16$ |
| 13. 256 | 14. 0.0625 |

Problems 15 – 20, find each real number root.

| | | |
|---|---|---|
| 15. $\sqrt{121}$ | 16. $\sqrt{0.04}$ | 17. $-\sqrt{225}$ |
| 18. $\sqrt[3]{-64}$ | 19. $-\sqrt[4]{0.0081}$ | 20. $\sqrt{-0.49}$ |

Problems 21 – 32, simplify each radical expression. Use absolute value symbols when needed.

| | | |
|---|---|---|
| 21. $\sqrt{144x^4}$ | 22. $\sqrt{0.64x^6}$ | 23. $\sqrt[3]{-8m^9}$ |
| 24. $\sqrt[4]{16x^8y^{12}}$ | 25. $\sqrt[5]{-32y^{10}}$ | 26. $\sqrt[3]{125b^{12}c^3}$ |
| 27. $\sqrt[3]{0.343m^6}$ | 28. $\sqrt{64(x+2)^6}$ | 29. $\sqrt[3]{\frac{27x^9}{216}}$ |
| 30. $-\sqrt{576x^3y^4z^5}$ | 31. $\sqrt[3]{(8x-3)^3}$ | 32. $\sqrt[5]{-32y^{15}}$ |

Problems 33 – 34, find the two real solutions of each equation.

| | |
|---|---|
| 33. $x^2 = 196$ | 34. $x^4 = \frac{81}{16}$ |

Problems 33 – 36, solve.

33. A cardboard box in the shape of a cube used for mailing packages has volume of $V = s^3$, where $s$ is the length of the side. What is the side length for a cube with volume 2744 in$^3$?

34. The formula for the volume of a cone with a base of radius $r$ is $V = \frac{\pi}{3}r^3$. Find the radius to the nearest hundredth centimeter, if the volume is 60 cm$^3$.

35. A chocolate manufacturer wants to ship fudge squares in cartons that will have 3 layers of fudge squares in $4 \times 4$ layers. The weight of the fudge square is related to its diameter by the formula $w = \frac{d^2}{5}$, where $d$ is the diameter, in inches, and $w$ is the weight, in ounces.

A. Find the range of the diameter of fudge squares that weigh between 10 to 12 ounces.

B. What size cartons should the manufacturer order to accommodate a shipment of the largest fudge squares? Assume the cartons are in whole-inch dimensions.

# Practice Sets

# Lesson Warm-Ups

3. Simplify. $\sqrt{-64x^2y^3}$

4. Simplify, $\sqrt[3]{-27m^3n^9}$

Name _____ Date _____ Period _____

**Lesson 6.1: Roots & Radical Expressions** **Warm-Up #2**

1. Simplify. $\pm\sqrt{144x^8}$

2. Simplify. $-\sqrt{(m^5-2)^6}$

3. Simplify. $\sqrt{-49x^3y^4}$

4. Simpli

**6.1 Roots and Radical Expressions** **Form A**

Name _____

Date _____ Period _____

**1.** Find the real cube root of $-0.027$

**2.** Find the real root. $\sqrt[4]{16}$

**3-5: Simplify, use absolute value symbols when needed.**

**3.** $\sqrt{49\,x^6}$

**4.** $\sqrt[3]{\frac{27}{64}}$

**5.** $\sqrt[4]{81\,x^{12}y^8}$

**6.1 Roots and Radical Expressions** **Form B**

Name _____

Date _____ Period _____

**1.** Find the real cube root of 0.125

**2.** Find the real root. $\sqrt[4]{81}$

**3-5: Simplify, use absolute value symbols when needed.**

**3.** $\sqrt{36\,x^2}$

**4.** $\sqrt[3]{\frac{125}{8}}$

**5.** $\sqrt[4]{16\,x^{20}y^8}$

# Daily Lesson Quiz