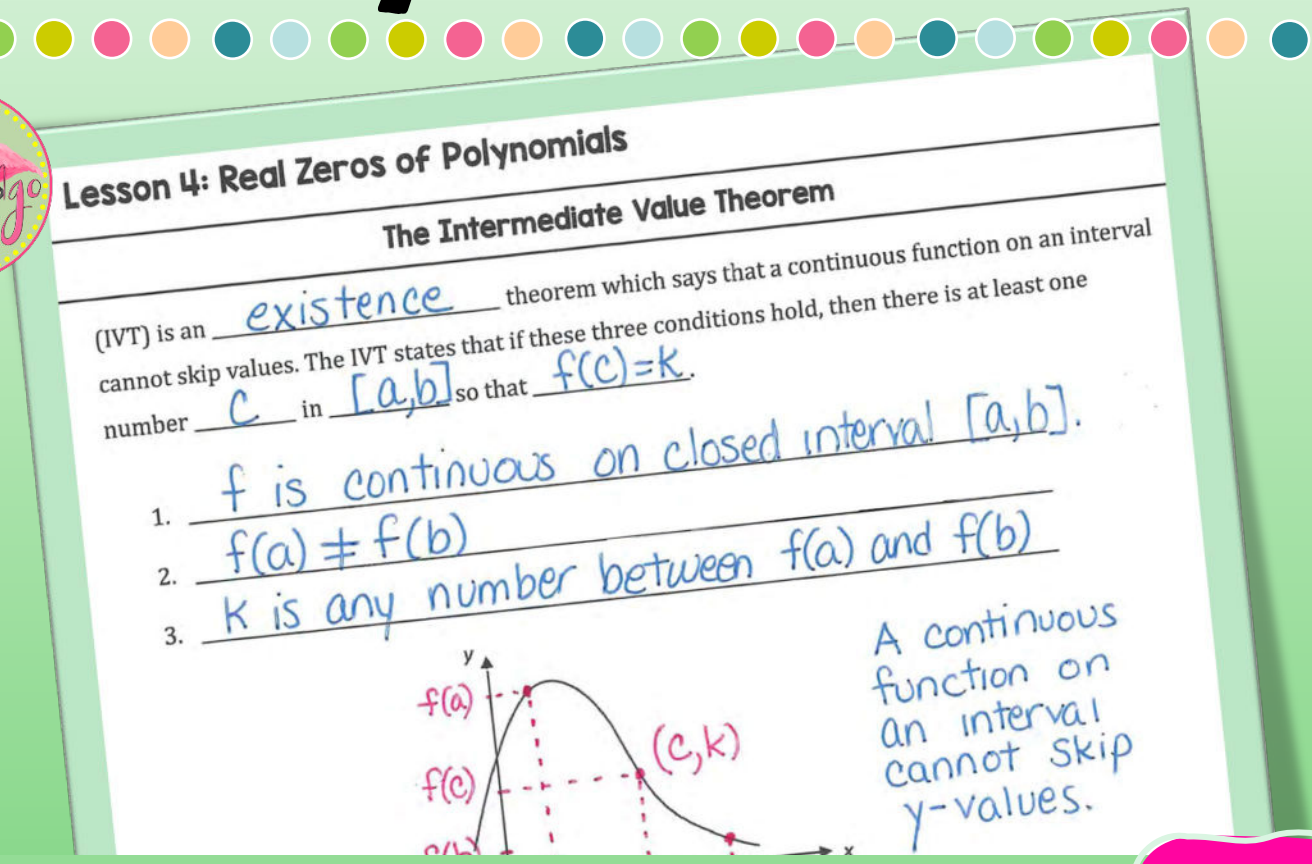
Precalculus Honors

Real Zeros of Polynomials



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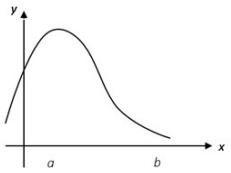
2.4

Lesson 4: Real Zeros of Polynomials

The Intermediate Value Theorem

(IVT) is an _____ theorem which says that a continuous function on an interval cannot skip values. The IVT states that if these three conditions hold, then there is at least one number _____ in _____ so that _____.

1. _____
2. _____
3. _____



Practice

EX #1: (Revisiting homework question) Would you b
on the interval $[1, 3]$? Why or why not?

EX #2: Use the Intermediate Value Theorem to show
$[0, 1]$.

For the rest of this lesson, we will explore techniques for finding the real zeros of a polynomial function. Recall from Algebra 2, that if b is a real zero of a polynomial function f then $f(b) = 0$, b is an x-intercept of the graph of f and b is a solution of the equation $f(x) = 0$.

The Division Algorithm

If $f(x)$ and $g(x)$ are polynomial functions and $g(x) \neq 0$, then there are unique polynomials $q(x)$ and $r(x)$ such that

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or}$$

$$f(x) = q(x) \cdot d(x) + r(x)$$

Dividend = (Quotient)(Divisor) + Remainder

Long Division vs. Synthetic Division

EX #3: Divide using Long Division
$(5x^2 - 17x - 12) \div (x - 4)$

EX #4: Divide using Synthetic Division
$(5x^2 - 17x - 12) \div (x - 4)$

Factor Theorems

ynomial function. . .

) is divided by _____, then the remainder is _____.

$f(k) = 0$, then $(x - k)$ is a _____ of $f(x)$.

$(x - k)$ is a factor of $f(x)$, then _____.

EX #5: Use the remainder theorem to evaluate $f(x) = 4x^2 - 10x - 21$ when $x = 5$.

EX #6: Is $(x + 3)$ a factor of $f(x) = 3x^3 + 4x^2 - 18x - 3$? Explain.

EX #7: Verify that $(x + 3)$ and $(x - 2)$ are factors of $f(x)$
Then find all the zeros of f.

The Rational Zeros

If f is a polynomial of degree _____ in descen
organized list of potential rational numbers

Possible Rational Zeros = $\frac{\text{factors of}}{\text{factors of}}$

Using the Ration

1. Find one root by using synthetic division.
2. Continue finding roots and dividing until you ha
3. Use any method for solving Quadratics to find th

EX #8: Find the rational zeros of $f(x) = x^3 - 7x -$

$\frac{p}{q} =$

EX #9: Find the rational zeros of $f(x) = 2x^3 + 3x^2$

$\frac{p}{q} =$

Note: When counting zeros of a polynomial, it should be noted that you count each zero as many times as its multiplicity. A polynomial function cannot have more real zeros than its degree.

Sometimes it is helpful to know the number and location of real zeros of a polynomial function written in descending powers of x. Descartes' Rule of Signs gives this information. This is optional to learn but helpful if needed. It will be necessary to count the number of variations in the sign of coefficients for $f(x)$ and, also for $f(-x)$.

Descartes' Rule of Signs

The number of positive real roots of a polynomial $P(x)$ with real coefficients is:

1. The same as the number of sign changes of $P(x)$, or
2. less than the number of sign changes of $P(x)$ by a positive even integer.

The number of negative real roots of a polynomial $P(x)$ with real coefficients is:

1. The same as the number of sign changes of $P(-x)$, or
2. less than the number of sign changes of $P(-x)$ by a positive even integer.

That is a lot to absorb, but it is not as complicated as it seems. Let's look at an example:

EX #11: Consider the polynomial $P(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$
What does Descartes' Rule of Signs tell you about the real roots?

NOTES

2.4 **Real Zeros of Polynomials Homework**

Name _____

Date _____ Period _____

Problems 1 – 5, Use synthetic division to divide. When finished, write the answer in the form $q(x) + \frac{r(x)}{d(x)}$ where q is the quotient polynomial, r is the remainder and d is the divisor.

1. $2x^2 + 5x - 12$ by $x + 4$

2. $5x^2 - 13x - 6$ by $x - 3$

3. $x^3 - 4x^2 - 17x + 6$ by $x - 3$

4. $(2x^3 + 14x^2 - 20x + 7) + (x + 6)$

5. $(4x^3 + 16x^2 - 23x - 15) + (2x + 1)$

Problems 6 – 9, List the potential rational zeros of each polynomial function. ***Do not attempt to find the zeros.***

6. $g(x) = 6x^4 - x^2 + 9$

7. $f(x) = 4x^5 - 8x^4 - x + 6$

8. $h(x) = x^3 + 8x^2 + 11x - 20$

9. $f(x) = 3x^4 - 2x^3 + x - 18$

Problems 10 – 11, Write the function in the form $f(x) = (x - k) \cdot q(x) + r(x)$ for the given value of k.

10. $f(x) = x^3 - x^2 - 15x + 8$ for $k = 4$

11. $f(x) = 12x^4 - 8x^3 - 7x^2 - 14$ for $k = \frac{3}{2}$

Problems 12 – 15, Verify that the given value of x is a solution of the polynomial, then find the remaining factors. **Use your results to write the complete factorization of *f*.**

12. $f(x) = 3x^3 + 2x^2 - 19x + 6;\ x = -3$

13. $f(x) = 4x^3 + 9x^2 - 49x + 30;\ x = 2$

14. $f(x) = 2x^3 + x^2 - 6x - 3;\ x = -\frac{1}{2}$

15. $f(x) = 6x^3 + 41x^2 - 9x - 14;\ x = -\frac{1}{2}$

Problems 16 – 19, What does Descartes' Rule of Signs say about the number of positive real roots and negative real roots for each polynomial function?

16. $P(x) = 3x^3 + x^2 - 6x - 18$

17. $P(x) = x^3 + 4x^2 + x - 6$

18. $P(x) = 2x^4 - x^3 - 5x + 9$

19. $P(x) = 2x^5 - x^4 - x^3 + 4x^2 - 3$

Problems 20 – 21, Using the Rational Zeros Theorem and synthetic division, find all the real zeros of the polynomial function.

20. $f(x) = x^3 - 4x^2 - 4x + 16$

21. $h(x) = 4x^3 + 20x^2 + 31x + 15$

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____

Lesson 2.5: Complex Zeros **Warm-Up # 1**

1. Write the polynomial in standard form.
$f(x) = (x + 2i)(x - 2i)(x + 3)$

2. Write the linear factorization of a polynomial whose degree is 3 and zeros are $x = 3$ (multiplicity of 1) and $x = \sqrt{3}$.

3. Find the complex zeros for:
$f(x) = x^4 + 17x^2 + 16$

4. Use the given zero to find the remaining zeros.
$f(x) = x^3 - 5x^2 + 9x - 45$; zero: $3i$

Name _____ Date _____ Period _____

Lesson 2.5: Complex Zeros **Warm-Up # 2**

1. Write the polynomial in standard form.
$f(x) = (x + i)(x - i)(x - 2)$

2. Write the linear factorization of a polynomial whose degree is 4 and zeros are $x = -3$ (multiplicity of 2) and $x = \sqrt{5}$.

3. Find the complex zeros for:
$f(x) = x^4 + 5x^2 + 4$

4. Use the given zero to find the remaining zeros.
$f(x) = 2x^3 + 2x^2 - 4x - 4$; zero: $\sqrt{2}$

2.4 Real Zeros of Polynomial Functions **Form A**

Name _____

Date _____ Period _____

1. List the possible rational zeros of:
$f(x) = 6x^3 + 11x^2 - 37x + 3$

2. Find $f(-4)$ using the Remainder Theorem for $f(x) = 3x^3 + 7x^2 - 13$

3.. Divide $(2x^4 + 4x^3 + 5x^2 - 3x - 2)$ by $(x + 2)$. Write answer as $q(x) + \frac{f(x)}{d(x)}$.

4. Write a polynomial function with a leading coefficient of -3 that has zeros at $x = -2, 0, 1$.

Grade:

2.4 Real Zeros of Polynomial Functions **Form B**

Name _____

Date _____ Period _____

1. List the possible rational zeros of:
$f(x) = 2x^3 + 16x^2 - 23x - 15$

2. Find $f(3)$ using the Remainder Theorem for $f(x) = 3x^3 + 5x^2 - 10x + 1$

3.. Divide $(2x^4 - 3x^3 + 4x^2 - 5x - 1)$ by $(x + 3)$. Write answer as $q(x) + \frac{f(x)}{d(x)}$.

4. Write a polynomial function with a leading coefficient of -2 that has zeros at $x = -3, 0, 1$.

Grade:

Daily Lesson Quiz