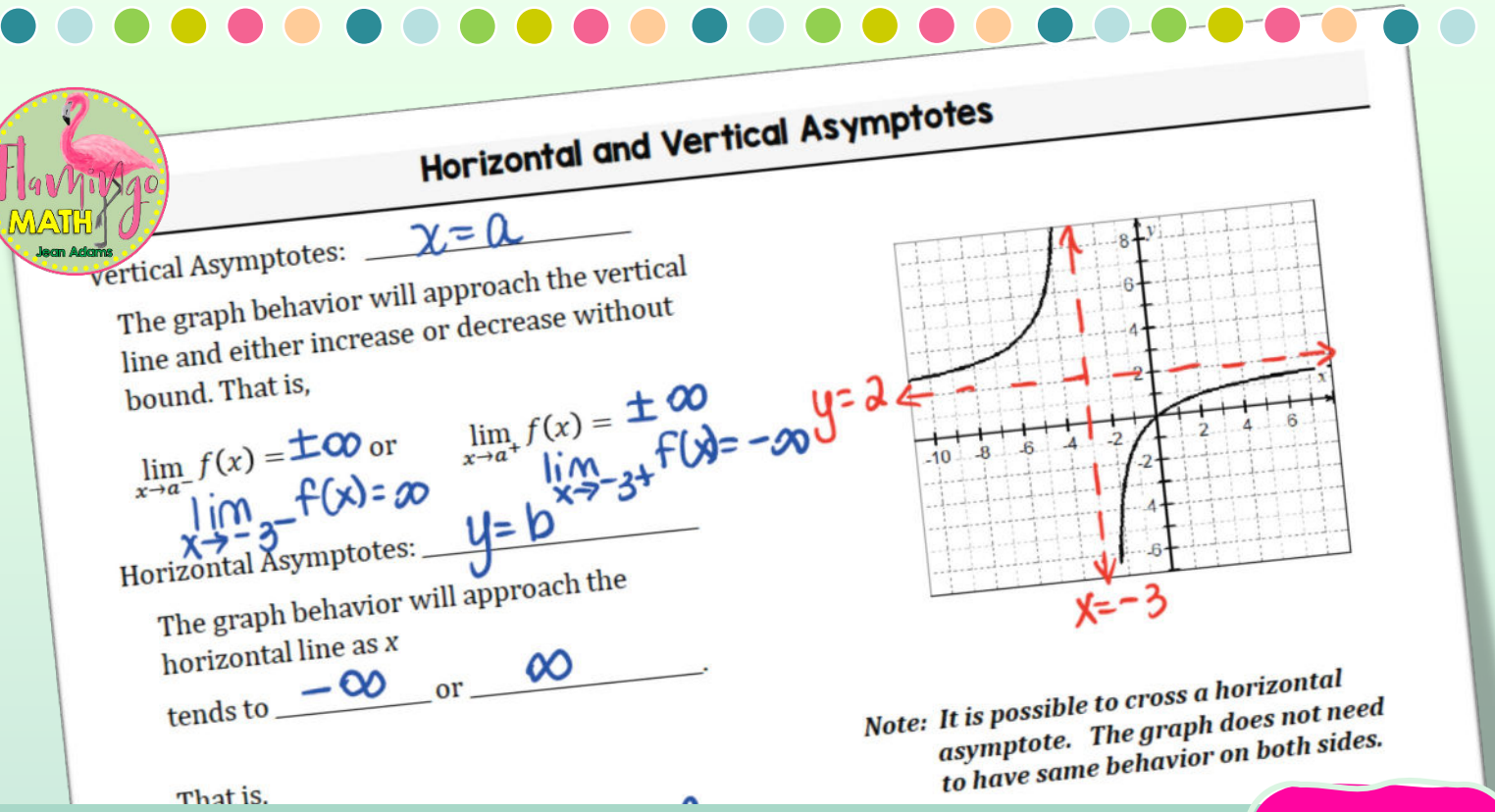
AP® Precalculus

Rational Functions and End Behavior



Completely Editable

1.7

Lesson 7-A: Rational Functions and End Behavior

The definition of a rational number is a ratio of integers. Similarly, ratios of polynomial functions are known as ***rational functions***. The domain for polynomial functions is all real numbers, but the domain of a rational function will be restricted from any values of x that cause ***division by zero***.

Rational Functions

Let $N(x)$ (numerator) and $D(x)$ (denominator) be polynomial functions, where $D(x) \neq 0$.

Then, a Rational Function is:

Finding Domain

EX #1: Find the domain for $f(x) = \frac{x+3}{x^2-8x+12}$.

Reading Arrow Notation

Throughout this course we will use **arrow notation** to show that x or $f(x)$ is approaching a particular value. The table below is a summary of proper notation and how it is read.

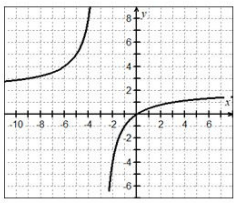
Symbol	How to Read and Interpret
$x \to a^-$	x approaches a from the left ($x < a$ but close to a)
$x \to a^+$	x approaches a from the right ($x >$ a but close to a)
$x \to \infty$	x approaches infinity (x increases without bound)
$x \to -\infty$	x approaches negative infinity (x decreases without bound)
$f(x) \to a$	The output ($y-value$ of the function) is approaching a
$f(x) \to \infty$	The output approaches infinity (the output increases without bound)
$f(x) \to -\infty$	The output approaches negative infinity (the output decreases without bound)

Horizontal and Vertical Asymptotes

Vertical Asymptotes: _____

The graph behavior will approach the vertical line and either increase or decrease without bound. That is,

$\lim_{x \to a^-} f(x) =$ or $\lim_{x \to a^+} f(x) =$



Horizontal Asymptotes: _____

The graph behavior will approach the horizontal line as x

tends to _____ or _____.

That is,

$\lim_{x \to -\infty} f(x) =$ or $\lim_{x \to \infty} f(x) =$

Note: It is possible to cross a horizontal asymptote. The graph does not need to have same behavior on both sides.

Finding Horizontal Asymptotes

Case:	Example:
(1) degree of numerator < degree of denominator The line _____ is the horizontal asymptote.	
(2) degree of numerator = degree of denominator The _____ of the leading coefficients is the horizontal asymptote. That is, $y =$ ———	
(3) degree of numerator > degree of denominator The graph has _____ horizontal asymptote.	

EX #2: Find the horizontal asymptotes for the given functions and provide a reason.

A. $f(x) = \frac{1}{2x^2+3x-15}$ **B.** $f(x) = \frac{3x-3}{x+3}$ **C.** $f(x) = \frac{3x^2+3x-18}{3x-5}$

Analyzing More End-Behavior of Rational Functions

The end behavior of a rational function will be affected most by the polynomial with the greater degree, as its values will dominate the values of the ration function for input values of large magnitude. For input values of large magnitude, a polynomial is dominated by its leading term. Therefore, the end behavior of a rational function can be understood by examining the corresponding quotient of the leading terms.

EX #3: Describe the end behavior for the graph of the rational functions. Analyze using limit notation to explain your answer.

A. $f(x) = \frac{2x^5 - 4x}{x^3 + 2}$ **B.** $g(x) = \frac{3x - 5x^4}{x + 2}$

Guided Notes

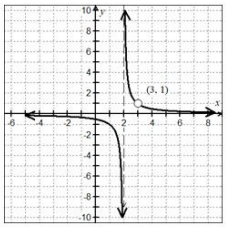
1.7 **Rational Functions and End Behavior Practice Set 1**

Name _____

Date _____ Period _____

Problems 1 – 4, Use the graph of each function to complete the limit notation statements.

1. $f(x) = \frac{x-3}{x^2-5x+6}$



There is a hole in the graph at: _____

Domain: _____

$\lim_{x\to-\infty} f(x) =$ _____

$\lim_{x\to\infty} f(x) =$ _____

$\lim_{x\to2^-} f(x) =$ _____

$\lim_{x\to2^+} f(x) =$ _____

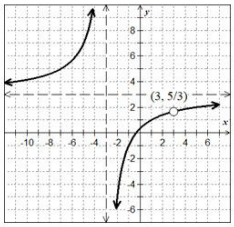
$\lim_{x\to3^-} f(x) =$ _____

$\lim_{x\to3^+} f(x) =$ _____

Horizontal Asymptote:

_____

2. $g(x) = \frac{3x^2-8x-3}{x^2-9}$



There is a hole in the graph at: _____

Domain: _____

$\lim_{x\to-\infty} f(x) =$ _____

$\lim_{x\to\infty} f(x) =$ _____

$\lim_{x\to3^-} f(x) =$ _____

$\lim_{x\to3^+} f(x) =$ _____

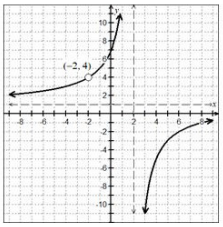
$\lim_{x\to3^-} f(x) =$ _____

$\lim_{x\to3^+} f(x) =$ _____

Horizontal Asymptote:

_____

3. $h(x) = \frac{x^2-12x-28}{x^2-4}$



There is a hole in the graph at: _____

Domain: _____

$\lim_{x\to-\infty} f(x) =$ _____

$\lim_{x\to\infty} f(x) =$ _____

$\lim_{x\to-2^-} f(x) =$ _____

$\lim_{x\to-2^+} f(x) =$ _____

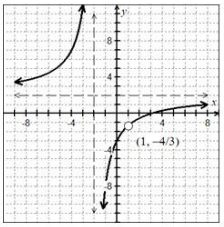
$\lim_{x\to2} f(x) =$ _____

$\lim_{x\to2^+} f(x) =$ _____

Horizontal Asymptote:

_____

4. $k(x) = \frac{2x^2-8x+6}{x^2+x-2}$



There is a hole in the graph at: _____

Domain: _____

$\lim_{x\to-\infty} f(x) =$ _____

$\lim_{x\to\infty} f(x) =$ _____

$\lim_{x\to-2^-} f(x) =$ _____

$\lim_{x\to-2^+} f(x) =$ _____

$\lim_{x\to1^-} f(x) =$ _____

$\lim_{x\to1^+} f(x) =$ _____

Horizontal Asymptote:

_____

Problems 5 – 8, Find the horizontal asymptotes for the given functions and provide a reason.

5. $f(x) = \frac{3x^2+5x-2}{x^2-5x+6}$

6. $g(x) = \frac{x^2+6x+8}{x^2-1}$

7. $h(x) = \frac{x-5}{x+4}$

8. $k(x) = \frac{2x^3+8}{4x^3-12}$

Problems 9 – 12, Find the slant asymptote equation, if one exists, or give a reason if not possible.

9. $g(x) = \frac{x^2-5x-6}{x+2}$

10. $k(x) = \frac{2x^3+5x-3}{x-4}$

Problems 25-28, Describe the end behavior for the graph of the rational functions. Analyze using limit notation to explain your answer.

13. $f(x) = \frac{x^4+6x^2}{x-1}$

14. $g(x) = \frac{8x-x^6}{3+x^2}$

15. $h(x) = \frac{4x^3+3x}{7-x^2}$

16. $k(x) = \frac{2x^3+x^4}{3x^2-5}$

Practice Sets

1.7 Rational Functions and End Behavior Set 1 **Form A**

Name _____

Date _____Period _____

Use the following rational function for the problems.

$$f(x) = \frac{3x - 4}{x + 2}$$

2. Determine the horizontal asymptote. Give a reason.

3. Use limit notation to describe the end behavior of the graph of $f(x)$.

4. Find the slant asymptote equation for the function. Show your work:

$$g(x) = \frac{2x^2 - 3x + 8}{x - 5}$$

Grade:

1.7 Rational Functions and End Behavior Set 1 **Form B**

Name _____

Date _____Period _____

Use the following rational function for the problems.

$$f(x) = \frac{5 - 2x}{x - 1}$$

2. Determine the horizontal asymptote. Give a reason.

3. Use limit notation to describe the end behavior of the graph of $f(x)$.

4. Find the slant asymptote equation for the function. Show your work:

$$g(x) = \frac{3x^2 + 5x - 7}{x + 4}$$

Grade:

Daily Lesson Quiz