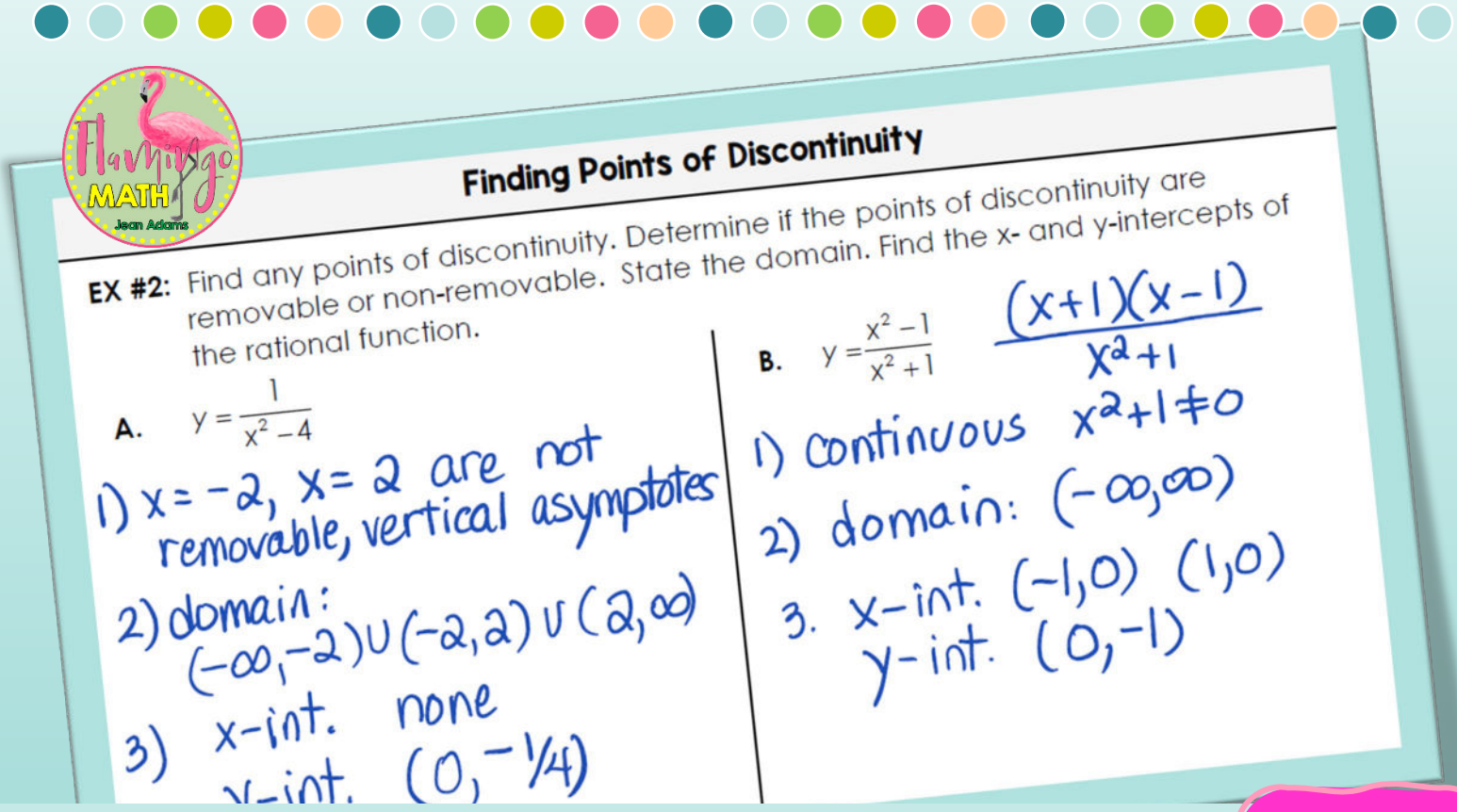
Algebra 2 Honors

Rational Functions



Guided Notes 8

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ALGEBRA 2 CURRICULUM

Scope and Sequence

Unit 8: Rational Functions

Lesson	Skills & Learning Objectives
[8.1] Inverse Variation	❑ Interpret, analyze and graph models of inverse variations ❑ Interpret and analyze models of combined variations ❑ Graph translations of inverse variations ❑ Solve real-world problems using inverse variations
[8.2] Reciprocal Functions Family	❑ Graph reciprocal functions ❑ Analyze and graph translations of reciprocal functions
[8.3] Rational Functions and Their Graphs	❑ Analyze and describe properties of rational functions ❑ Graph rational functions ❑ Identify the impact of parameters on graphs of rational functions
[8.4] Rational Expressions	❑ Simplify rational expressions ❑ Multiply and divide rational expressions ❑ State domain restrictions and understand when expressions are equivalent
[8.5] Adding & Subtracting Rational Expressions	❑ Find Least Common Multiples, Least Common Denominators ❑ Simplify complex fractions
[8.6] Solving Rational Equations & Inequalities	❑ Solve rational equations ❑ Use rational equations to solve real-world problems
[8.7] Work, Motion & Mixture Problems	❑ Solve work and motion problems

Lesson 1: Inverse Variation

Inverse variation is a relation represented by an equation of the forms:

$$xy = k,\ y = \frac{k}{x}, \text{ or } x = \frac{k}{y}, \text{ where } k \neq 0.$$

Joint variation is a relation in which one variable varies directly with respect to each of two or more variables.

EX. $z = 8xy$

$T = kPV$

Combined variation is a relation in which one variable varies with respect to each of two or more variables.

Combined Variation	Equation Form
z varies jointly with x and y	
z varies jointly with x and y and inversely with w	
z varies directly with x and inversely with the product wy	

EX #1: Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for the direct and inverse variations.

A.

x	y
0.2	8
0.5	20
1.0	40
1.5	60

B.

x	y
0.2	40
0.5	16
1.0	8.0
2.0	4.0

Lesson 2: Reciprocal Functions

Reciprocal function – any function that belongs to the family whose parent function is $f(x)=\frac{1}{x}$ where $x \neq 0$.

You can write a reciprocal function in the form $f(x)=\left(\frac{a}{x-h}\right)+k$, where $a \neq 0$ and $x \neq h$.

Branch – each piece of a discontinuous graph is called a branch.

The Reciprocal Function Family

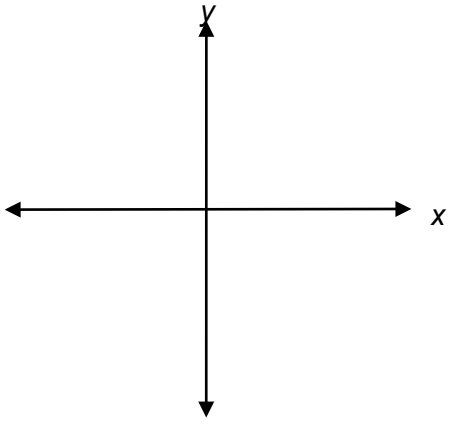
<table>
<tr><td colspan="2">Parent Function</td><td>$f(x)=\frac{1}{x};x\neq 0$</td></tr>
<tr><td colspan="2">Stretch</td><td rowspan="3">$f(x)=\frac{a}{x};x\neq 0$</td></tr>
<tr><td colspan="2">Shrink</td></tr>
<tr><td colspan="2">Reflection across x-axis</td></tr>
<tr><td colspan="2">Translation</td><td rowspan="2">$y=\frac{1}{x-h}+k;x\neq h$</td></tr>
<tr><td>Horizontal</td><td>Vertical</td></tr>
<tr><td colspan="2">Combined</td><td>$y=\frac{a}{x-h}+k;x\neq h$</td></tr>
</table>

The Parent Function

EX #1: Graph the Parent Function. State the domain, range, vertical asymptote and horizontal asymptote.

$f(x)=\frac{1}{x}$

Domain:	Vertical Asymptote:
Range:	Horizontal Asymptote:



Lesson 5: Add & Subtract Rational Expressions

Vocabulary	EXAMPLES:
Complex fraction a rational expression that has a fraction in its numerator or denominator, or in both its numerator and denominator.	
Least Common Multiple the product of the prime factors, each raised to the greatest power that occurs in any of the expressions.	**EX:** $12xy^2$ and $15x^3y$

Adding and Subtracting Fractions

Same Denominators:	
Different Denominators:	

Finding the Least Common Multiple

To find the LCM of two expressions, factor them. Then, use each factor the greatest number of times it occurs in any one expression.

EX #1: Find the LCM of each expression.

A. $2x + 4$ and $x^2 - x - 6$

B. $x^2 + 3x - 4$, $x^2 + 2x - 8$,and $x^2 - 4x + 4$

Lesson 6: Solving Rational Equations & Inequalities

A **rational equation** is any equation that contains one or more rational expressions. The preferred method for solving such equations is to eliminate the fractions by multiplying each side by the least common denominator.

Example: $\frac{2}{3}+\frac{5}{6}=\frac{1}{x}$

Procedure to Solve Rational Equations:

1. Clear Fractions by multiplying both sides of the equation by LCD of the rational expressions.
2. Collect like terms.
3. Isolate the variable and solve.
4. Check for extraneous solutions

NOTE: By multiplying each side of an equation by an algebraic expression, it is possible to introduce extraneous solutions.



BE SURE TO CHECK ALL SOLUTIONS IN THE ORIGINAL EQUATION TO CONFIRM THAT THEY ARE SOLUTIONS.

What causes the possibility of an extraneous solution?

Solving a Rational Equation

EX #1: Solve and check the solutions.

A. $\frac{x-2}{5}-\frac{x-5}{4}=-2$

B. $\frac{x-1}{x+2}=\frac{x^2+2x-3}{x+2}$