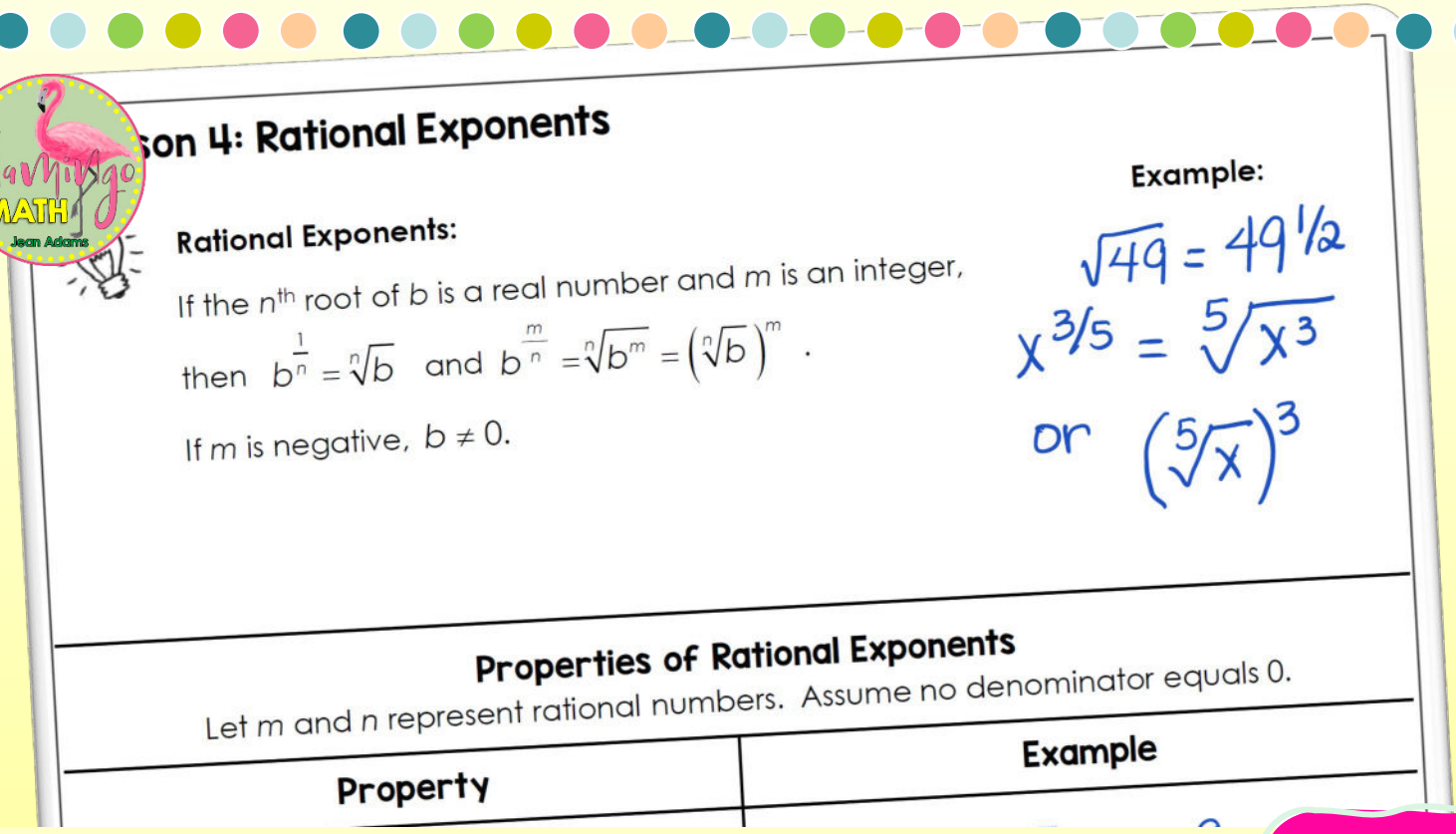
Algebra 2 Honors

Rational Exponents



son 4: Rational Exponents

Rational Exponents:

If the nth root of b is a real number and m is an integer,

then $b^{\frac{1}{n}} = \sqrt[n]{b}$ and $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$.

If m is negative, $b \neq 0$.

Example:

$\sqrt{49} = 49^{1/2}$

$x^{3/5} = \sqrt[5]{x^3}$

or $\left(\sqrt[5]{x}\right)^3$

Properties of Rational Exponents

Let m and n represent rational numbers. Assume no denominator equals 0.

Property	Example

Completely Editable

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Lesson 6.4: Rational Exponents

Rational Exponents:

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Example:

Properties of Rational Exponents

Let m and n represent rational numbers. Assume no denominator equals 0.

Property	Example
$b^m \bullet b^n = b^{m+n}$	
$\left(b^m\right)^n = b^{mn}$	
$(ab)^m = a^m b^m$	
$b^{-m} = \frac{1}{b^m}$	
$\frac{b^m}{b^n} = b^{m-n}$	
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	

Simplifying Expressions

EX #1: Find the simplified form.

A. $3^{\frac{1}{2}} \bullet 12^{\frac{1}{2}}$

B. $64^{\frac{1}{3}}$

C. $4^{\frac{1}{4}} \bullet 64^{\frac{1}{4}}$

D. $(-32)^{\frac{3}{5}}$

Converting Between Forms

EX #2: Rewrite each of the following in radical form.

A. $w^{0.2}$

B. $w^{-\frac{4}{6}}$

EX #3: Rewrite each of the following in exponential form.

A. $\sqrt[4]{x^3}$

B. $\left(\sqrt[5]{y}\right)^4$

C. $\left(\sqrt{6x}\right)^4$

D. $\sqrt[4]{\left(5xy^2\right)^3}$

Using Rational Exponents

EX #4: Kepler's Third Law of Orbital Motion shows how you can approximate the period P (in Earth years) it takes a planet to complete one orbit of the sun. The function $P = d^{\frac{3}{2}}$, where d is the distance from the planet to the sun in astronomical units (AU – where d is the distance from Earth to the sun.)

A. If Mars is 1.52 AU from the sun, how many Earth years does it take Mars to orbit the sun?

B. Find the length, in Earth years, of a Venusian year if Venus is 0.72 AU from the sun.

C. Find the length, in Earth years, of a Jovian year if Jupiter is 5.46 AU from the sun.

Combining Radical Expressions

C. $\frac{\sqrt{x^3}}{\sqrt[3]{x^2}}$

Simplifying Numbers with Rational Exponents

EX #6: Write each number in simplest form.

A. $32^{-\frac{4}{5}}$

B. $16^{\frac{3}{4}}$

C. $9^{-3.5}$

Writing Expressions in Simplest Form

EX #7: Write each expression in simplest form.

A. $\left(8x^{15}\right)^{-\frac{1}{3}}$

$\left(y^{-\frac{3}{3}}\right)$

NOTES

Name _____ Date_____

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Rational Exponents Practice

Problems 1 – 6, determine whether each statement is true or false. If it is false, explain why.

1. $27^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} = (27 \cdot 8)^{\frac{1}{3}}$
2. $27^{\frac{1}{3}} - 8^{\frac{1}{3}} = (27 - 8)^{\frac{1}{3}}$
3. $\sqrt{\sqrt[3]{5}} = 5^{\frac{1}{6}}$
4. $-81x^{-\frac{1}{4}} = \frac{-3}{\sqrt[4]{x}}$
5. $\left(25^{\frac{1}{2}} + 9^{\frac{1}{2}}\right)^3 = (5+3)^3$
6. $\left(\frac{1}{2}\right)^{-2} \cdot \left(\frac{x}{y^{-2}}\right)^2 = 4x^2y^6$

Problems 7 – 10, identify the exponent.

7. $\sqrt{\sqrt{7}} = 7^P$
8. $4\sqrt[3]{16} = 4^P$
9. $\sqrt[3]{\sqrt{5}} = 5^P$
10. $(0.01)^{\frac{1}{2}} = 10^P$

Problems 11 – 14, write in radical form.

11. $x^{\frac{2}{3}}$
12. $m^{-\frac{1}{2}}$
13. $y^{1.2}$
14. $t^{-\frac{3}{5}}$

Problems 15 – 18, write in exponential form.

15. $\sqrt{y^3}$
16. $\sqrt[6]{m}$
17. $\sqrt[5]{x^2}$
18. $\sqrt[5]{9x^{-1}}$

Problems 19 – 36, simplify each expression. Assume that all variables are positive.

19. $64^{\frac{2}{3}}$
20. $32^{\frac{1}{5}}$
21. $(-8)^{\frac{2}{3}}$
22. $9^{1.5}$
23. $(-27)^{\frac{2}{3}}$
24. $(-125)^{-\frac{1}{3}}$
25. $x^{-\frac{2}{3}} \cdot x^{\frac{1}{2}}$
26. $m^{\frac{5}{7}} \div m^{\frac{3}{14}}$
27. $6(16)^{-\frac{3}{4}}$
28. $7\left(x^{\frac{2}{3}}\right)^{-1}$
29. $(-6 \cdot 36)^{-\frac{2}{3}}$
30. $32^{1.2}$
31. $\left(6x^{\frac{2}{5}}\right)\left(2x^{\frac{1}{4}}\right)$
32. $(9x^8y^{-4})^{\frac{1}{2}}$
33. $7^{\frac{1}{2}} \cdot 28^{\frac{1}{2}}$
34. $\left(x^{\frac{2}{3}} \cdot y^{\frac{1}{6}}\right)^{-3}$
35. $\left(\frac{64x^{12}}{125y^{15}}\right)^{\frac{2}{3}}$
36. $\left(\frac{x^{-\frac{2}{7}}}{y^{-\frac{1}{5}}}\right)^{15}$

Problems 37 – 38, solve.

37. The formula for the ratio of radioactive carbon, R, to non-radioactive carbon left in a sample from an organism that died T years ago is approximated by the formula below , where A is the ratio of radioactive carbon to nonradioactive carbon in the living organism . What percent of A is left after 2500 years?

$R = A(2.7)^{-\frac{T}{8033}}$

Practice Sets

Lesson Warm-Ups

3. Write the expression in radical form. $(mn)^{\frac{1}{3}}$	4. Write the expression in exponential form. $\sqrt{5x}$

Name _____ Date _____ Period _____

Lesson 6.4: Rational Exponents **Warm-Up #2**

1. Simplify. $64^{\frac{1}{2}}$	2. Simplify $5^{\frac{1}{2}} \cdot 20^{\frac{1}{2}}$
3. Write the expression in radical form. $(2b)^{\frac{1}{4}}$	4. Write the ex $\sqrt[3]{m^2}$

6.4 Rational Exponents **Form A**

Name _____

Date _____ Period _____

1. Write in radical form. $(9x)^{\frac{3}{5}}$
2. Simplify. $5^{\frac{1}{2}} \times 20^{\frac{1}{2}}$
3. Multiply and simplify. $(\sqrt[5]{7})(\sqrt{7})$
4. Simplify. $(32)^{-\frac{4}{5}}$

6.4 Rational Exponents **Form B**

Name _____

Date _____ Period _____

1. Write in radical form. $(5x)^{\frac{2}{3}}$
2. Simplify. $7^{\frac{1}{2}} \times 28^{\frac{1}{2}}$
3. Multiply and simplify. $(\sqrt{3})(\sqrt[4]{3})$
4. Simplify. $(16)^{-\frac{3}{4}}$

Daily Lesson Quiz