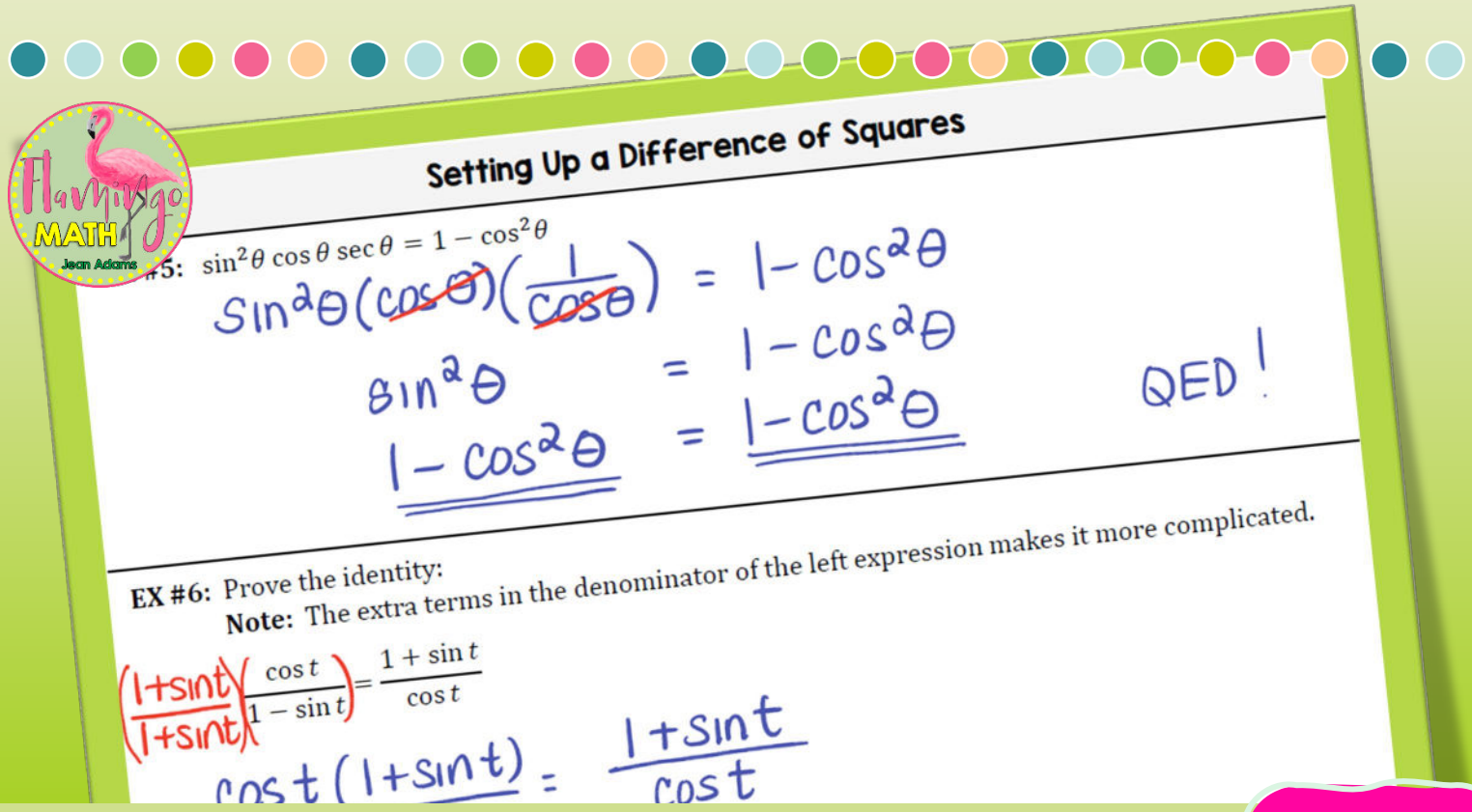
Precalculus Honors

Trigonometric Identities



Completely Editable

5.2

Lesson 2: Proving Trigonometric Identities

GUIDELINES TO VERIFYING TRIGONOMETRIC IDENTITIES

1. Pick one side of the equation to work with. Usually, it is easier to start with the more complicated side. Simplify the chosen side until it is identical to the other side. Do not work across the equal sign.
2. Change every expression to be in terms of sine or cosine.
3. Look for *algebraic* operations you can perform. Such as:
   - **A.** add fractions
   - **B.** factor something out
   - **C.** distribute
   - **D.** square terms
   - **E.** multiply by conjugates
   - **F.** multiply terms
   - **G.** multiply by a clever form of 1
4. Look for *trigonometric* things you can do. Such as:
   - **A.** If there are squares of functions, think about using Pythagorean properties.
   - **B.** Remember that $(\cos x)(\sec x) = (\sin x)(\csc x) = (\tan x)(\cot x) = 1$, ALWAYS!
5. Keep looking at the answer, that's your goal, to make sure you are on the right path.
6. Remember, there will often be more than one way to verify an identity. Don't quit! If one idea doesn't work, try something else.
7. When you are finished put Q.E.D. at the end. (Quod Erat Demonstrandum – "which was to be demonstrated.")

Proving an Algebraic Identity

EX #1: Prove the algebraic identity: $\frac{x^2-1}{x-1} - \frac{x^2-1}{x+1} = 2$

Proving Identities

EX #2: Prove the identity.

Note: *Work with the left side unless the right side is obviously more complicated.*

$\frac{1}{\cos^2\beta} - 1 = \tan^2\beta$

EX #3: Prove the identity. $\tan x + \cot x = \sec x \csc x$

EX. #4: Match the expression with one of the following:

A.) $2\cot x \csc x$ **B.)** $\frac{1}{\sec x}$

$\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1}$

Setting Up a Difference of Squares

EX. #5: $\sin^2\theta \cos\theta \sec\theta = 1 - \cos^2\theta$

EX #6: Prove the identity:

Note: The extra terms in the denominator of the left expression makes it more complicated.

$\frac{\cos t}{1 - \sin t} = \frac{1 + \sin t}{\cos t}$

EX #7: Prove the identity: $1 - \frac{\sin^2 x}{1+\cos x} = \cos x$

Identities Useful in Calculus

EX #8: Prove: $\csc\theta - \sin\theta = \cot\theta \cos\theta$

EX #9: Prove: $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

NOTES

5.2 **Proving Trigonometric Identities Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, Write the expression in terms of sine and cosine only.

1. $\tan\theta \cdot \csc\theta$

2. $\cot\theta \cdot \csc\theta$

3. $\sec\theta \cdot \cot\theta$

4. $\frac{\cot\theta}{\csc\theta}$

Problems 5 – 6, Multiply each expression by the indicated fraction and simplify.

5. $\frac{\tan\theta}{\sec\theta-1}$ by $\frac{\sec\theta+1}{\sec\theta+1}$

6. $\frac{\cot\theta}{1+\csc\theta}$ by $\frac{1-\csc\theta}{1-\csc\theta}$

Problems 7 – 9, Tell whether the statement is an identity or not.

7. $\frac{\sin^2 x+\cos^2 x}{\csc x} = \sin x$

8. $\sin x \sec x = \cot x$

9. $\frac{\tan x}{\sec x} = \sin x$

Problems 10 – 27, Prove the identity.

10. $\sin x(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

11. $(1-\tan x)^2 = \sec^2 x - 2\tan x$

12. $\sin\theta \csc\theta - \sin^2\theta = \cos^2\theta$

13. $\cos x(\tan x + \cot x) = \csc x$

14. $\csc x - \cot x = \frac{\sin x}{1+\cos x}$

15. $\frac{\sin^2\theta}{1-\cos\theta} - 1 = \cos\theta$

16. $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2\tan x$

17. $\frac{\tan\theta}{1+\sec\theta} + \frac{1+\sec\theta}{\tan\theta} = 2\csc\theta$

18. $\tan y - \frac{\sec^2 y}{\tan y} = -\cot y$

19. $\frac{\cos\theta+1}{\cos\theta-1} = \frac{1+\sec\theta}{1-\sec\theta}$

20. $\frac{\sin x}{\sin x-\cos x} = \frac{1}{1-\cot x}$

21. $\frac{\sec x+\tan x}{\cot x+\cos x} = \tan x \sec x$

22. $\frac{\cos\theta}{\sec\theta-\tan\theta} = 1+\sin\theta$

23. $\frac{\csc\alpha}{1+\cot^2\alpha} = \sin\alpha$

24. $\sin\theta + \cos\theta\cot\theta = \csc\theta$

25. $\cos x + \sin x \tan x = \sec x$

26. $\frac{\sec x-1}{1-\cos x} = \sec x$

27. $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____

Lesson 5.2: Proving Trigonometric Identities **Warm-Up # 1**

1. Verify:
$\sec^2(\theta) - \csc^2(\theta) = \tan^2(\theta) - \cot^2(\theta)$

2. Verify:
$\sec(x) - \tan(x)\sin(x) = \frac{1}{\sec(x)}$

3. Verify:
$\frac{\sec(\theta)\sin(\theta)}{\tan(\theta) + \cot(\theta)} = \sin^2(\theta)$

Name _____ Date _____ Period _____

Lesson 5.2: Proving Trigonometric Identities

2. Verify:
$(\csc^2(\theta))(\cos^2(\theta)) = \csc^2(\theta) - 1$

2. Verify:
$\frac{1 + \cos(x)}{\sin(x)} = \csc$

3. Verify:
$\frac{\sec(\theta)}{\cos(\theta)} - \frac{\tan(\theta)}{\cot(\theta)} = 1$

Daily Lesson Quiz

5.2 Verifying Trigonometric Identities **Form A**

Name _____

Date _____ Period _____

1. Complete the Reciprocal Identities.

$\sin u =$ $\csc u =$

$\frac{1}{\sec u} =$ $\tan u =$

$\frac{1}{\cos u} =$ $\frac{1}{\cot u} =$

2. Complete the Quotient Identities:

$\tan u =$

$\cot u =$

3. Write the Pythagorean Identities:

Grade:

5.2 Verifying Trigonometric Identities **Form B**

Name _____

Date _____ Period _____

1. Complete the Reciprocal Identities.

$\cos u =$ $\csc u =$

$\frac{1}{\cot u} =$ $\sec u =$

$\frac{1}{\tan u} =$ $\frac{1}{\sin u} =$

2. Complete the Quotient Identities:

$\tan u =$

$\cot u =$

3. Write the Pythagorean Identities:

Grade: