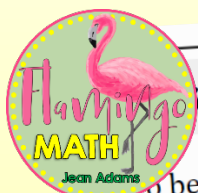


Precalculus Honors

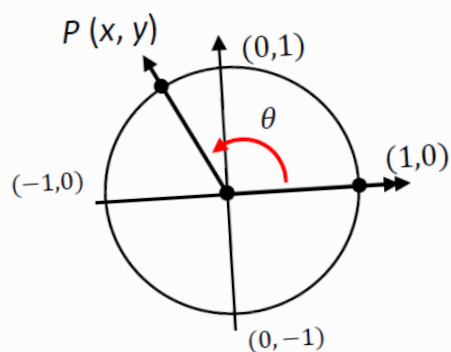
Properties of Trigonometric Functions



Lesson 5: Properties of the Trigonometric Functions

To be successful in the study of calculus it is important to know some of the basic properties that occur with the six trigonometric functions

Ratios on the Unit Circle



EX #1: In the sketch, at left, θ is in standard position and $P(x, y)$ is a point on the unit circle that corresponds to θ . Complete the following in terms of (x, y) :

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{1}{y}$$

$$\sec \theta = \frac{1}{x}$$

$$\cot \theta = \frac{x}{y}$$

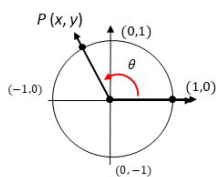
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4.5

Lesson 5: Properties of the Trigonometric Functions

To be successful in the study of calculus it is important to know some of the basic properties that occur with the six trigonometric functions

Ratios on the Unit Circle



EX #1: In the sketch, at left, θ is in standard position and $P(x, y)$ is a point on the unit circle that corresponds to θ . Complete the following in terms of (x, y) :

$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
$\csc \theta =$	$\sec \theta =$	$\cot \theta =$

Domain and Range of the Trigonometric Functions

Function	Undefined on $[0, 2\pi)$	Domain	Range
sine			
cosine			
tangent			
cosecant			
secant			
cotangent			

Periodic Functions and Finding Exact Values

The behavior of sine and cosine repeats after one revolution. This repetitive (or cyclic) pattern is called **periodic**.

Therefore, one cycle for sine and cosine has a period _____, while the tangent function has a period _____.

EX #2: Evaluate without a calculator by using the period of the function and values on the unit circle.

A. $\sin\left(\frac{23\pi}{2}\right)$	B. $\cos\left(\frac{17\pi}{4}\right)$
C. $\tan\left(-\frac{9\pi}{2}\right)$	D. $\sin\left(-\frac{29\pi}{6}\right)$

Trigonometric Identities

Look back at the ratios from Example 1, can you name the reciprocal identities and quotient identities in terms of a trigonometric function?

Reciprocal Identities	Quotient Identities
$\csc \theta =$	$\tan \theta =$
$\sec \theta =$	$\cot \theta =$
$\cot \theta =$	

Finding Exact Values Given a Terminal Point

When given the terminal point of an angle rotated on a coordinate plane, you should create a right triangle by dropping a perpendicular line back to the x-axis and use the Pythagorean Theorem to find the hypotenuse.

EX #3: Find the exact values of each of the six trigonometric functions of an angle θ if $(-2, 5)$ is a point on its terminal side.

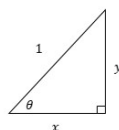
Using Fundamental Identities

EX #4: Given $\sin \theta = \frac{2\sqrt{13}}{13}$ and $\cos \theta = \frac{3\sqrt{13}}{13}$, find the exact values of the four remaining trigonometric functions of θ using identities.

EX #5: Use properties of the trigonometric functions to find the exact value of each expression.

A: $\tan 50^\circ - \frac{\sin 50^\circ}{\cos 50^\circ}$ B: $(\cos 40^\circ)(\sec 40^\circ)$

Pythagorean Identities



Pythagorean Identities

If $P = (x, y)$ is the point on the unit circle that corresponds to the angle θ , then the equation of the unit circle is _____. We know that $y = \sin \theta$ and $x = \cos \theta$, so we can substitute these into our equation to give us

There are two more Pythagorean Identities we can derive . . .

Divide each side of the equation by $\cos^2 \theta$:

Divide each side of the equation by $\sin^2 \theta$:

Using Identities

EX #6: Given that $\tan \theta = -\frac{1}{2}$ and $\sin \theta > 0$, find the exact value of each of the remaining five trigonometric

NOTES

4.5

Properties of Trigonometric Functions Homework

Name _____

Date _____ Period _____

Problems 1 – 8, Use the periodic properties of the trigonometric functions to find the exact value of each expression. Do not use a calculator.

1. $\sin\left(\frac{17\pi}{4}\right)$	2. $\cos(21\pi)$	3. $\tan\left(\frac{19\pi}{6}\right)$	4. $\csc 390^\circ$
5. $\tan 405^\circ$	6. $\sec\left(\frac{25\pi}{6}\right)$	7. $\csc\left(\frac{9\pi}{2}\right)$	8. $\cos(420^\circ)$

Problems 9 – 12, Name the quadrant in which the angle θ lies.

9. $\cos \theta < 0, \sin \theta > 0$	10. $\sin \theta > 0, \tan \theta < 0$
11. $\cot \theta > 0, \sec \theta < 0$	12. $\sin \theta < 0, \cos \theta > 0$

Problems 13 – 16, Given sine and cosine values, find the exact values for each of the four remaining trigonometric functions.

13. $\sin \theta = \frac{2\sqrt{2}}{3}, \cos \theta = -\frac{1}{3}$

14. $\sin \theta = \frac{3}{5}, \cos \theta = -\frac{4}{5}$

15. $\sin \theta = -\frac{\sqrt{5}}{5}, \cos \theta = \frac{2\sqrt{5}}{5}$

16. $\sin \theta = -\frac{\sqrt{3}}{2}, \cos \theta = -\frac{1}{2}$

Problems 17 – 20, Find the exact value of the remaining trigonometric functions of θ .

17. $\sin \theta = \frac{5}{13}; \theta$ in Quadrant II

18. $\tan \theta = \frac{4}{3}; \cos \theta < 0$

19. $\sec \theta = 2; \tan \theta < 0$

20. $\cos \theta = -\frac{\sqrt{10}}{10}; \pi < \theta < \frac{3\pi}{2}$

Problems 21 – 26, Use properties of the trigonometric functions to find the exact values of each expression.

21. $\sin(75^\circ) \csc(75^\circ)$

22. $\sec^2 47^\circ - \tan^2 47^\circ$

23. $\cot 25^\circ - \frac{\cos 25^\circ}{\sin 25^\circ}$

24. $\cos\left(-\frac{\pi}{18}\right) \cdot \sec\left(\frac{19\pi}{18}\right)$

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 4.5 - Properties of Trigonometric Functions Warm-Up # I	
1. Given $\theta = \frac{15\pi}{4}$, find $\sin \theta$ _____ $\cos \theta$ _____ $\tan \theta$ _____	2. Name the quadrant in which angle θ lies if $\tan \theta < 0$, and $\sin \theta > 0$.
3. Given $\sin \theta = \frac{2\sqrt{3}}{3}$, $\cos \theta = -\frac{\sqrt{3}}{3}$, find the remaining four functions.	

Name _____ Date _____	
Lesson 4.5 - Properties of Trigonometric Functions	
1. Given $\theta = \frac{17\pi}{6}$, find $\sin \theta$ _____ $\cos \theta$ _____ $\tan \theta$ _____	2. Name the quadrant in which angle θ lies if $\cos \theta < 0$, and $\tan \theta > 0$.
3. Given $\sin \theta = \frac{\sqrt{6}}{6}$, $\cos \theta = -\frac{5\sqrt{6}}{6}$, find the remaining four functions.	

4.5 Properties of Trigonometric Functions Form A		
Name _____		
Date _____ Period _____		
1. Given $\theta = \frac{31\pi}{4}$, find		
$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
2. Name the quadrant in which the angle θ lies if $\cos \theta < 0$, $\tan \theta > 0$		
3. Given $\sin \theta = \frac{2\sqrt{5}}{5}$, $\cos \theta = -\frac{\sqrt{5}}{5}$ find remaining four trigonometric functions		

4.5 Properties of Trigonometric Functions Form B		
Name _____		
Date _____ Period _____		
1. Given $\theta = \frac{19\pi}{6}$, find		
$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
2. Name the quadrant in which the angle θ lies if $\sin \theta < 0$, $\sec \theta > 0$		
3. Given $\sin \theta = \frac{\sqrt{10}}{10}$, $\cos \theta = -\frac{3\sqrt{10}}{10}$ find remaining four trigonometric functions		

Daily Lesson Quiz

Grade: _____	Grade: _____
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