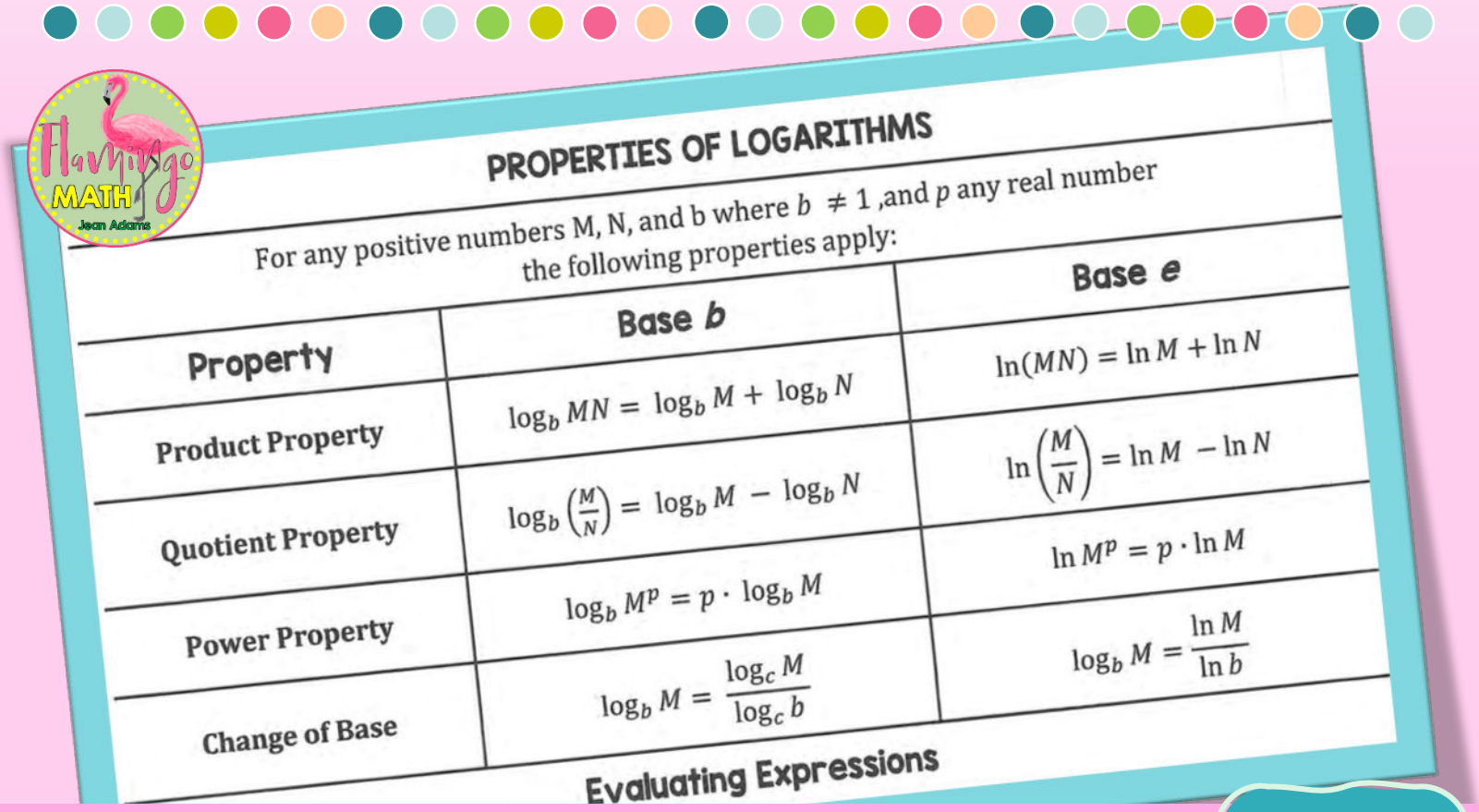
Precalculus Honors

Properties of Logarithmic Functions



PROPERTIES OF LOGARITHMS

For any positive numbers M, N, and b where $b \neq 1$, and p any real number the following properties apply:

Property	Base b	Base e
Product Property	$\log_b MN = \log_b M + \log_b N$	$\ln(MN) = \ln M + \ln N$
Quotient Property	$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$	$\ln \left(\frac{M}{N}\right) = \ln M - \ln N$
Power Property	$\log_b M^p = p \cdot \log_b M$	$\ln M^p = p \cdot \ln M$
Change of Base	$\log_b M = \frac{\log_c M}{\log_c b}$	$\log_b M = \frac{\ln M}{\ln b}$

Evaluating Expressions

Completely Editable

3.3

Lesson I: Exponential and Logistic Functions

Definition

Let a and b be real number constants. An _____ function in x is a function that can be written in the form, _____ where a is _____, b is _____, and _____. The constant a is the _____ value of f (the value at $x = 0$), and b is the _____.

Ex #1: Which of the following are exponential functions? Why?

Identifying Exponential Functions

Function	Initial Value	Base	Why?
$f(x) = 2^x$			
$g(x) = 4x^{-3}$			
$h(x) = 5(e^{-x})$			
$f(x) = 3 \cdot 4^x$			

Ex #2: Compute each of the following without a calculat

$f(x) = -5(8)^x$ for $x = \frac{1}{3}$

Finding an Exponential Function from its Table of Values

Ex #3: Determine the formulas for the functions from values given in the table below.

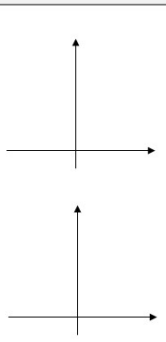
Values for Two Exponential Functions		
x	$f(x)$	$g(x)$
-2	$\frac{3}{4}$	20
-1	$\frac{3}{2}$	10
0	3	5
1	6	$\frac{5}{2}$
2	12	$\frac{5}{4}$

Exponential Growth and Decay

any real number x,

and is an exponential

and is an exponential



Exponential Population

If a population P is changing at a constant percentage rate r each year, then

where P_0 is the _____ population, r is the _____ expressed as a decimal, and t is _____ in years.

Note: If $r > 0$ then $(1 + r)$ is the _____ factor for the population;

If $r < 0$ then $(1 + r)$ is the _____ factor for the population.

Ex #4: Tell whether the population is an exponential growth function or an exponential decay function and find the constant percentage rate of growth or decay.

A. Surfside: $P(t) = 102{,}324(1.025)^t$

B. Mountain View: $P(t) = 86{,}527(0.92)^t$

The Natural B

The 2nd most famous irrational number is e; it is the sum

Definition:

Any exponential function $f(x) = a \cdot b^x$ can be rewritten

chosen real constant k. This form is an:

exponential growth function: If

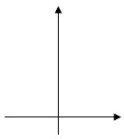
exponential decay function: If

Ln means NATURAL LOG

Transforming Expone

Ex #5: Describe how to transform the graph of $f(x) =$

A. $g(x) = 2^{x+1}$



_____

_____

_____

EX #6: Describe how to transform the graph of $f(x) = e^x$ into the graph of the given function.

A. $k(x) = 4 \cdot e^x$

B. $g(x) = e^{-2x}$

_____ _____

_____ _____

Logistics Growth Functions

Let a, b, c, and k be _____ constants, with _____. A _____ **function** in x is a function that can be written in the form:

or

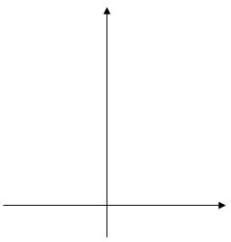
here the constant c is the _____ or _____.

Ex #7: Graph the function. Find the y-intercept and the horizontal asymptote(s).

$f(x) = \frac{18}{1 + 5 \cdot 0.2^x}$

y-intercept:

Horizontal asymptotes:



EX #8: An exponential function of the form $G(x) = a \cdot b^{-x} + c$ is shown in the table below. Use the table to answer the following questions.

C. What conclusion can be made about the value of a ? Why?

D. What is the value of c ? Give a reason.

NOTES

3.1 **Exponential and Logistics Functions Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, determine the exact value of the function without using a calculator.

1. $f(x) = 5 \cdot 4^x \; for \; x = -1$

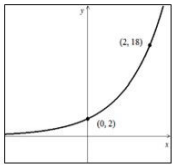
2. $g(x) = 3 \cdot 2^x \; for \; x = 5$

3. $f(x) = 6\left(\frac{1}{2}\right)^x \; for \; x = 3$

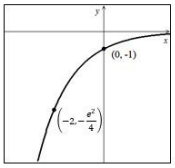
4. $f(x) = -2 \cdot 5^x \; for \; x = -2$

Problems 5 – 6, write an exponential function that describes each graph.

5.



6.



Problems 7 – 10, describe the transformation of the graph of $f(x)$ into the graph of $g(x)$.

7. $f(x) = 2^x; \; g(x) = 2^{x-4}$

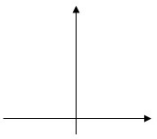
8. $f(x) = \left(\frac{1}{2}\right)^x; \; g(x) = 3\left(\frac{1}{2}\right)^{-2x}$

9. $f(x) = e^x; \; g(x) = \left(\frac{1}{3}\right)e^{x+2}$

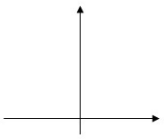
10. $f(x) = e^x; \; g(x) = 4e^{2x} - 3$

Problems 11 – 12, use a graphing utility to graph each function and then sketch it. Find the y-intercept, describe the end behavior using limits.

11. $f(x) = \frac{14}{1+3(0.5)^x}$

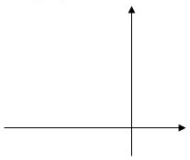


12. $g(x) = \frac{12}{1+2e^{-x}}$



Problems 13 – 14, graph the function and analyze completely.

13. $f(x) = 4(e^{2x})$



Domain: Range:

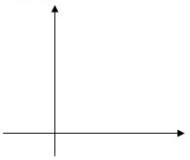
Continuity: Increase / Decrease

Symmetry: Boundedness:

Extrema: Asymptotes:

End-behavior:

14. $g(x) = 3(0.5)^x$



Domain: Range:

Continuity: Increase / Decrease

Symmetry: Boundedness:

Extrema: Asymptotes:

End-behavior:

Problems 15 – 17, Solve.

15. Bacteria is being grown in a petri dish. The number of bacteria B after t hours can be modeled by $B(t) = 150e^{0.585t}$.

A. What is the initial number of bacteria present?

B. How many bacteria will be in the petri dish after 12 hours?

16. An archaeologist found that an artifact contained 18 grams of radioactive material. The decay model $A(t) = 18e^{-0.000122t}$ shows the amount of the radioactive matter, in grams, after t years. How many grams will be present after 5500 years?

17. The logistic growth function describes the population $P(t)$ of an endangered species of turtles t years after being introduced into a safe habitat area. $P(t) = \frac{263}{1+3.62e^{-0.144t}}$

C) What is the limiting size of the habitat that will sustain this population of turtles?

Free Response Question

The table below is a representation of values for the function $f(x) = e^{x-3} + 2$.

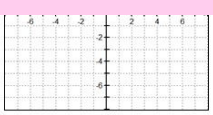
x	−6	−1	0	1	4	7
$f(x)$	2.000	2.018	2.049	2.135	4.718	56.598

A. What is the equation of the horizontal asymptote for $f(x)$. Justify your answer.

B. Does the graph of $f(x)$ lie above or below the horizontal asymptote. Explain your reasoning.

C. Let $g(x)$ represent the inverse of $f(x)$. Find the equation of the inverse function $f^{-1}(x) = g(x)$. Show your work to support your answer.

D. What is the vertical asymptote of $y = g(x)$? Justify your answer.



Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____

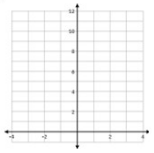
Lesson 3.I: Exponential and Logistic Functions: Warm-Up # 1

1. Find the exact value of the function without using a calculator. Given:
$f(x) = 4 \cdot 2^x$ for $x = -2$

2. Describe the transformation of $f(x) = e^x$ to $g(x) = -2e^x + 3$

3. State the initial value and the base of $f(x) = 2(\frac{1}{4})^x$

4. Sketch the graph from #3.



Name _____ Date _____ Period _____

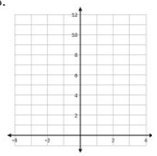
Lesson 3.I: Exponential and Logistic Functions: Warm-Up # 2

1. Find the exact value of the function without using a calculator. Given:
$f(x) = 6 \cdot 3^x$ for $x = -3$

2. Describe the transformation of $f(x) = e^x$ to $g(x) = 3e^{x-2} + 1$

3. State the initial value and the base of $f(x) = 3(\frac{1}{3})^x$

4. Sketch the graph from #3.



3.1 Exponential & Logistic Functions **Form A**

Name _____

Date _____ Period _____

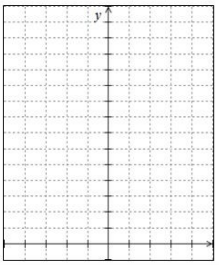
1. Find the exact value of the function without using a calculator. Given:
$f(x) = 6 \cdot 4^x$ *for* $x = -2$

2. Describe the transformation of $f(x) = e^x$ to $g(x) = -e^{x+2}$

Problems 3 – 4: $f(x) = 3\left(\frac{1}{2}\right)^x$

3. State the initial value and the base.

4. Sketch



Grade:

3.1 Exponential & Logistic Functions **Form B**

Name _____

Date _____ Period _____

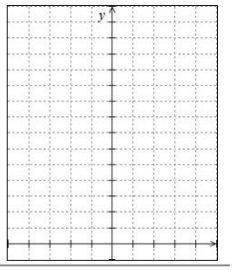
1. Find the exact value of the function without using a calculator. Given:
$f(x) = 3 \cdot 2^x$ *for* $x = -3$

2. Describe the transformation of $f(x) = e^x$ to $g(x) = 2e^x - 3$

Problems 3 – 4: $f(x) = \frac{1}{2}(2)^x$

3. State the initial value and the base.

4. Sketch



Grade:

Daily Lesson Quiz