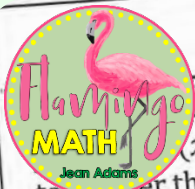


Precalculus Honors

Properties of Functions

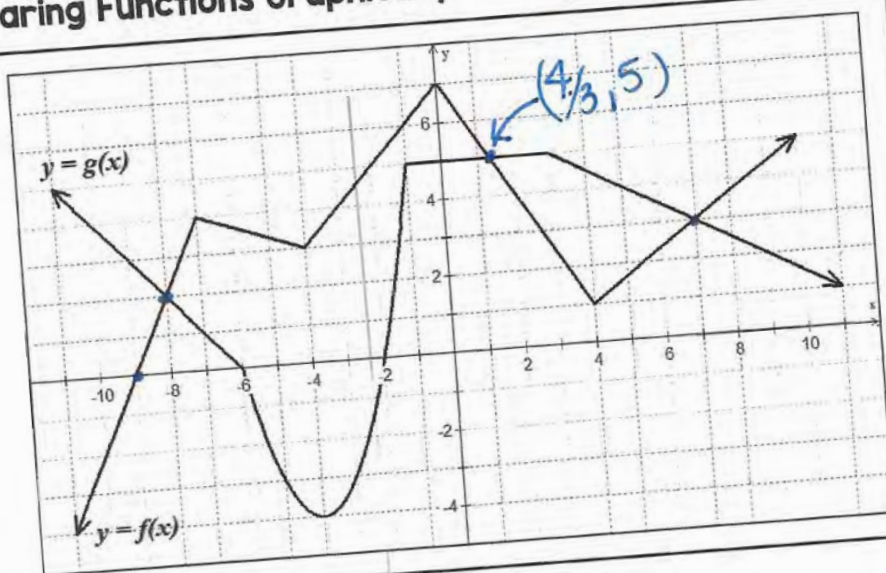


Comparing Functions Graphically



The graphs of functions $f(x)$ and $g(x)$, shown at right, to answer the following.

$$\begin{aligned} f &= 7 - \frac{3}{2}x \\ g &= 5 \\ 7 - \frac{3}{2}x &= 5 \\ \frac{3}{2}x &= 2 \\ x &= \frac{4}{3} \end{aligned}$$



A. The domain for each function.

$$f(x) \quad (-\infty, \infty)$$

B. The value(s) of x such that

$$f(x) = 0 \quad x = -9$$

Completely Editable

1.3

Lesson 3: Properties of Functions

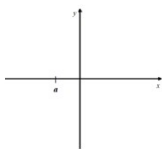
This lesson will focus on continuity, asymptotes and end-behavior. We will be introduced to limit notation that will help describe these function characteristics needed for analysis. Our goal is to be able to quickly graph a function by knowing key elements and characteristics of function behavior. These ideas will serve you well into Calculus and beyond.

Continuity

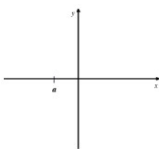
Continuity is a key concept for analyzing function behavior. Graphically, a function is continuous at a point if the graph does not "come apart" at that point. Through our course, we will learn to define a function f to be continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$.

Jump discontinuities will be discussed in depth in the lesson on piecewise functions.

Polynomial functions are continuous at all x .



Removable discontinuity



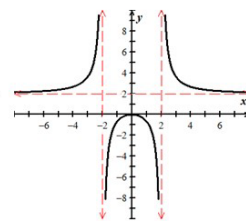
Jump discontinuity

Infinite discontinuity

Asymptotes

Asymptotes describe the behavior of the graph at its vertical or horizontal extremities.

The graph of the function $f(x) = \frac{2x^2}{x^2 - 4}$ is shown at right. Notice that as the values of x become unbounded in the positive direction (as $x \rightarrow \infty$, read as "x approaches positive infinity") the function values are approaching 2. In a similar fashion, as the graph approaches the vertical lines $x = -2$ and $x = 2$, the y -values are unbounded either upward or downward. We will use limit notation as the preferred method to describe this "limiting" behavior:



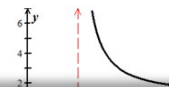
NOTE: $x \rightarrow c^+$ means "x approaches c from the right," and $x \rightarrow c^-$ means "x approaches c from the left."

Horizontal and Vertical Asymptotes

Vertical Asymptotes: _____

The graph behavior will approach the vertical line and either increase or decrease without bound. The graph of the function will never intersect a vertical asymptote.

That is, $\lim_{x \rightarrow c} f(x) = \infty$ or $\lim_{x \rightarrow c} f(x) = -\infty$



Finding Horizontal Asymptotes:

Case:

Example:

Case #1: numerator degree < denominator degree

The line _____ is the horizontal asymptote.

Case #2: numerator degree = denominator degree

The _____ of the leading coefficients is the horizontal asymptote.
That is, $y =$ _____

Case #3: numerator degree > denominator degree

The graph has _____ horizontal asymptote.

EX #2: Identify any vertical or horizontal asymptotes of the graph of each function.

End Behavior

The behavior of the graph of a function for very large values of x , either positive or negative, is commonly known as the **end behavior**. Recall that the graph of a polynomial resembles the graph of the power function for the highest-powered term of the polynomial.

EX #3: State the power function that the graph of f resembles for large values of x . Then find the end behavior for the function. Write your findings using limit notation.

A. $y = 4 - 2x - x^2$

B. $y = (x - 1)^3$

C. $y = -\frac{1}{2}x^3(x + 2)$

D. $y = 3x^2(1 - x^3)$

Comparing Functions and Interpreting Graphs

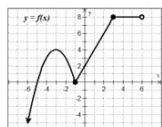
We now turn our focus and attention to the comparison between two functions. Oftentimes, we want to find y -values where a function is positive, negative or zero.

EX #5: Write a mathematical statement for functions f and g to represent each verbal statement. Use function notation $f(x)$ and $g(x)$ as well as appropriate symbols, $>$, $<$, $=$.

- A point $(x, 0)$ is located on the graph of f _____
- Two functions f and g intersect at a given point _____
- A point lies above the x -axis on function g _____
- A point lies below the x -axis on function f _____
- The graph of f lies above the graph of g _____
- The graph of f lies below the graph of g _____

Function Operations with Multiple Representations

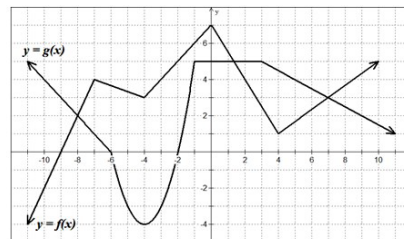
EX #6: The graph of $f(x)$ and the table of values for the discrete function, $g(x)$ are shown below.



x	$g(x)$
-6	4
-3	1
0	-3
2	-7

Comparing Functions Graphically

EX #7: Use the graphs of functions $f(x)$ and $g(x)$, shown at right, to answer the following.



- The domain for each function.
 $f(x)$ _____
 $g(x)$ _____
- The value(s) of x such that $f(x) = 0$ _____
 $g(x) = 0$ _____
- The value(s) of x such that $f(x) = g(x)$ _____
- The value(s) of x such that $f(x) > g(x)$ _____
- The value(s) of x such that $f(x) < g(x)$ _____
- The value(s) of x such that $g(x) > 0$ _____

K. The value(s) of x such that $f(x) > 0$

L. The value(s) of x such that $f(x)$ decreases

NOTES

1.3

Properties of Functions Practice

Name _____

Date _____ Period _____

Problems 1 – 4, Identify points of discontinuity and classify the type of each discontinuity.

1. $y = \frac{2x}{x+1}$

2. $y = \frac{x+3}{x^2-9}$

3. $y = \frac{x^2-x-20}{x^2+2x-35}$

4. $y = \frac{8-2x}{x^2-16}$

Problems 5 – 8, State the vertical and horizontal asymptotes of the graph.

5. $y = \frac{-2x}{x^2-2}$

6. $g(x) = \frac{x^2+x-6}{x^2-4}$

7. $h(x) = \frac{5x^2}{2x^2-8}$

8. $f(x) = \frac{x^2-1}{2x^2+x-3}$

Problems 9 – 12, State the power function that the graph of f resembles for large values of x . Find the end-behavior for the function. Write your results using limit notation.

9. $y = x^3 - 4x^2 + 3$

10. $y = 2x^2(x-5)^2$

11. $y = (x^2+1)(2-x^3)$

12. $y = 3x - x^2 + 4x^5 - x^6$

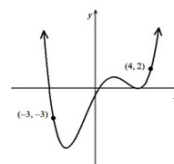
Problems 13 – 16, Find the average rate of change. Show your work for each calculation.

13. Find the average rate of change of $f(x) = x^2 + 7x + 12$ on the interval $[2, 2.01]$

14. The table shows some values of $g(x)$. The average rate of change of $g(x)$ over the interval $1 \leq x \leq 4$ is 6. What is the missing value in the table?

x	1	2	3	4
$g(x)$	3	7	16	?

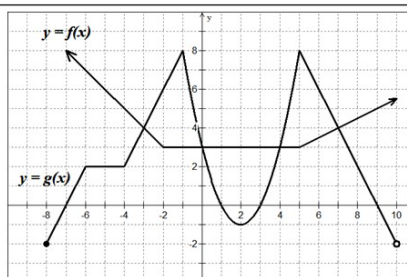
15. Find the average rate of change for the interval $[-3, 4]$.



16. Find the average rate of change on the interval $[1, 5]$. What does this value tell you about the function on the interval?

x	1	2	2.6	3.2	4.7	5
y	25	18	16	12	14	10

Problems 17 – 24, Use the graphs of $f(x)$ and $g(x)$ shown at right, to complete the following.



17. The value(s) of x such that $f(x) = g(x)$

18. The value(s) of x such that $f(x) = 0$

19. The value(s) of x such that $g(x) = 0$

20. The value(s) of x such that $f(x) > g(x)$

21. The value(s) of x such that $f(x) < g(x)$

22. The value(s) of x such that $g(x) < 0$

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 1.3: Properties of Functions Warm-Up # 1	
1. State the points of discontinuity and classify the type of each. $g(x) = \frac{x^2 + 2x - 3}{x^2 - 1}$	2. Using the function from #1, state the horizontal and vertical asymptotes.
3. Describe the end behavior of the function using limit notation, $y = 2x^4 + 3x^2 + 7x + 1$.	4. Find the average rate of change on the interval $-2 \leq x \leq 1$, $g(x) = 3 - x^2$

Name _____ Date _____ Period _____	
Lesson 1.3: Properties of Functions Warm-Up # 2	
1. State the points of discontinuity and classify the type of each. $g(x) = \frac{x^2 - x - 20}{x^2 - 25}$	2. Using the function from #1, state the horizontal and vertical asymptotes.
3. Describe the end behavior of the function using limit notation, $y = -2x^5 + x^4 + 3x - 2$.	4. Find the average rate of change on the interval $-1 \leq x \leq 1$, $g(x) = 3 - x^3$

1.3 Properties of Functions	Form A
Name _____	
Date _____ Period _____	
Use the function for problems 1 & 2	
$f(x) = \frac{x^2 + 3x - 10}{x^2 - 4}$	
1. State the points of discontinuity and classify the type of each.	
2. State the vertical and horizontal asymptotes	
3. Describe the end behavior of the function using limit notation. $y = x^4 - 3x^2 + 2$	
4. Find the average rate of change on the interval on $-1 \leq x \leq 2$. $g(x) = x^3 - 2x$	
Grade: _____	

1.3 Properties of Functions	Form B
Name _____	
Date _____ Period _____	
Use the function for problems 1 & 2	
$f(x) = \frac{x^2 + 2x - 15}{x^2 - 9}$	
1. State the points of discontinuity and classify the type of each.	
2. State the vertical and horizontal asymptotes	
3. Describe the end behavior of the function using limit notation. $y = x^5 - 2x^3 + 3x$	
4. Find the average rate of change on the interval on $-1 \leq x \leq 2$. $g(x) = 3x - x^3$	
Grade: _____	

Daily Lesson Quiz