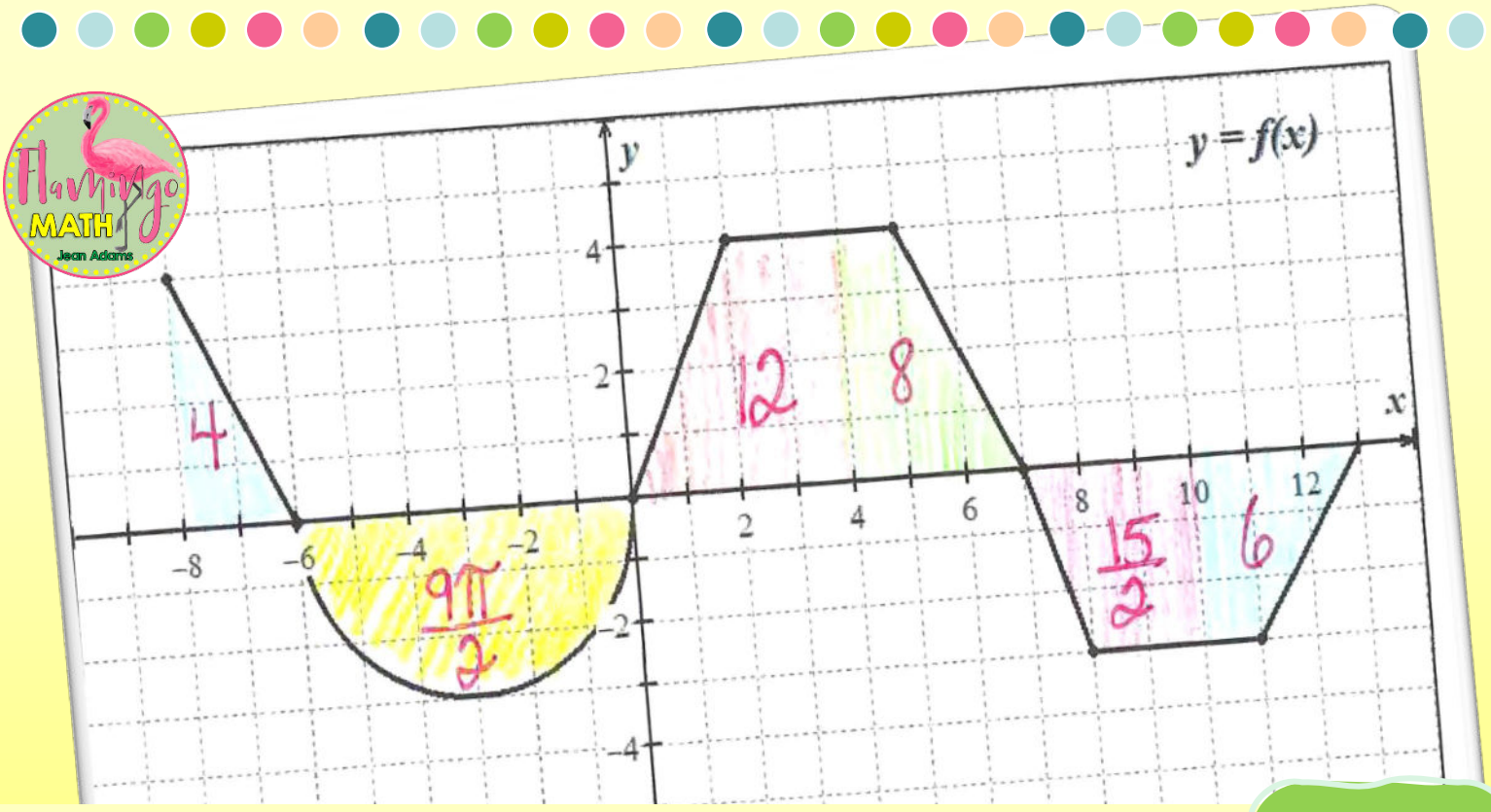
AP® Calculus

Applying Properties of Definite Integrals



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Completely Editable 6.4

Lesson 6.4: Applying Properties of Definite Integrals

Topic 6.6: Applying Properties of Definite Integrals

In this section, we move beyond finite sums (rectangles) and explore what happens in the limit, as the terms become infinitely small, and the number of rectangles approaches infinity. We will look at some geometric functions that may contain negative values (below the x-axis), as well as positive values (above the x-axis), and zero.

Dealing With Negative Area:

At times we will integrate functions that go below the x-axis (negatives).
Here are a few pointers for when that happens:

- Area below the x-axis counts as *negative area*.
- The total area between a and b for some curve is really a *net area*, where the total area below the x-axis (and above the curve) is subtracted from the total area above the x-axis (and below the curve).

Definite Integral Notation:

Leibniz introduced the notation for the definite integral. The Greek letters of summation become Roman letters in the limit.

The notation $\int_a^b f(x)\,dx$ is read as:

The component part

$\int_a^b f($

The definite integral can

Properties of Definite Integrals:

The definition of the definite integral of f on the interval $[a, b]$ specifies that $a < b$. Now, however, it is convenient to extend the definition to cover cases in which $a = b$ or $a > b$.

1. If f is defined at $x = a$, then $\int_a^a f(x)\,dx = 0$
2. If f is integrable on $[a, b]$, then $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

Additive Interval Property:

3. If f is integrable on the three closed intervals determined by a, b, and c, then

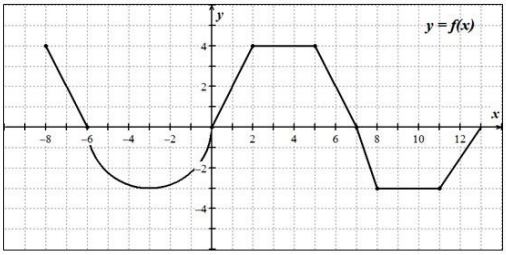
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Properties of Definite Integrals:

If f and g are integrable on $[a, b]$ and k is a constant, then

4. $\int_a^b k \cdot f(x)\,dx = k\int_a^b f(x)\,dx$
5. $\int_a^b k\,dx = k(b-a)$
6. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

EX #1: Use the graph below to evaluate each of the following. Note that the graph of $f(x)$ consists entirely of line segments and a semi-circle on the interval $-8 \le x \le 13$.



A. $\int_0^7 f(x)\,dx =$	B. $\int_0^{13} f(x)\,dx =$
C. $\int_{-6}^0 f(x)\,dx =$	D. $\int_3^3 f(x)\,dx =$
	F. $\int_{10}^4 f(x)\,dx =$
	H. $f'(-6)$
	J. Average rate of change in f over $[0, 5]$.
	L. Over which values of x is $f(x)$ not differentiable?

EX #2: Use the properties of integrals to evaluate each of

$\int_a^b f(x)\,dx = 8$ $\int_a^b g(x)\,dx = -3$

A. $\int_b^a g(x)\,dx =$	B.
C. $\int_a^b 5\,dx =$	D.
E. $\int_a^b [f(x) - h(x)]\,dx =$	F.

EX #3: Find the following. If $\int_1^5 [3 \cdot f(x) + 4]\,dx =$

EX #4: Even and Odd functions: Consider if $f(x)$

Even Function	Where $\int_0^3 f(x)\,dx = 8$, the
Odd Function	Where $\int_0^3 f(x)\,dx = 8$, the

In some cases, a definite integral can be evaluated by using geometry.

EX #5: Sketch the region defined by the definite integral. Then evaluate each integral using a geometric formula.

A. $\int_1^4 (x+3)\,dx$	B. $\int_{-2}^2 \sqrt{4-x^2}\,dx$
C. $\int_{-2}^4 (\lvert x \rvert + 3)\,dx$	D. $\int_{-4}^4 \sqrt[3]{x}\,dx$
E. $\int_1^4 (3x+2)\,dx$	F. $\int_0^2 \left(1 + \sqrt{4-x^2}\right)\,dx + \int_2^5 (3-x)\,dx$

NOTES

6.4 **Properties of Definite Integrals Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, Given $\int_1^5 f(x)\,dx = 8$ and $\int_1^5 g(x)\,dx = -3$ find the values of the following definite integrals, if possible, by rewriting the given integral using the properties of integrals.

1. $\int_1^5 [f(x) + g(x)]\,dx$

2. $\int_1^5 [3f(x) - 4g(x)]\,dx$

3. $\int_1^5 [3g(x)]\,dx$

4. $\int_1^5 \left[f(x) + \frac{1}{2}g(x)\right]\,dx$

Problems 5 – 10, Express each integral as a single integral.

5. $\int_0^2 f(x)\,dx + \int_2^5 f(x)\,dx$

6. $\int_3^8 f(x)\,dx - \int_5^8 f(x)\,dx$

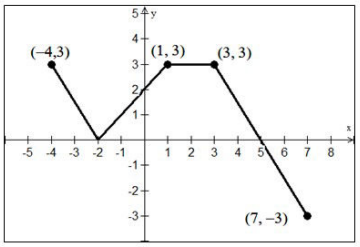
7. $\int_2^9 f(x)\,dx - \int_2^6 f(x)\,dx$

8. $\int_5^1 f(x)\,dx + \int_8^5 f(x)\,dx$

9. $\int_c^d f(x)\,dx + \int_k^c f(x)\,dx$

10. $\int_c^{d+h} f(x)\,dx - \int_c^h f(x)\,dx$

Problems 11 – 16, Below is the graph of $f(x)$. Use the graph to find the values of each of the following definite integrals.



11. $\int_{-4}^3 f(x)\,dx$

12. $\int_0^5 f(x)\,dx$

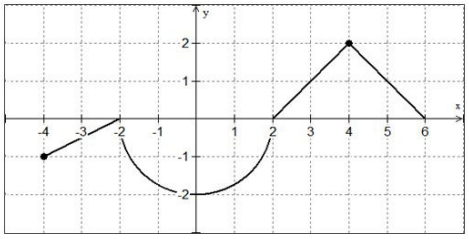
13. $\int_{-2}^7 f(x)\,dx$

14. $\int_7^0 f(x)\,dx$

15. $\int_{-2}^5 f(x)\,dx$

16. $\int_{-4}^7 f(x)\,dx$

Problems 17 – 24, The graph below consists of line segments and a semi-circle. Evaluate each definite integral by using geometric formulas



17. $\int_0^2 f(x)\,dx$

18. $\int_2^6 f(x)\,dx$

19. $\int_{-4}^0 f(x)\,dx$

20. $\int_{-4}^2 f(x)\,dx$

21. $\int_{-4}^6 f(x)\,dx$

22. $\int_0^{-4} f(x)\,dx$

Problems 25 – 30, Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

25. $\int_1^5 [6 - x]\,dx$

26. $\int_{-3}^2 (2 - |x|)\,dx$

27. $\int_{-2}^2 \left[3 + \sqrt{4 - x^2}\right]\,dx$

28. $\int_{-1}^3 (4 + |x|)\,dx$

29. $\int_{-1}^5 (2x - 3)\,dx$

30. $\int_0^a \sqrt{a^2 - x^2}\,dx,\ a > 0$

Practice Sets

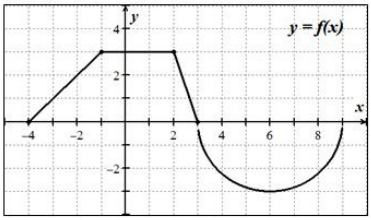
6.4 Properties of Definite Integrals

Form A

Name _____

Date _____ Period _____

Use the graph below to evaluate the following:



1. $\int_{-4}^{3} f(x)\,dx$

2. $\int_{9}^{0} f(x)\,dx$

Use the properties of integrals to evaluate each of the following, given that:

$\int_{a}^{b} f(x)\,dx = 7$ $\int_{a}^{b} g(x)\,dx = -5$

3. $\int_{a}^{b} [2 + f(x)]\,dx$

4. $\int_{a}^{b} [3f(x) - g(x)]\,dx$

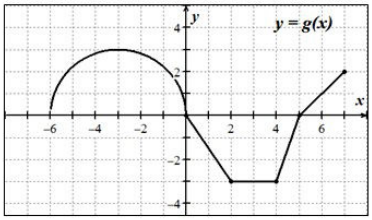
6.4 Properties of Definite Integrals

Form B

Name _____

Date _____ Period _____

Use the graph below to evaluate the following:



1. $\int_{-6}^{0} g(x)\,dx$

2. $\int_{7}^{0} g(x)\,dx$

Use the properties of integrals to evaluate each of the following, given that:

$\int_{a}^{b} f(x)\,dx = 9$ $\int_{a}^{b} g(x)\,dx = -4$

3. $\int_{a}^{b}$ [

4. $\int_{a}^{b}$ [

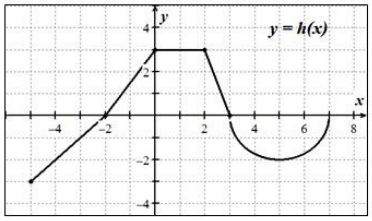
6.4 Properties of Definite Integrals

Form C

Name _____

Date _____ Period _____

Use the graph below to evaluate the following:



1. $\int_{-5}^{3} h(x)\,dx$

2. $\int_{7}^{3} h(x)\,dx$

Use the properties of integrals to evaluate each of the following, given that:

$\int_{a}^{b} f(x)\,dx = -6$ $\int_{a}^{b} g(x)\,dx = 4$

3. $\int_{a}^{b} [4 - f(x)]\,dx$

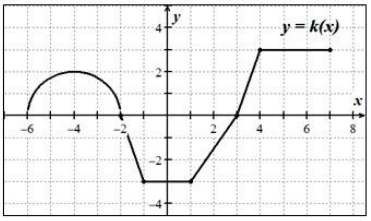
6.4 Properties of Definite Integrals

Form D

Name _____

Date _____ Period _____

Use the graph below to evaluate the following:



1. $\int_{-6}^{0} k(x)\,dx$

2. $\int_{7}^{0} k(x)\,dx$

Use the properties of integrals to evaluate each of the following, given that:

$\int_{a}^{b} f(x)\,dx = -8$ $\int_{a}^{b} g(x)\,dx = 3$

3. $\int_{a}^{b} [5 + g(x)]\,dx$

Daily Lesson Quiz