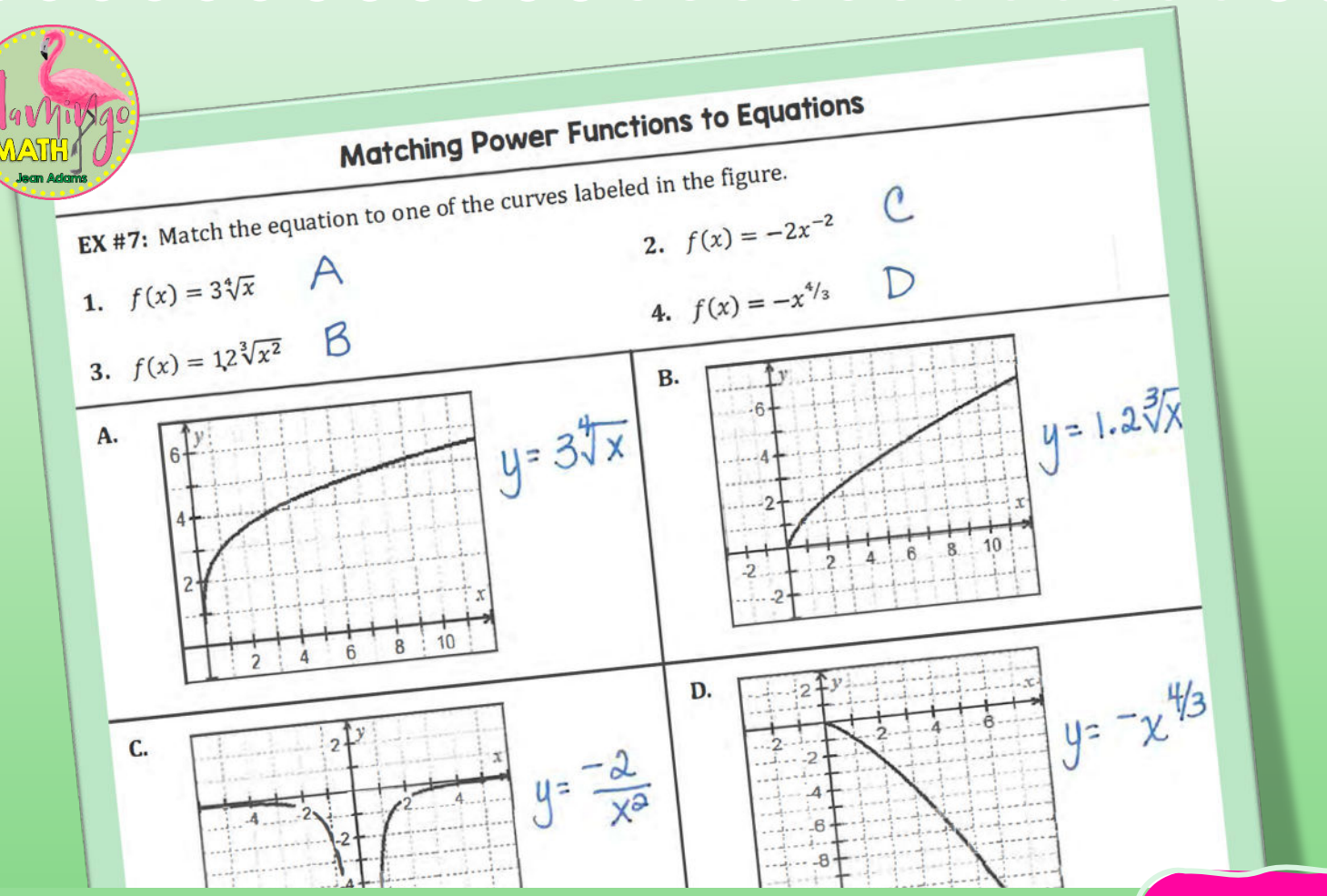
Precalculus Honors

Power Functions



Completely Editable 2.2

Lesson 2: Power Functions

Any function that can be written in the form _____, where k and a are nonzero constants, is a _____ **function**. The constant _____ is the **power**, and _____ is the **constant of** _____, or **constant of** _____. We say $f(x)$ **varies as** the a^{th} power of x, or $f(x)$ is _____ to the a^{th} power of x.

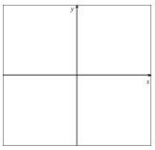
Common Formulas from Geometry and Science

Name	Formula	Power	Constant of Variation
Circumference	$C = 2\pi r$		
Area of a circle	$A = \pi r^2$		
Force of Gravity	$F = \frac{k}{d^2}$		
Boyle's Law	$V = \frac{k}{P}$		

Writing and Analyzing Power Functions

EX #1: In the formula $d = \frac{1}{2}gt^2$, state the power and the constant of variation.

EX #5: Sketch a graph $g(x) = 3x^3$. State whether the function is even or odd, the end-behavior and the leading coefficient.



Identifying End-Behavior of Power Functions

Theorem: For large values of x, either positive or negative, the graph of the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + ax + a_0$ resembles the graph of the power function $y = a_n x^n$.

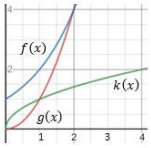
EX #6: For power functions of the form $f(x) = k \cdot x^n$, n is a positive integer, fill in the table:

	Even Power	Odd Power
		

Relative Rates of Growth

It is important to understand that many functions tend to ∞ as $x \to \infty$. It is equally important to understand that they tend towards ∞ at different rates. We will look specifically at two power functions and one exponential function now.

EX #9: Let: $f(x) = 2^x$, $g(x) = x^2$, $k(x) = \sqrt{x}$



[4x4 window]

a) Describe what you see in the interval [0, 1].

b) Describe what you see in the interval [1, 2].

c) How can we determine what happens as x grows larger?

EX #10: Complete the table.

	$x = 1/2$	$x = 1$	$x = 4$	$x = 9$	$x = 50$	$x = 100$
$f(x) = 2^x$						
$g(x) = x^2$						
$k(x) = \sqrt{x}$						

Put the functions in order from smallest growth rate to greatest growth rate.

B) $x^{\quad}$ or $x^{\quad} + e^{\quad}$

C) e^x or π^x

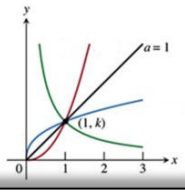
Monomial Functions and their Graphs

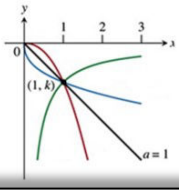
Any function that can be written as $f(x) = k$ or $f(x) = k \cdot x^n$, where k is a constant and n is a positive integer, is a _____.

For any power function $f(x) = k \cdot x^a$, one of the following three things happens when $x < 0$.

1. _____
2. _____
3. _____

The graphs below represent the four shapes that are possible for general power functions of the form $f(x) = k \cdot x^a$ for $x \geq 0$.



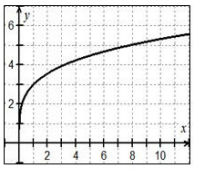


Matching Power Functions to Equations

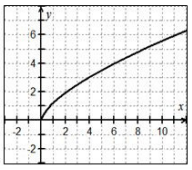
EX #7: Match the equation to one of the curves labeled in the figure.

1. $f(x) = 3\sqrt[4]{x}$
2. $f(x) = -2x^{-2}$
3. $f(x) = 1.2\sqrt[3]{x^2}$
4. $f(x) = -x^{4/3}$

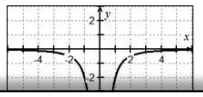
A.



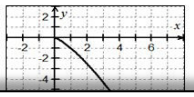
B.



C.



D.



Facts About Relative Rates of Growth

The idea of relative rates of growth will be helpful in future courses. Here are some important facts to consider.

1. Any two polynomial functions of equal degree grow at the same rate.
2. x^m grows slower than x^n if $m < n$.
3. a^x grows slower than b^x if $a < b$.
4. Logarithms grow slower than polynomials which grow slower than (increasing) exponentials.

Growth at the Same Rate

EX #12: Consider functions $f(x) = 4x^2$ and $g(x) = x^2 + 5x - 2$ for sufficiently large positive values of x. We know the end-behavior for the graph of $g(x)$ as $x \to +\infty$ will resemble $y = x^2$ from the Theorem stated in EX #6. Compare the ratio of the two functions for these large values of x. You can show that the two functions are growing at the same rate using algebra.

EX #13: Use properties of exponents to compare the ratio of $f(x) = 2^x$ and $g(x) = e^x$ and verify Rule #3 above.

NOTES

2.2 **Power Functions Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, Determine whether each of the functions are power functions. If so, identify k and a. If not, explain why.

1. $y = -5x^{-3}$

2. $S = 4\pi r^2$

3. $y = 6x^5$

4. $f(x) = 4x^{2/3}$

Problems 5 – 9, Write the statement as a power function equation. Use the constant of variation given.

5. The current in an electrical circuit I, is inversely proportional to the resistance R, with constant of variation V.

6. The weight of an object W, varies directly with its mass m, and constant of variation g.

7. The distance d traveled by an object falling for time t varies directly with the square of the time and constant of variation $\frac{1}{2}g$.

8. The volume of a cone V, is directly proportional to the square of its radius r and height h and constant of variation $\frac{1}{3}\pi$.

9. Boyle's Gas Law states that the volume V, of an enclosed gas at a constant temperature, varies inversely as the pressure P, and constant of variation k.

Problems 10 – 11, Describe the end behaviors of the functions and explain how you reached your conclusion.

10. $f(x) = 3x^6 - 5x + 3$

11. $g(x) = -x^3 + 7x^2 - 8x + 9$

Problems 12 – 13, State the domain, range, x- and y-intercepts and end behaviors of the power functions. Then sketch a graph.

12. $f(x) = 5x^{-4}$

Domain:

Range:

x-intercept(s):

y-intercept(s):

End-behavior:

13. $g(x) = 8x^{-\frac{4}{3}}$

Domain:

Range:

x-intercept(s):

y-intercept(s):

End-behavior:

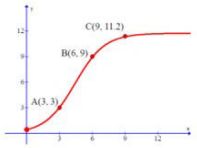
14. Which of the following functions grow faster than x^2? Which grow at the same rate as x^2?

e. x^3e^{-x}	Faster	Same	Slower
f. $10x^2$	Faster	Same	Slower

Applications

15. A gas law states that the volume of an enclosed gas varies directly as the temperature and inversely as the pressure. The pressure of a gas is 0.7 kg/cm² when the temperature is 294 K and the volume is 7400 cm³. Find the pressure when the temperature is 306 K and the volume is 6500 cm³.

16. Using the graph below, compute the average rate of change from A to B, from B to C, and from A to C.



SHOW WORK:

a) Which one gives the largest average rate of change?

b) By looking at the graph, how can you tell which one has the smallest rate of change?

c) Sketch a graph that has the following characteristics:

i) 4 points labeled A, B and C and the origin O: (0, 0)



Practice Sets

Lesson Warm-Ups

2.2 Power Functions **Form A**

Name _____

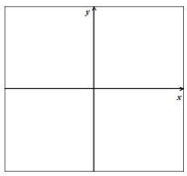
Date _____ Period _____

1. Write the statement as a power function equation. Use k for the constant of variation.

"The volume V of a circular cylinder with fixed height is proportional to the square of its radius r."

For problems 2 – 3, Use $f(x) = -2x^3$

2. State the power and constant of variation; then graph.



3. Analyze Completely.

Domain:

Range:

Other Characteristics:

Grade:

2.2 Power Functions **Form B**

Name _____

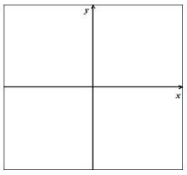
Date _____ Period _____

1. Write the statement as a power function equation. Use k for the constant of variation.

"Charles's Gas Law states the volume V of an ideal gas enclosed at a constant pressure varies directly as the absolute temperature T."

For problems 2 – 3, Use $f(x) = 2x^4$

2. State the power and constant of variation; then graph.



3. Analyze Completely.

Domain:

Range:

Other Characteristics:

Grade:

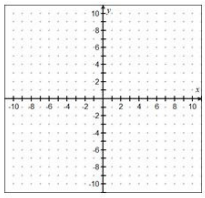
Name _____ Date _____ Period _____

Lesson 2.2: Power Functions **Warm-Up # 1**

1. Write the statement as a power function equation. Use k as the constant of variation.

Acceleration of a vehicle is inversely proportional to time.

2. Use $f(x) = -x^3$, state the power and constant of variation and graph.



3. Analyze the graph from #2 completely:

Domain:

Range:

Other Characteristics:

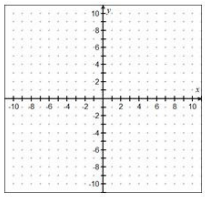
Name _____ Date _____ Period _____

Lesson 2.2: Power Functions **Warm-Up # 2**

1. Write the statement as a power function equation. Use k as the constant of variation.

The acceleration of a car is directly proportional to the velocity.

2. Use $f(x) = -x^4$, state the power and constant of variation and graph.



3. Analyze the graph from #2 completely:

Domain:

Range:

Other Characteristics:

Daily Lesson Quiz