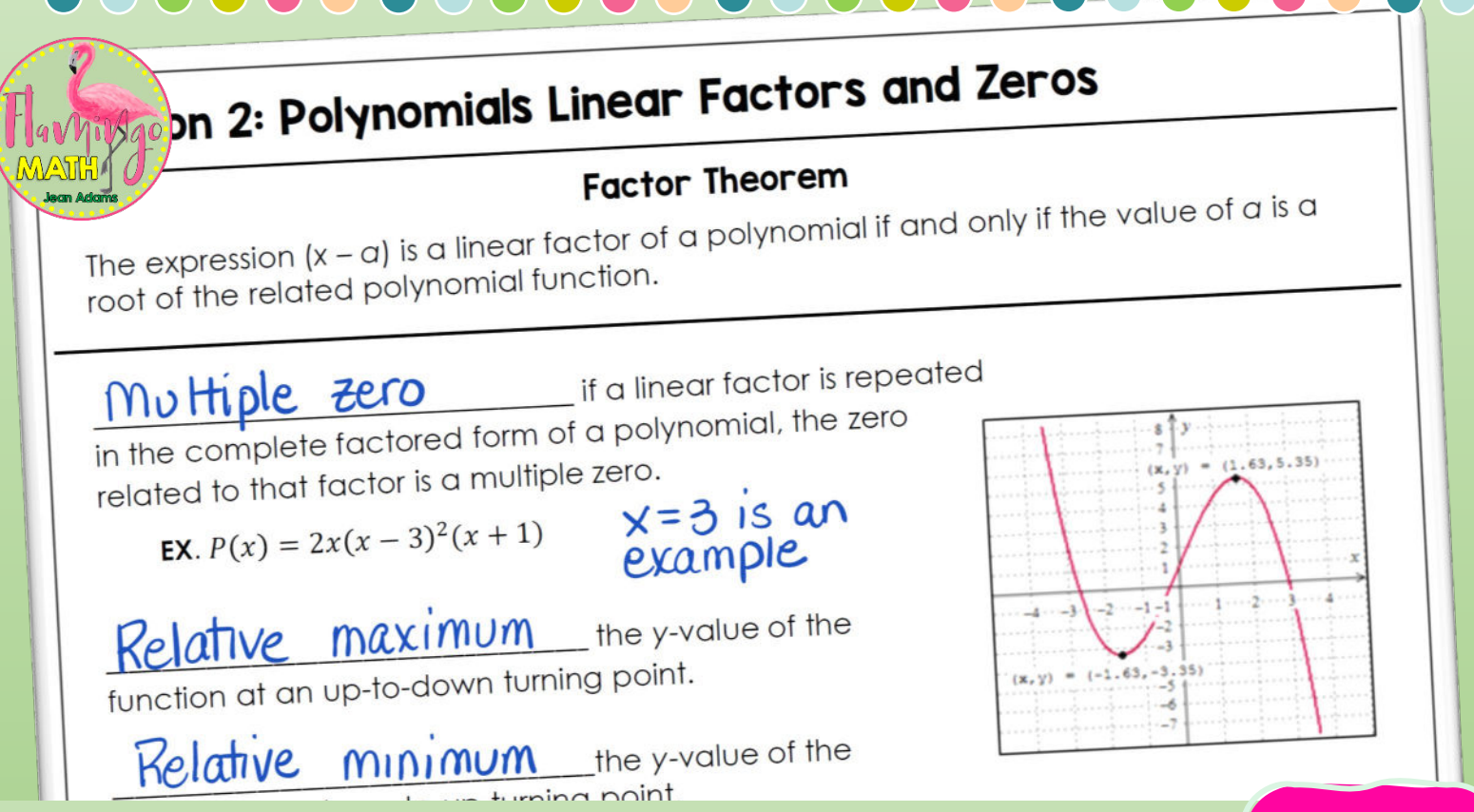
Algebra 2 Honors

Polynomials
Linear Factors & Zeros



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5.2

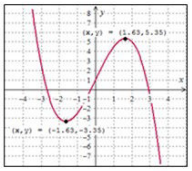
Lesson 5.2: Polynomials Linear Factors and Zeros

Factor Theorem

The expression $(x - a)$ is a linear factor of a polynomial if and only if the value of a is a root of the related polynomial function.

_____ if a linear factor is repeated in the complete factored form of a polynomial, the zero related to that factor is a multiple zero.

EX. $P(x) = 2x(x-3)^2(x+1)$



_____ the y-value of the function at an up-to-down turning point.

_____ the y-value of the function at a down-to-up turning point.

Roots, Zeros, and x-intercepts

Key Concepts

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

- ❑ _____ is a linear factor of the polynomial $P(x)$
- ❑ b is a _____ of the polynomial function $y = P(x)$
- ❑ b is a _____ (solution) of the polynomial equation $P(x) = 0$
- ❑ b is an _____ of the graph of $y = P(x)$

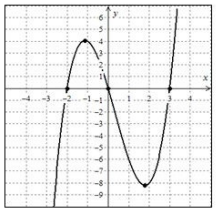
EX #1: What is the factored form of $x^3 - x^2 - 12x$?

EX #2: Write in factored form for $x^4 - 5x^2 + 4$

Finding Zeros of a Polynomial

EX #3: Finding Zeros of a Polynomial Function

The graph of $y = x(x+2)(x-3)$ is shown at right:



A. Find the zeros

B. Determine the end behavior

C. Estimate the relative maximum and minimum.

D. Use a graphing calculator to find a better estimate.

The Factor Theorem

The expression $(x - a)$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

EX. $f(x) = (x+1)(x+3)(x-2)$

Writing a Polynomial from its Zeros

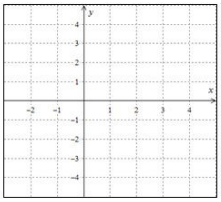
EX #4: What is a cubic polynomial function in standard form with zeros -2,3 and 5?

Multiple Zeros

If a is a zero of multiplicity n in the polynomial function $y = p(x)$, then the behavior of the graph at the x-intercept will be close to linear if n =1, close to quadratic if n = 2, close to cubic if n = 3, and so on.

EX #5: Find the zeros of $f(x) = x^3 - 4x^2 + 4x$

A. Sketch the graph.



B. Describe the behavior of the graph at each zero and the multiplicity.

Maximum and Minimum Values

EX #6: The graph of $y = -x^3 + x^2 + 6x$ is shown below. Use a graphing calculator to find the maximum and minimum points.



EX #7: What are the relative maximum and minimum values of $f(x) = 3x^3 + x^2 - 5x$?

Where do those maximums and minimums occur?

Minimum occurs when _____

Maximizing a Real-World Situation

EX #8: The design of a digital box camera maximizes the volume while keeping the sum of the dimensions at 6 inches. If the length must be 1.5 times the eight, what should each dimension be?

A. Define a variable x.

x =

B. Determine the length and width.

Length =

Width =

C. Model the volume $V = (length)(width)(height)$

D. Graph the polynomial function and use the **MAXIMUM** feature to find the maximum volume and where that occurs.

NOTES

Name _____ Date_____

5.2

Polynomials Linear Factors and Zeros Practice

Problems 1 – 4, write each polynomial in factored form. Check by multiplication.

1. $f(x) = 2x^2 + 8x + 6$

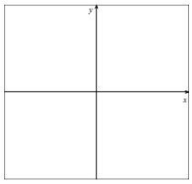
2. $f(x) = x^3 - 5x^2 + 6x$

3. $g(x) = -3x^3 + 30x^2 - 75$

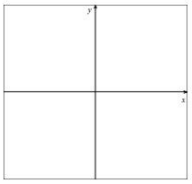
4. $h(x) = 3x^3 + 5x^2 + 9x + 15$

Problems 5 – 8, find the zeros of each function. Then graph the function.

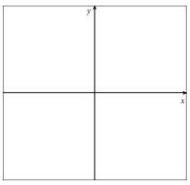
5. $f(x) = (x+1)(x-1)(x-3)$



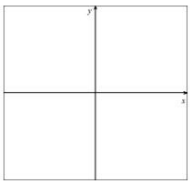
6. $f(x) = x(x+3)(x-3)$



7. $f(x) = (x+2)^2(x-3)$



8. $f(x) = -(5-x)(x+3)$



Problems 9 – 12, Write a polynomial function in standard form with the given zeros.

9. $x = -1, 2, 5$

10. $x = 4, 0, 0, 3$

11. $x = 3, 2, -2, 0$

12. $-1, -3, 4$

Problems 13 – 18, find the zeros of each function. State the multiplicity of multiple zeros.

13. $y = (x-4)^3$

14. $y = x(x+5)^2$

15. $y = (x-7)(x+5)^3$

16. $y = x^4 - 8x^3 + 16x^2$

17. $y = x^3 + 3x^2 - 18x$

18. $y = 2x^3 + 5x^2 - 12x$

Problems 19 - 22, find the relative maximum and relative minimum of the graph of each function.

19. $y = x^3 - 5x + 12$

20. $y = x^3 - 2x^2 - 4x + 8$

Problems 23 – 25, solve.

23. A rectangular box has a base whose length is 3 inches more than the width. Let the width of the box be represented by x The height of the box is expressed as $h = 2 - x$

A. Write a polynomial function in factored form modeling the volume V of the box.

B. What is the maximum possible volume of the box?

24. Write a polynomial function with the following characteristics: it has three distinct zeros, one of the zeros is 1, and another zero has a multiplicity of 2.

25. What is the original length of the edge of a cube if, after a one-inch-thick slice is cut from one of the sides the volume that remains is 180 cubic inches?

Practice Sets

Lesson Warm-Ups

3. State the zeros for the polynomial in question #1.	**4.** Write a polynomial with zeros at $x = 2$ and $x = -2$

Name _____ Date _____ Period _____

Lesson 5.2: Polynomials, Linear Factors & Zeros **Warm-Up #2**

1. Factor the polynomial $2x^3 - 2x^2 - 24x$	**2.** State the number of turning points and the end behavior of the polynomial in question #1.
3. State the zeros for the polynomial in question #1.	**4.** Write a po… $x = 3$ and…

5.2 Polynomials Linear Factors and Zeros **Form A**

Name _____

Date _____ Period _____

1. Write the polynomial in factored form.

$x^3 + 6x^2 + 9x$

2. Find the zeros of the function and state the multiplicity of multiple zeros.

$y = x(x + 3)^2$

Zeros: _____

Multiplicity: _____

3. Write a polynomial function in standard form with zeros at $x = 3, 1, -1$

GRADE:

5.2 Polynomials Linear Factors and Zeros **Form B**

Name _____

Date _____ Period _____

1. Write the polynomial in factored form.

$9x^3 + 6x^2 - 3x$

2. Find the zeros of the function and state the multiplicity of multiple zeros.

$y = (x - 2)(x + 1)^3$

Zeros: _____

Multiplicity: _____

3. Write a polynomial function in standard form with zeros at $x = 1, 2, -2$

GRADE:

Daily Lesson Quiz