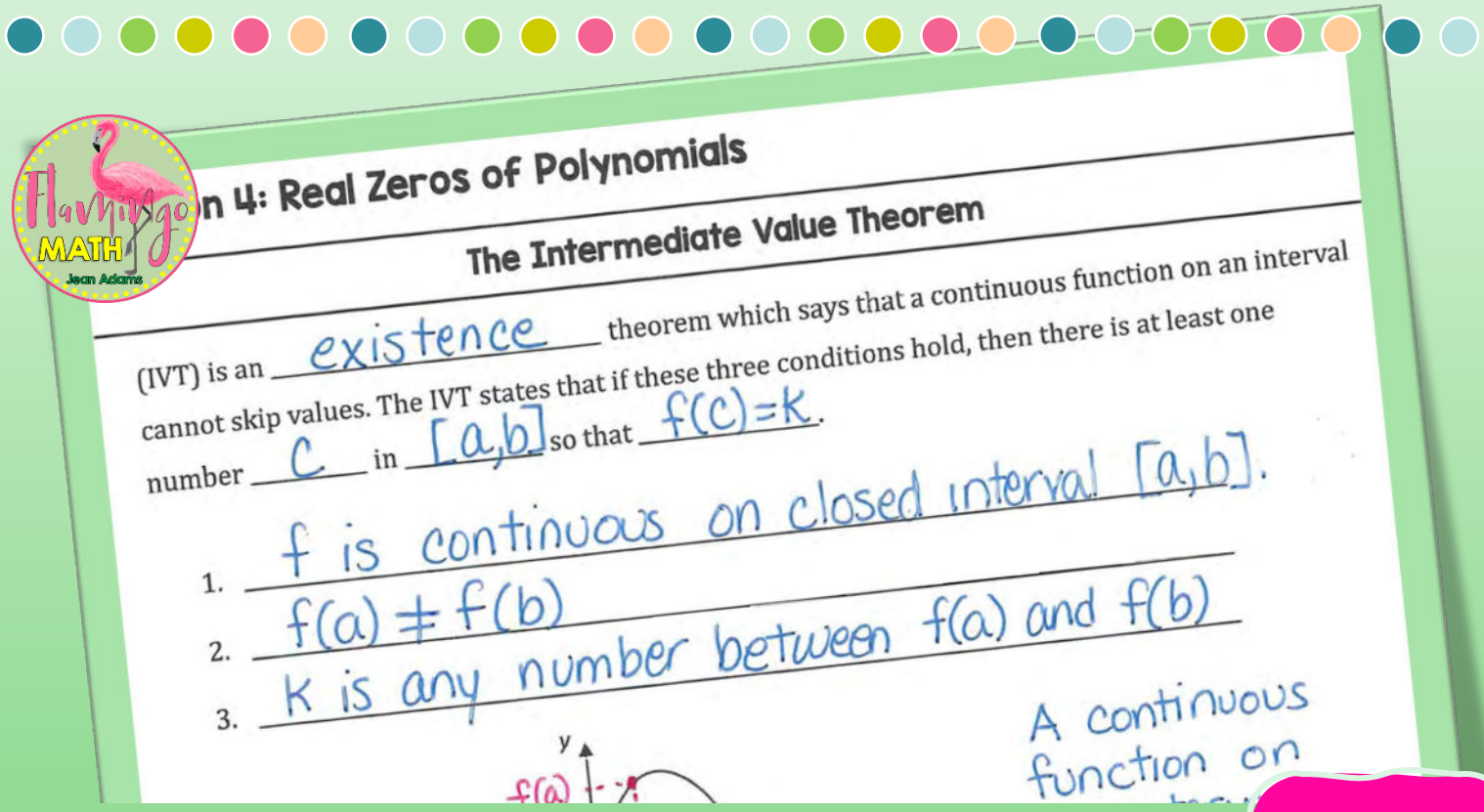
Precalculus Honors

Polynomial Power & Rational Functions



Guided Notes 2

Completely Editable

Unit 2: Polynomial Power & Rational Functions

This ***fully-editable and customizable resource*** is designed for the Precalculus Honors course.

The **Ready-to-Go Guided Student Notes** cover all of the topics for an Honors Precalculus course divided into 8 lessons.

Product Overview

Each individual lesson includes

- PowerPoint© Editable version of Notes and Warm-Ups
- Google Slides® for presentation
- A PDF student version of Guided Notes and Warm-Ups
- A video lesson link for the examples
- Answer keys and full solution sets

Teacher Notes – Using the Resource

- ❑ There is a PDF version of the Learning Objectives and Standards, as well as an editable PowerPoint version.

- ❑ You will have an editable planning guide for 21 days, designed for a 45-50-minute class setting.
- ❑ On page 4, click the lesson image to make a copy of the Google Slides.
- ❑ On page 4, click the Video Link to go to my **Unit 2 You Tube Video Lessons**.

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PRECALCULUS CURRICULUM

Scope and Sequence

Unit 2: Polynomials, Power & Rational Functions

Lesson	Skills & Learning Objectives
[2.1] Linear & Quadratic Functions	▪ Recognize and graph linear and quadratic functions ▪ Describe qualities and characteristics of linear and quadratic functions ▪ Graph linear and quadratic functions by recognition of equations ▪ Choose, write and analyze an algebraic model given real-world scenarios
[2.2] Power Functions	▪ Graph and use power functions to solve real-world problems ▪ Describe the basic characteristics of power functions, including power and constant of variation ▪ Determine relative rates of growth, concavity, and more for power functions
[2.3] Polynomial Functions and Their Graphs	▪ Graph and use the leading term test to sketch the general shape of a polynomial ▪ Describe the basic characteristics of a polynomial from the equation (increase, decrease, constant, concavity, inflection points) ▪ Find zeros, multiplicity of zeros, extrema, and end behavior ▪ Use the Intermediate Value Theorem to find zeros ▪ Estimate points of inflection
[2.4] Real Zeros of Polynomial Functions	▪ Apply the Remainder Theorem, Factor Theorem, and Rational Zeros Theorem ▪ Use the Remainder and Factor Theorems ▪ Divide polynomials using long division and synthetic division. ▪ Find both real zeros and rational zeros of polynomials. ▪ Explain the connections of characteristics such as roots, zeros, factors, and x-intercepts

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Scope and Sequence

Unit 2: Polynomials, Power & Rational Functions

Lesson	Skills & Learning Objectives
[2.5] Complex Zeros & the Fundamental Theorem of Algebra	▪ Use the Fundamental Theorem of Algebra to factor polynomials ▪ Write polynomials in standard form given zeros, degree, multiplicity, and factors ▪ Find all zeros both real and complex using the Fundamental Theorem of Algebra ▪ Write a linear factorization of polynomials given equations, zeros, and other key characteristics.
[2.6] Graphs of Rational Functions	▪ Graph transformations of the reciprocal function, by finding vertical and horizontal asymptotes and intercepts. ▪ Factor and simplify rational expressions ▪ Find removable and non-removable points of discontinuity ▪ Analyze rational functions completely, including using limit notation to describe behavior at both horizontal and vertical asymptotes
[2.7] Analyzing Rational Functions	▪ Analyze rational functions completely, including using limit notation to describe behavior at both horizontal and vertical asymptotes ▪ Use test intervals to establish function behavior for analysis ▪ Solve applied problems for rational functions
[2.8] Solving Rational Equations & Inequalities	▪ Solve equations involving fractions using both algebraic and graphical techniques ▪ Find where rational functions are zero and the intervals where the polynomial is positive or negative by using a sign chart. ▪ Solve polynomial and rational inequalities, identify extraneous solutions ▪ Model and predict real-world scenarios with given parameter values

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Lesson 1: Linear and Quadratic Functions

DEFINITION OF A POLYNOMIAL:

A polynomial function of *x* with degree *n* where the leading coefficient is a_n:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0 \; ; \; a_n \neq 0$$

… AVERAGE RATE OF CHANGE …

$$m = \frac{f(b) - f(a)}{b - a} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \text{velocity}$$

Different Forms of Linear Equations

1. General Form:	
2. Slope-Intercept Form:	
3. Point-Slope Form:	

Identifying a Polynomial Function

EX#1: Find the degree and leading coefficient, if possible, for the functions below. If the equation is not a polynomial, explain why not.

A. $f(x) = 5x - 9x^4$

Degree: _____

Leading Coefficient: _____

Is this a polynomial? Yes No

B. $g(x) = \sqrt{25x^4 + 9x^2}$

Degree: _____

Leading Coefficient: _____

Is this a polynomial? Yes No

C. $h(x) = 8x^{-3} + 2$

Degree: _____

Leading Coefficient: _____

Is this a polynomial? Yes No

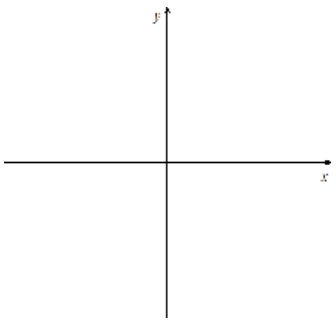
D. $k(x) = 3x^2 - 5x^3 + 2x - \frac{1}{3}$

Degree: _____

Leading Coefficient: _____

Is this a polynomial? Yes No

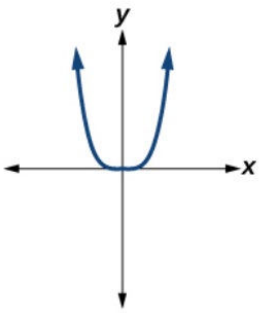
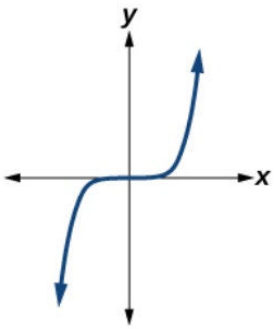
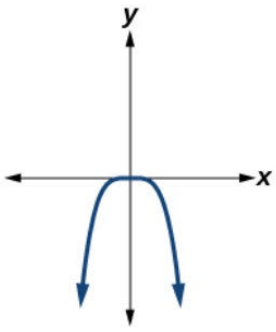
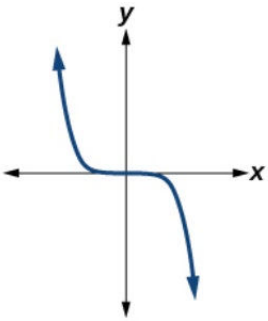
EX #5: Sketch a graph $g(x) = 3x^3$. State whether the function is even or odd, the end-behavior and the leading coefficient.



Identifying End-Behavior of Power Functions

Theorem: For large values of *x*, either positive or negative, the graph of the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + ax + a_0$ resembles the graph of the power function $y = a_n x^n$.

EX #6: For power functions of the form $f(x) = k \cdot x^n$, n is a positive integer, fill in the table:

	Even Power	Odd Power
Positive Constant k > 0	 $As\ x \rightarrow \quad f(x) \rightarrow$ $As\ x \rightarrow \quad f(x) \rightarrow$	 $As\ x \rightarrow \quad f(x) \rightarrow$ $As\ x \rightarrow \quad f(x) \rightarrow$
Negative Constant k < 0	 $As\ x \rightarrow \quad f(x) \rightarrow$ $As\ x \rightarrow \quad f(x) \rightarrow$	 $As\ x \rightarrow \quad f(x) \rightarrow$ $As\ x \rightarrow \quad f(x) \rightarrow$

Lesson 3: Polynomial Functions of Higher Degree

Each monomial in a sum is called a _____ . Terms of a polynomial written in descending degree is said to be in _____.

A _____ is the constant (number) value of each term. The highest degreed monomial in a polynomial is called the _____ . The _____ is a number without a variable.

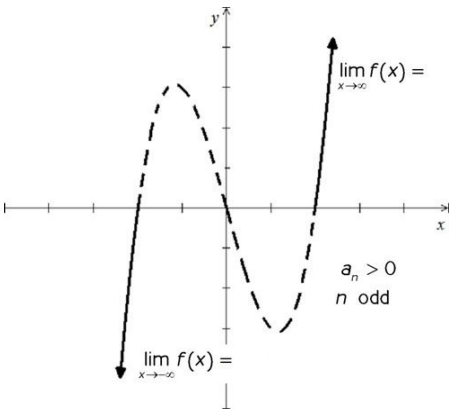
Leading Coefficient Test Using Limits

Power functions eventually rise or fall without bound as *x* moves toward infinity (left or right). This behavior can be determined by

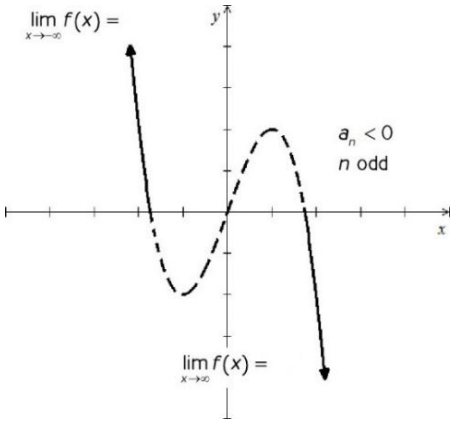
1. _____

2. _____

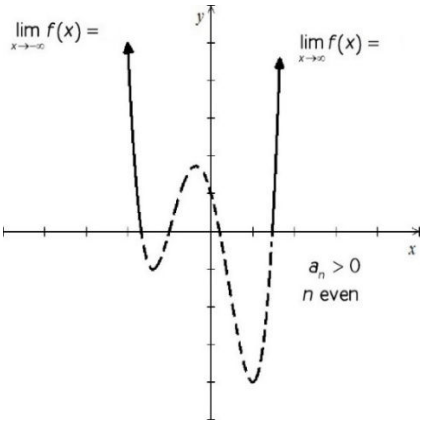
Coefficient sign: _____



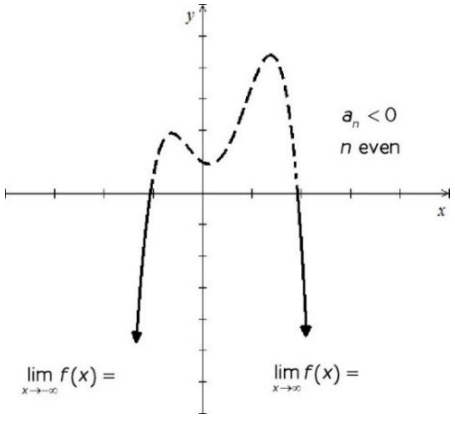
Coefficient sign: _____



Coefficient sign: _____

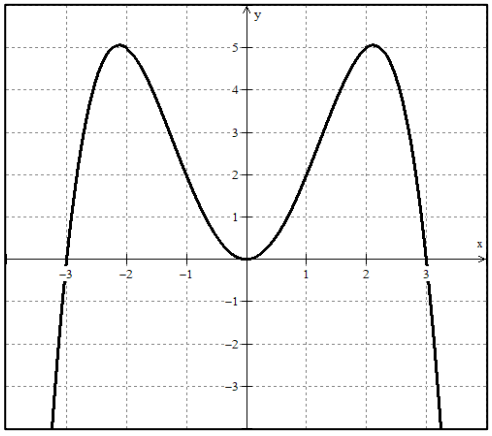


Coefficient sign: _____



Maxima and Minima

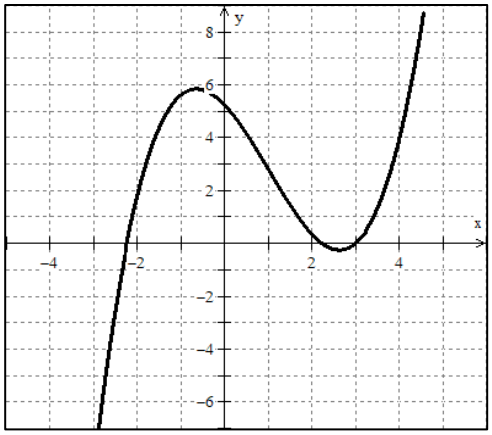
For any function f, if there exists an interval (x_1, x_2) that contains c so that, for all $x \neq c$, in the interval:



1. Then f has a **local maximum at c** when $f(c) > f(x)$, and $f(c)$ is called a _____ **maximum** of f.

2. Then f has a **local minimum at c** when $f(c) < f(x)$, and $f(c)$ is called a _____ **minimum** of f.

EX #3:



A. At a local max the graph changes from increasing to decreasing. On the graph at right shade the intervals of increase in green.

B. At a local min the graph changes from decreasing to increasing. On the graph at right shade the intervals of decrease in yellow.

EX #4: Using a graphing utility, determine the relative maximum or relative minimum of the function below. State the intervals where the function is increasing or decreasing. What is the connection between the relative extrema and the intervals?

$$g(x) = -2x^5 + 12x^2 + 18x$$

Relative Max:

Relative Min:

Increases on:

Decreases on:

Connection:

Sketch:

Lesson 6: Graphs of Rational Functions

Rational Functions

Let $N(x)$ (numerator) and $D(x)$ (denominator) be polynomial functions, where $D(x) \neq 0$.

Then, a Rational Function is:

Finding Domain

The domain of *x* includes _____ except the values that cause _____

by _____.

(Domain restrictions come from the values that make the _____.)

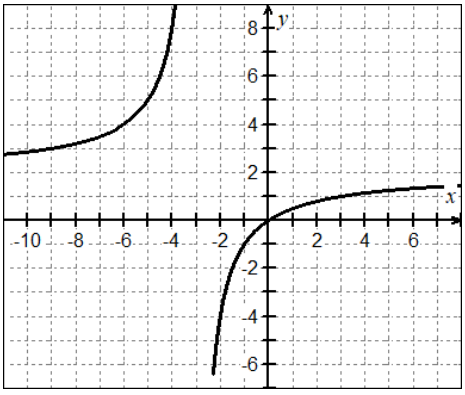
EX #1: Find the domain for $f(x) = \frac{x+7}{x^2-3x+2}$.

Horizontal and Vertical Asymptotes

Vertical Asymptotes: _____

The graph behavior will approach the vertical line and either increase or decrease without bound. That is,

$\lim_{x \to a^-} f(x) =$ or $\lim_{x \to a^+} f(x) =$



Horizontal Asymptotes: _____

The graph behavior will approach the horizontal line as *x* tends to _____ or _____.

That is,

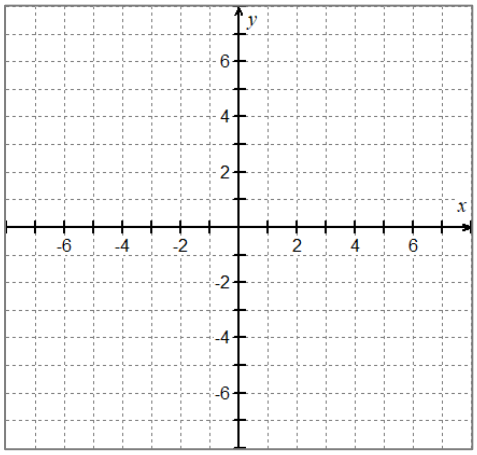
$\lim_{x \to -\infty} f(x) =$ or $\lim_{x \to \infty} f(x) =$

Note: It is possible to cross a horizontal asymptote. Graph does not need to have same behavior on both sides.

Rational Functions with a Hole

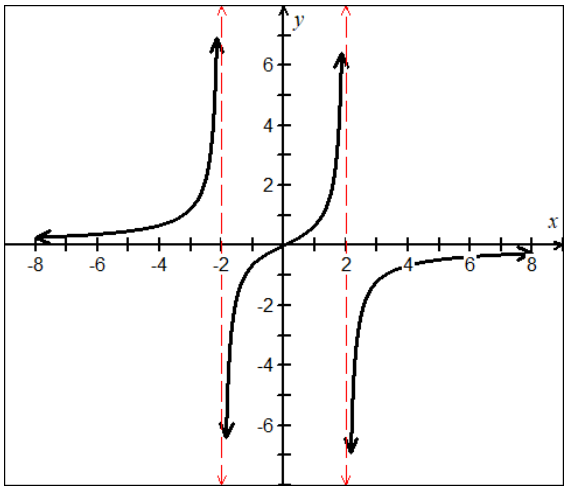
EX #4: Analyze and graph. $h(x) = \frac{2x^2-5x+2}{x^2-x-2}$

Interval
Test Point x
Value of $f(x)$
Sign of $f(x)$
Above/Below x**-axis**



Constructing a Function from its Graph

EX #5: Find a rational function that could have the graph shown below. Describe how you know.



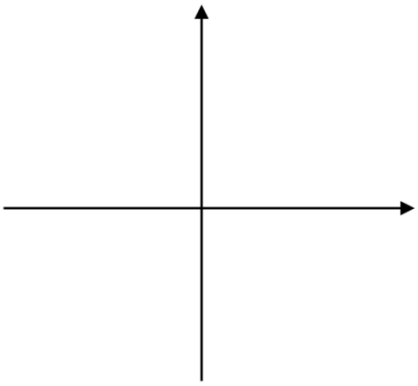
Finding Where a Rational Expression is Zero, Positive, or Negative

EX #5: Let $f(x) = \frac{(x+3)(x-5)}{(2x-1)}$. Determine the real number values of x that cause $f(x)$ to be:

A. Zero	**B.** Undefined
C. Positive	**D.** Negative

Sign Chart:





Solving Rational Inequalities

Similar to what we did above with polynomial inequalities, we can solve rational inequalities by using sign charts. Here are a few procedures to follow:

Step 1: Write the inequality in standard form; that is, the rational expression f is on the left side and zero is on the right. It should look like one of the following forms:

$f(x) > 0$	$f(x) \geq 0$	$f(x) < 0$	$f(x) \leq 0$

*****Be sure you have a common denominator.*****

Step 2: Find the value that makes your expression zero or undefined. These critical values are the numbers where the expression can change signs.

Step 3: Use the numbers found in Step 2 to separate the real number line into intervals.

Step 4: Pick a value in each interval to test the sign of the standard form expression. You want to know if an interval is positive or negative.

- ❑ If the value of f is positive, then $f(x) > 0$ for all numbers x in that interval.
- ❑ If the value of f is negative, then $f(x) < 0$ for all numbers x in that interval.
- ❑ Include the solutions of $f(x) = 0$ if the inequality is not strict.