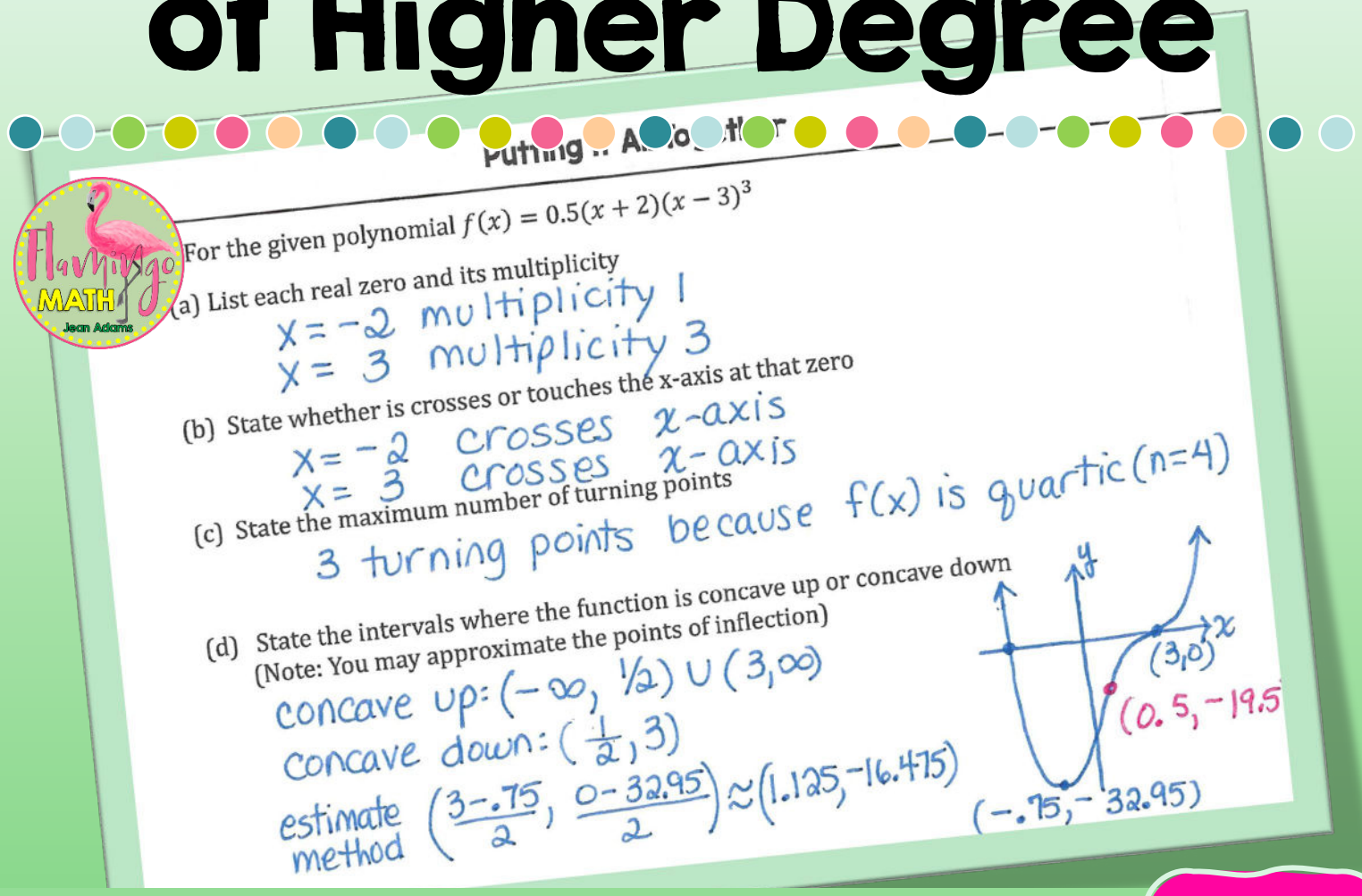
Precalculus Honors

Polynomial Functions of Higher Degree



Completely Editable

2.3

Lesson 3: Polynomial Functions of Higher Degree

Each monomial in a sum is called a _____ . Terms of a polynomial written in descending degree is said to be in _____.

A _____ is the constant (number) value of each term. The highest degreed monomial in a polynomial is called the _____ . The _____ is a number without a variable.

Leading Coefficient Test Using Limits

Power functions eventually rise or fall without bound as x moves toward infinity (left or right). This behavior can be determined by

1. _____
2. _____

Coefficient sign: _____

Coefficient sign: _____

End-Behavior of Polynomial Functions

Consider the domain and range for each "degree" family.

- ❑ As $x \to \infty$ and $x \to -\infty$, what do you notice about $f(x)$ or the y - value of the graphs?

_____

_____

- ❑ How does the value of the leading coefficient "a" affect the value as $x \to \infty$ and $x \to -\infty$?

_____

End-Behavior, Turning Points and Real Zeros

EX #1: Use a calculator, sketch a graph of the function and determine its end behavior.

A. $f(x) = -x^3 + 2x^2 + x + 1$

n is _____

a_n is _____

as $x \to -\infty, f(x) \to$ _____

as $x \to \infty, f(x) \to$ _____

B. $f(x) = x^4 - 3x^3 + 2x$

n is _____

a_n is _____

as $x \to -\infty, f(x) \to$ _____

as $x \to \infty, f(x) \to$ _____

Even and Odd Multiplicity

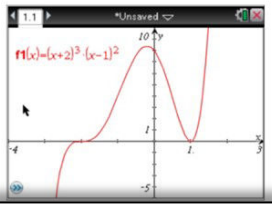
If a polynomial function has a factor that is repeated, these duplicate "roots" are labeled as a _____.

If f has a real zero c of **odd multiplicity**, then the graph of f will _____ the x-axis at _____.

If f has a real zero c of **even multiplicity**, then the graph of f will _____ the x-axis at _____.

Zeros of Odd and Even Multiplicity

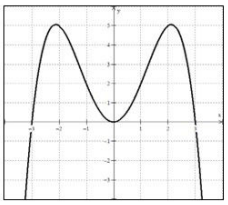
EX#2: Use a calculator to sketch: $f(x) = (x+2)^3(x-1)^2$. In the graph, at right, label the zeros and describe the behavior of the roots.



Maxima and Minima

For any function f, if there exists an interval (x_1, x_2) that contains c so that, for all $x \neq c$, in the interval:

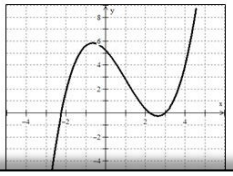
1. Then f has a **local maximum at c** when $f(c) > f(x)$, and $f(c)$ is called a _____ **maximum** of f.
2. Then f has a **local minimum at c** when $f(c) < f(x)$, and $f(c)$ is called a _____ **minimum** of f.



EX #3:

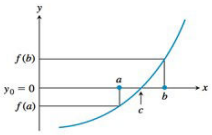
A. At a local max the graph changes from increasing to decreasing. On the graph at right shade the intervals of increase in green.

B. At a local min the graph changes from decreasing to increasing. On the graph at right shade the intervals of decrease in yellow.



Intermediate Value Theorem

In easy to understand language . . . This theorem states that if you can find two y-values where the signs change, then a *zero of the function* will fall between those two *x-values of the function.*

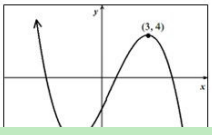


EX #5: Use the Intermediate Value Theorem to explain why the function $g(x) = x^2 + 3x - 6$ must have a root (x-intercept) on the closed interval $[1, 2]$.

Concavity and Points of Inflection

Looking ahead to Calculus, let's explore concavity. Our goal is to understand the terminology. Later we will have rules for finding these intervals.

A function f is _____ on an open interval if the graph looks like a U or part of a smile. Shade the graph at right in **green** for this behavior.



1. A ***zero*** with multiplicity three.
2. Estimate a possible point of inflection ***between two consecutive extreme values.*** (At the midpoint between the relative max/relative min values.)
3. Estimate a possible point of inflection ***between a zero, multiplicity three, and the closest relative extreme value.***

Putting it All Together

EX #6: For the given polynomial $f(x) = 0.5(x+2)(x-3)^3$

(a) List each real zero and its multiplicity

(b) State whether is crosses or touches the x-axis at that zero

(c) State the maximum number of turning points

(d) State the intervals where the function is concave up or concave down (Note: You may approximate the points of inflection)

EX #7: Using a graphing calculator, analyze the given function by stating the domain, range, x- and y-intercepts, intervals of decrease or increase, intervals of continuity, intervals where function is concave up or down and end behavior.

$k(x) = -\frac{3}{x^3}$

NOTES

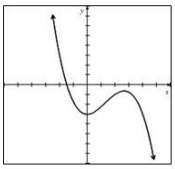
2.3 **Polynomials of Higher Degree Homework**

Name _____

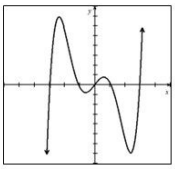
Date _____ Period _____

Problems 1 – 6, match the polynomial function with its graph.

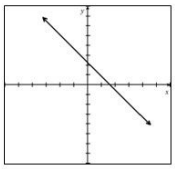
A.



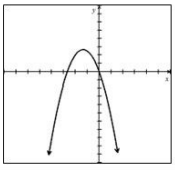
B.



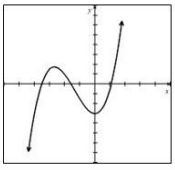
C.



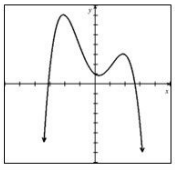
D.



E.



F.



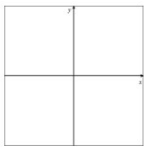
1. $f(x) = -1.4x + 2.3$
2. $f(x) = -\frac{3}{2}x^2 + 4x$
3. $f(x) = \frac{1}{2}x^3 + 2x^2 - 3$
4. $f(x) = \frac{1}{5}x^5 - 2x^3 + 2x$
5. $f(x) = -\frac{1}{4}x^4 + 2x^2 - x + 1$
6. $f(x) = -\frac{1}{4}x^3 + x^2 - 3$

Problems 7 – 10, Use the Leading Coefficient Test to describe the right-hand and left-hand behavior of the graph of the polynomial function. Use a graphing utility to check your results.

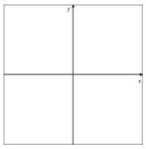
7. $g(x) = -3x^4 + 2x^2 + 1$
8. $h(x) = 4$
9. $f(x) = 4 - 3.2x^2 - 1.5x^3$
10. $s(t) = 9.4t^2 + 42t + 150$

Problems 11 – 12, State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of the polynomial by hand.

11. $g(x) = x^2(x-2)^3$



12. $f(x) = (x^2 - 1)(4 - x^2)$



Problems 13 – 16, Use a graphing utility to determine the x-value(s) for the relative maximum or relative minimum. State the intervals where the function is increasing or decreasing and turning points.

13. $y = 3x^3 - 9x^2$

Relative Maximum(s):

Relative Minimum(s):

Increasing:

Decreasing:

Turning point(s):

14. $y = -x^4 + 9x^2 - 20$

Relative Maximum(s):

Relative Minimum(s):

Increasing:

Decreasing:

Turning point(s):

15. $y = x^4 + 4x^3 + 3x^2$

Relative Maximum(s):

Relative Minimum(s):

Increasing:

Decreasing:

Turning point(s):

16. $y = x^3 + 3x^2 - 9x - 27$

Relative Maximum(s):

Relative Minimum(s):

Increasing:

Decreasing:

Turning point(s):

Problem 17 - 20, Use a graphing calculator to analyze each of the following functions by stating the domain, range, intercepts, intervals of increase or decrease, intervals of continuity, absolute extrema and end behavior.

17. $y = \frac{2}{x^2}$

Domain:

Range:

Zero(s):

y-intercept:

Increasing:

Decreasing:

Continuity:

Absolute Extrema:

End behavior:

18. $y = -\frac{1}{2}x^3$

Domain:

Range:

Zero(s):

y-intercept:

Increasing:

Decreasing:

Continuity:

Absolute Extrema:

End behavior:

19. $y = \frac{5}{2}x^4$

Domain:

Range:

Zero(s):

y-intercept:

End behavior:

20. $y = \frac{1}{2}x^3 + x^2 - 3x + 1$

Domain:

Range:

Zero(s):

y-intercept:

End behavior:

Problem 21 - 24, For each polynomial function do the following: (a) list each real zero and it's multiplicity; (b) state whether is crosses or touches the x-axis at that zero; (c) state the maximum number of turning points; and (d) state the intervals where the function is concave up/down. *Note: *You may approximate the points of inflection.**

21. $f(x) = 2(x-5)(x+2)^3$

Zero(s) and Behavior:

Maximum turning points:

Concave Up:

Concave Down:

22. $f(x) = x(x+3)^2(x-1)$

Zero(s) and Behavior:

Maximum turning points:

Concave Up:

Concave Down:

23. $f(x) = (x+\sqrt{2})(x-6)^2$

Zero(s) and Behavior:

Maximum turning points:

Concave Up:

Concave Down:

24. $f(x) = 2x(x^2 - 5)$

Zero(s) and Behavior:

Maximum turning points:

Concave Up:

Concave Down:

Free Response Question

25. (A) Use the Intermediate Value Theorem (IVT) to explain why the function $h(x) = x^2 + 2x - 5$ must have a root (x-intercept) in the closed interval $[-4, -3]$.

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____

Lesson 2.4: Real Zeros of Polynomial Functions **Warm-Up # 1**

1. List all the possible rational zeros of:
$f(x) = 3x^3 + 2x^2 + 9x + 9$

2. Find $f(-3)$ using the Remainder Theorem for
$f(x) = 3x^3 + 4x^2 + 8$

3. Divide $(2x^4 + 3x^3 + x - 7)$ by $(x - 2)$. Write answer as $q(x) + \frac{f(x)}{d(x)}$.

4. Write a polynomial with a leading coefficient of 4 that has zeros at x = -1, 0, 1

Name _____ Date _____ Period _____

Lesson 2.4: Real Zeros of Polynomial Functions **Warm-Up # 2**

1. List all the possible rational zeros of:
$f(x) = 2x^3 + 3x^2 + 5x + 8$

2. Find $f(2)$ using the Remainder Theorem for
$f(x) = 5x^3 - 2x^2 + 3$

3. Divide $(3x^4 + 2x^3 - x + 2)$ by $(x + 1)$. Write answer as $q(x) + \frac{f(x)}{d(x)}$.

4. Write a polynomial with a leading coefficient of 2 that has zeros at x = -3, 0, 2

2.3 Polynomials of Higher Degree **Form A**

Name _____

Date _____ Period _____

1. Describe the right and left hand behavior of $f(x) = -3x^3 + 4x + 5$.

2. At most how many turning points does a 5^{th} degree polynomial have?

3. Find all real zeros and state the multiplicity of repeated zeros. $y = x^4 - 4x^3 + 4x^2$.

4. Find a polynomial function that has the given zeros. $-3, 4, 0$

Grade:

2.3 Polynomials of Higher Degree **Form B**

Name _____

Date _____ Period _____

1. Describe the right and left hand behavior of $f(x) = 2x^4 - 3x^2 + x$.

2. At most how many turning points does a 4^{th} degree polynomial have?

3. Find all real zeros and state the multiplicity of repeated zeros. $y = x^4 - x^3 - 20x^2$.

4. Find a polynomial function that has the given zeros. $-2, 3, 0$

Grade:

Daily Lesson Quiz