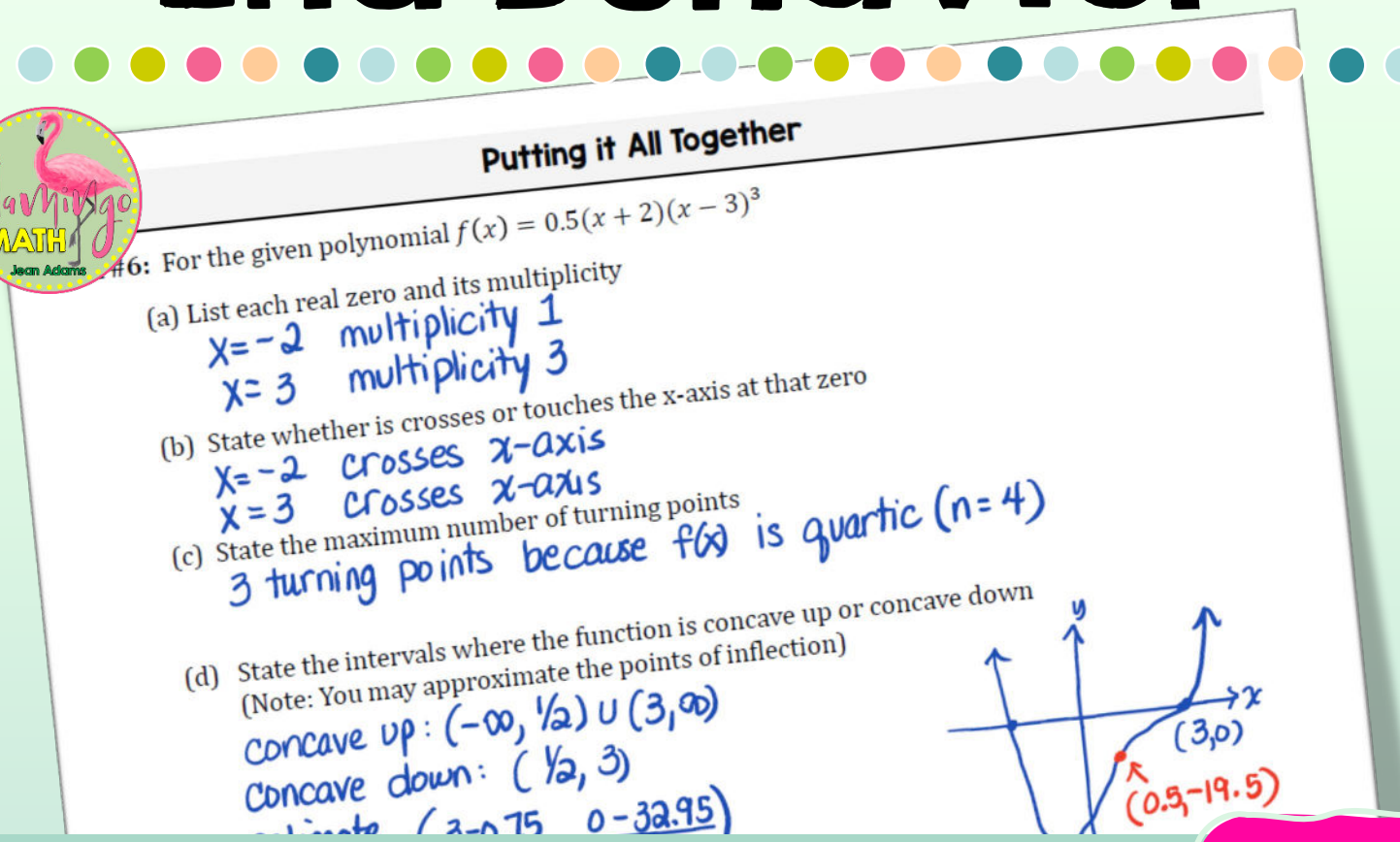
AP® Precalculus

Polynomial Functions

End Behavior



Completely Editable

1.6

Lesson 6: Polynomial Functions and End Behavior

By knowing the degree of a polynomial function, it is possible to predict the **end behavior**. By using **the leading term of the polynomial function**, we can determine the behavior at the extremes. Since the power of the leading term is the highest degree, that term will grow at a rate which is significantly faster than any of the remaining terms as x gets very large or very small. This means the end behavior of the polynomial will correspond with the end behavior of the term of the highest degree.

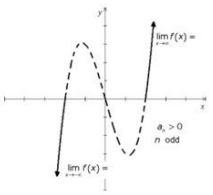
Leading Coefficient Test Using Limits

Power functions eventually rise or fall without bound as x moves toward infinity (left or right). This behavior can be determined by

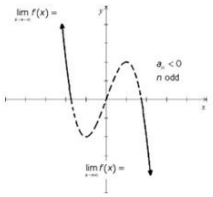
1. _____

2. _____

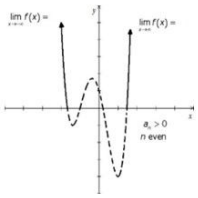
Coefficient sign: _____



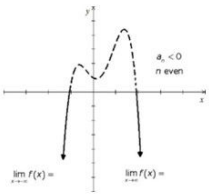
Coefficient sign: _____



Coefficient sign: _____



Coefficient sign: _____



Even and Odd Multiplicity

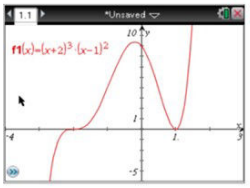
If a polynomial function has a factor that is repeated, these duplicate "roots" are labeled as a _____.

If f has a real zero c of **odd multiplicity**, then the graph of f will _____ the x-axis at _____.

If f has a real zero c of **even multiplicity**, then the graph of f will _____ the x-axis at _____.

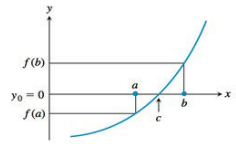
Zeros of Odd and Even Multiplicity

EX#2: Use a calculator to sketch: $f(x) = (x+2)^3(x-1)^2$. In the graph, at right, label the zeros and describe the behavior of the roots.



Intermediate Value Theorem

In, easy to understand, language . . . This theorem states that if you can find two y-values where the signs change, then a *zero of the function* will fall between those two *x-values of the function*.



EX #3: Use the Intermediate Value Theorem to explain why the function $g(x) = x^2 + 3x - 6$ must have a root (x-intercept) on the closed interval $[1, 2]$.

End-Behavior of Polynomial Functions

Consider the domain and range for each "degree" family.

- ❑ As $x \to \infty$ and $x \to -\infty$, what do you notice about $f(x)$ or the y - value of the graphs?

  _____

  _____

- ❑ How does the value of the leading coefficient "a" affect the value as $x \to \infty$ and $x \to -\infty$?

  _____

End-Behavior, Turning Points and Real Zeros

EX #1: Use a calculator, sketch a graph of the function and determine its end behavior.

A. $f(x) = -x^3 + 2x^2 + x + 1$

n is _____

a_n is _____

$\lim_{x \to -\infty} f(x) =$ _____

$\lim_{x \to \infty} f(x) =$ _____

Turning points: _____



B. $f(x) = x^4 - 3x^3 + 2x$

n is _____

a_n is _____

$\lim_{x \to -\infty} f(x) =$ _____

$\lim_{x \to \infty} f(x) =$ _____

Turning points: _____



1. The graph of f has at most, $(n-1)$ turning points, (points at which the graph changes from **increasing** to **decreasing** or vice versa.)
2. The function f has, at most, n real zeros.

Putting it All Together

EX #4: For the given polynomial $f(x) = 0.5(x+2)(x-3)^3$

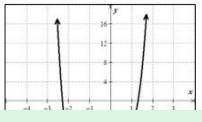
(a) List each real zero and its multiplicity

(b) State whether is crosses or touches the x-axis at that zero

(c) State the maximum number of turning points

(d) State the intervals where the function is concave up or concave down
(Note: You may approximate the points of inflection)

EX #5: Describe the end behavior of the polynomial function. Analyze the given function by stating the domain, range. What can you conclude about the polynomial represented by the graph shown based on its intercepts and turning points?



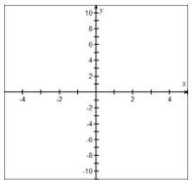
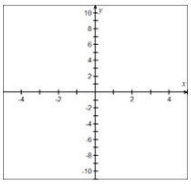
Guided Notes

1.6 **Polynomial Functions and End Behavior Practice Set 1**

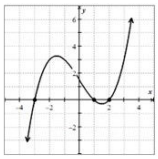
Name _____

Date _____ Period _____

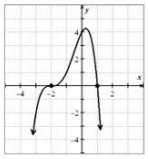
Problems 1 – 6, Use the factored form of the function to identify key characteristics and sketch.

$f(x) = x^2(x+3)$	$g(x) = -(x+2)(x-1)(x-3)$
1. Identify the zeros of $f(x)$ and state their multiplicities.	**2.** Identify the zeros of $g(x)$ and state their multiplicities.
3. State the end behavior of $f(x)$ using limit notation. Give a reason to justify your answer.	**4.** State the end behavior of $g(x)$ using limit notation. Give a reason to justify your answer.
5. Sketch a possible graph of $f(x)$ using the zeros, multiplicities and end behavior stated. 	**6.** Sketch a possible graph of $g(x)$ using the zeros, multiplicities and end behavior stated. 

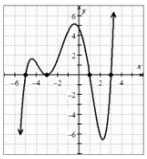
Problems 7 – 13, For each of the functions graphed below, answer the following questions.



Graph of $f(x)$



Graph of $g(x)$



Graph of $h(x)$

7. State the left-end behavior using limit notation for the functions, f, g, and h.

8. State the right-end behavior using limit notation for the functions, f, g, and h.

9. For each function, state the zeros and their multiplicity.

$f(x)$	$g(x)$	$h(x)$

10. For each function, state the degree and function name.

$f(x)$	$g(x)$	$h(x)$

11. Describe the behavior of the graph as it approaches a zero whose multiplicity is 1.

12. Describe the behavior of the graph as it approaches a zero whose multiplicity is 2.

13. Describe the behavior of the graph as it approaches a zero whose multiplicity is 3.

Problems 14 – 18, The table below shows function values of $g(x) = ax^3 + 5x^2 - 8x + c$. Use the equation of $g(x)$ and the table to answer the questions that follow.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(x)$	−214	−84	−16	8	6	−4	−4	24	98	236	456

14. Based on the degree and the end behavior of $g(x)$ what must be true about the sign of the leading coefficient of the equation? Justify your answer.

15. What is the value of c? Justify your answer.

16. Use your answer for the value of c and the point $(-1,6)$ to find the value of a. Show your work.

17. Between which two values in the table will a zero of the function occur? Explain your reasoning.

Problem 19 - 24, The table below shows function values of $h(x) = ax^3 - 2x^2 + 7.5x + c$. Use the equation of $g(x)$ and the table to answer the questions that follow. The only roots of $h(x)$ lie on the interval $-2 < x < 3$

x	−4	−2	0	2	4
$h(x)$	75	2	9	0	−121

19. Based on the degree and the end behavior of $h(x)$ what must be true about the sign of the leading coefficient of the equation? Justify your answer.

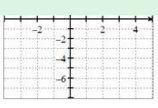
20. What is the value of c? Justify your answer.

21. Given that $h(-2) = 2$, What is the value of a? Show your work.

22. Use the Intermediate Value Theorem to show that the graph of the function has a zero in the interval $1 \le x \le 3$.

23. Find the rates of change on $[-2, -1.5]$ and $[-1.5, -1]$ and use the results to explain the

24. Find any extrema and estimate the point of inflection. Sketch the curve on the grid below.



Practice Sets

1.6 Polynomial Functions and End Behavior **Form A**

Name _____

Date _____Period _____

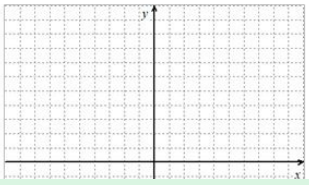
Use the factored form of the function to identify key characteristics and sketch.

$$f(x) = -x(x+2)^2(x-3)$$

1. Identify the zeros and state their multiplicities.

2. State the end behavior using limit notation.

3. Sketch a possible graph of $f(x)$ using the zeros, multiplicities and end behavior stated.



Grade:

1.6 Polynomial Functions and End Behavior **Form B**

Name _____

Date _____Period _____

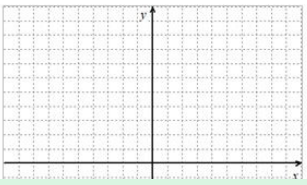
Use the factored form of the function to identify key characteristics and sketch.

$$f(x) = x(x-2)(x+3)^2$$

1. Identify the zeros and state their multiplicities.

2. State the end behavior using limit notation.

3. Sketch a possible graph of $f(x)$ using the zeros, multiplicities and end behavior stated.



Grade:

Daily Lesson Quiz