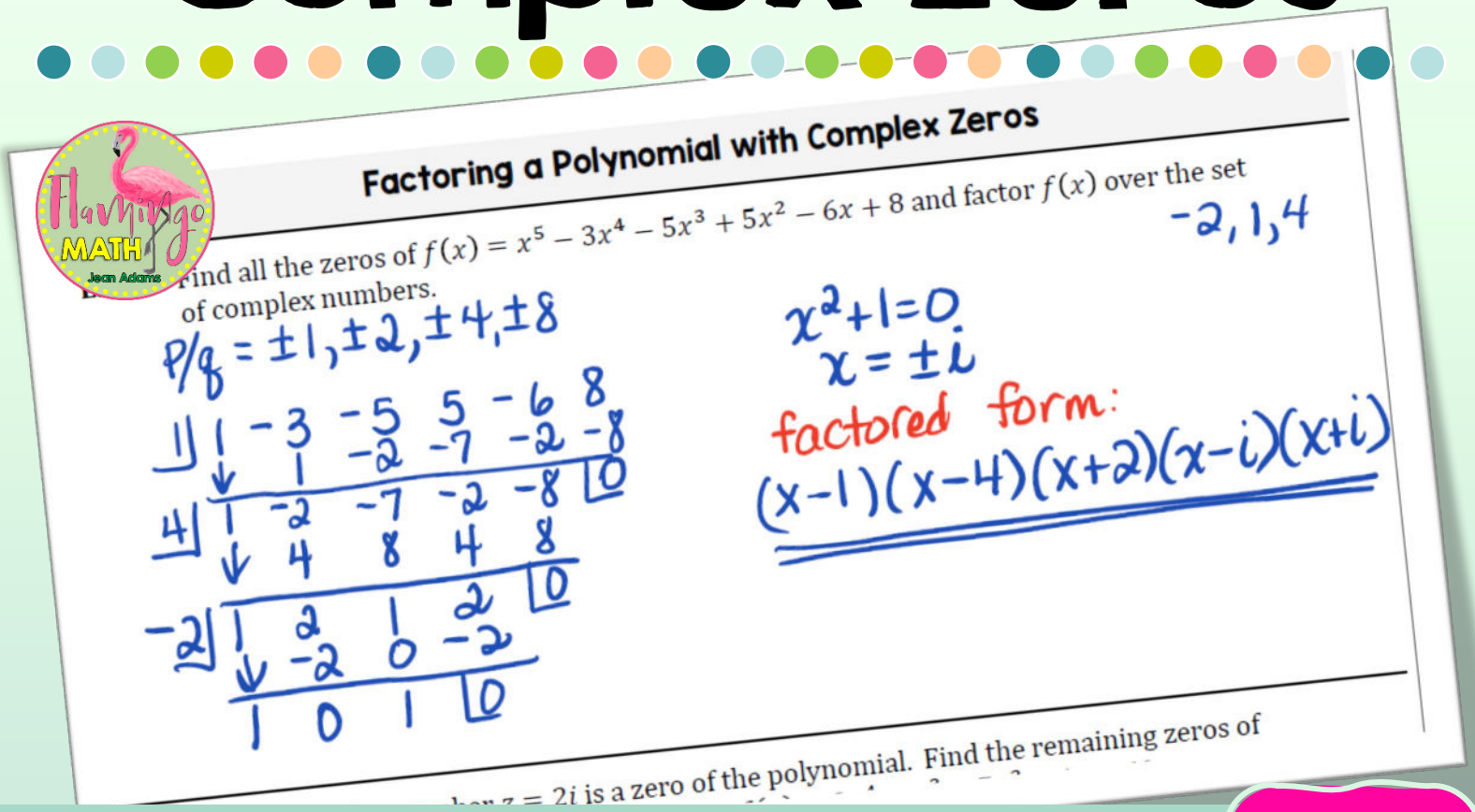
AP® Precalculus

Polynomial Functions

Complex Zeros



Completely Editable

1.5

Lesson 5-A: Polynomial Functions and Complex Zeros

We know how to find the principal square root for any non-negative real number. In earlier courses, we have explored how to find the square root of a negative number. When the value of the radicand is negative, the root is said to be an **imaginary number**. The imaginary number i is defined as the square root of negative 1. $\sqrt{-1} = i$. Recall that you can write the square root of any negative number as a multiple of i. Consider the process of evaluating the square root of -9.

$$\begin{aligned}\sqrt{-9} &= \sqrt{9 \cdot (-1)} \\ &= \sqrt{9} \cdot \sqrt{-1} \\ &= (3)(i) \\ &= 3i\end{aligned}$$

Imaginary and Complex Numbers

A **complex number** is a number of the form _____ where

- ❑ _____ is the real part of the complex number
- ❑ _____ is the imaginary part of the complex number

The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra guarantees that any polynomial function of x, with real number coefficients can be factored completely over the set of complex numbers.

Every complex polynomial function $f(x)$ of degree $n \geq 1$ can be factored into n linear factors

Factoring a Polynomial with Complex Zeros

EX #5: Find all the zeros of $f(x) = x^3 - 3x^2 + x - 3$ and factor $f(x)$ over the set of complex numbers.

EX #6: The complex number $z = 2i$ is a zero of the polynomial. Find the remaining zeros of $f(x)$ and factor over complex numbers: $f(x) = 4x^3 + 10x^2 + 16x + 40$

Real Zeros and Polynomial Inequalities

If a is a real root of a polynomial function p, then the graph of $y = p(x)$ has an x-intercept at the point $(a, 0)$. Consequently, real zeros of a polynomial can be endpoints for intervals satisfying polynomial inequalities. You can solve inequalities involving polynomials using both algebraic and graphic techniques. Let's explore some common connections between inequalities and the positive and negative sign of the expression.

To solve $f(x) > 0$ is to find the values of x that make $f(x)$ positive.
To solve $f(x) < 0$ is to find the values of x that make $f(x)$ negative.

Using Sign Charts

EX #13: Let $f(x) = (x + 3)(x - 4)^2$. Determine the real number values of x that cause $f(x)$ to be:

A. Zero	B. Positive	C. Negative
_____	_____	_____

Show work here:

EX #14: Solve the following polynomial inequality, $x^3 - 4x \leq 4x^2 - 16$ in the way indicated:

Factor: **Critical Values:**

Solutions:

Solutions:

EX #2: Factor completely using complex numbers: $4x^2y + 25y$

Conjugate Root Theorem

If $P(x)$ is a polynomial with rational coefficients, then the irrational roots of $P(x) = 0$ occur in conjugate pairs. That is, when $(a + \sqrt{b})$ is a root, then _____ is also a root.

Also, if $(a + bi)$ is a zero of a function, then _____ is also a zero.

Corollary: A polynomial function f of _____ degree with real coefficients has at least one real zero.

EX #3: A polynomial function f of degree 5 whose coefficients are real numbers has the zeros of $x = 2, 3i$, and $1 + i$. Find the remaining zeros of the function.

EX #4: **(A)** Find a polynomial function f of degree 4 whose coefficients are real numbers and that has the zeros $\{2, 2, -3 + i\}$. Assume $a = 1$ for the leading coefficient.

Lesson 5-B: Polynomial Functions and Complex Zeros

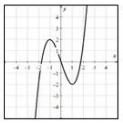
Symmetry

The terms even and odd describe the symmetry that exists for the graph of a function.

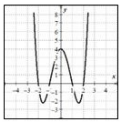
Even	Odd
A function f is even if and only if, when the point (x, y) is on the graph of f, then the point $(-x, y)$ is also on the graph.	A function f is odd if and only if, when the point (x, y) is on the graph of f, then the point $(-x, -y)$ is also on the graph.
$f(-x) = f(x)$	$f(-x) = -f(x)$

EX #8: Determine whether each graph is even, odd or neither.

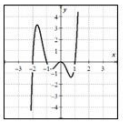
A.



B.



C.



EX #9: Algebraically determine whether each of the following functions is even, odd or neither.

A. $y = x^2 + x - 12$ **B.** $y = x^5 - 4x^3$ **C.** $y = 2x^4 - 6x^2$

Using Differences to Determine Degree

The **degree of a polynomial function** can be found by examining the successive differences of the output values over equal-interval input values. ***The degree of the polynomial function is equal to the least value n for which successive nth differences are constant***.

EX #11: Use the table below to determine the degree of the polynomial.

x	-3	-2	-1	0	1	2	3
y	-30	-14	-12	-12	-2	30	96

Sketching a Graph with Repeated Zeros

EX #12: Write a polynomial function of minimum degree whose zeros include the following, with $a > 0$:

A. $x = -3$, (multiplicity 2); $x = -1$ and $x = 4$ (multiplicity 1)

B. Complete the table to help determine the behavior of the function.

x	-4	-3	-2	-1	0	1	4	5

E. On what interval(s) is $f(x) < 0$

Guided Notes

1.5 **Polynomial Functions and Complex Zeros Practice Set 1**

Name _____

Date _____ Period _____

Problems 1 – 5, Find the remaining zeros of f using the given information about the polynomial. Then write the linear factorization of the polynomial.

1. Degree 3; zeros: $3, 2i$

2. Degree 4; zeros: $i, \sqrt{2}$

3. Degree 4; zeros: $-3i, 1 + i\sqrt{3}$

4. Degree 5; zeros: $-2, i, 3i$

5. Degree 6; zeros: $0, -2, \sqrt{5}, -2 + i$

Problems 6 – 9, Write a polynomial

6. Degree 4; zeros: $2 + i$; -2 (mult

8. Degree 4; zeros: $\sqrt{2}$; $3i$

Problems 10 – 12, Use the given zero to find the remaining zeros of each function.

10. $f(x) = x^3 - 2x^2 + 9x - 18$; zero: $3i$

11. $f(x) = 2x^3 + 8x^2 - 10x - 40$; zero: $-\sqrt{5}$

12. $f(x) = x^4 + 7x^2 - 144$; zero: $4i$

nplex zeros of each polynomial function. Use your results to write nial as a product of linear factors.

14. $f(x) = x^4 + 6x^2 + 8$

16. $f(x) = x^3 + 3x^2 - 5x - 15$

1.5 **Polynomial Functions and Complex Zeros Practice Set 2**

Name _____

Date _____ Period _____

Problems 17 – 19, For each of the table of values below, determine if the function represented is even, odd or neither. Justify your answer.

17.

x	-6	-4	-2	0	2	4	6
$f(x)$	-5	-2	1	0	-1	2	5

18.

x	-8	-5	-1	0	1	5	8
$g(x)$	-6	5	3	2	3	5	6

19.

x	-7	-4	-1	0	1	4	7

d, or neither.

$\frac{1}{4}x^6 - 5x^2 + 3x$

22. $h(x) = 6x^3 - 2x^5$

23. $k(x) = 2x^3 - x^5 + 1$

Problems 24 – 26, Use the table below to determine the degree of the polynomial.

24.

x	-3	-2	-1	0	1	2	3
$f(x)$	116	38	6	2	8	6	-22

25.

x	-3	-2	-1	0	1	2	3
$g(x)$	30	15	4	-3	-6	-5	0

26.

x	-2	-1	0	1	2	3	4
$h(x)$	91	1	-5	1	-5	1	91

f the polynomial inequalities by performing a sign analysis. nswers using a graphing calculator.

31. $x^3 + 3x^2 - 4x < 12$

Practice Sets

1.5 Polynomials and Complex Zeros Form A
Set 1

Name _____

Date _____Period _____

1. Find the remaining zeros of f using the given information about the polynomial. Then write the linear factorization of the polynomial.
   Degree: 5; zeros: $-3, 2+4i, -\sqrt{5}$

2. Write a polynomial with real coefficients having the given degree and zeros.
   Degree: 4; zeros: 4 (multiplicity 2), $-3i$

3. Find the complex zeros of the polynomial. Use your results to write the polynomial as a product of linear factors.
   $f(x) = x^4 + 5x^2 - 36$

Grade:

1.5 Polynomials and Complex Zeros Form B
Set 1

Name _____

Date _____Period _____

1. Find the remaining zeros of f using the given information about the polynomial. Then write the linear factorization of the polynomial.
   Degree: 5; zeros: $3-\sqrt{2}, 5, 3i$

2. Write a polynomial with real coefficients having the given degree and zeros.
   Degree: 4; zeros: 2 (multiplicity 2), $4i$

3. Find the c
   Use your
   product c

Grade:

1.5 Polynomials and Complex Zeros Form A
Set 2

Name _____

Date _____Period _____

For problems 1 and 2, determine whether each function is even, odd, or neither. Explain.

1. $f(x) = 5x^4 + 3x^2 + 2$

2. $f(x) = 6x^5 - 2x^3 + x$

3. Solve the polynomial inequality by performing a sign analysis.
   $x^3 + 2x^2 \geq 9x + 18$

Grade:

1.5 Polynomials and Complex Zeros Form B
Set 2

Name _____

Date _____Period _____

For problems 1 and 2, determine whether each function is even, odd, or neither. Explain.

1. $f(x) = 4x^5 - 3x^3 - x$

2. $f(x) = 7x^4 - 2x^2 + 5$

3. Solve the polynomial inequality by performing a sign analysis.
   $x^3 + 5x^2 < 4x + 20$

Grade:

Daily Lesson Quiz