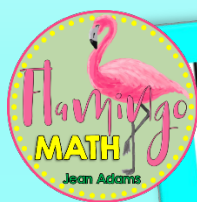


Precalculus Honors

Polar Coordinates and Vectors

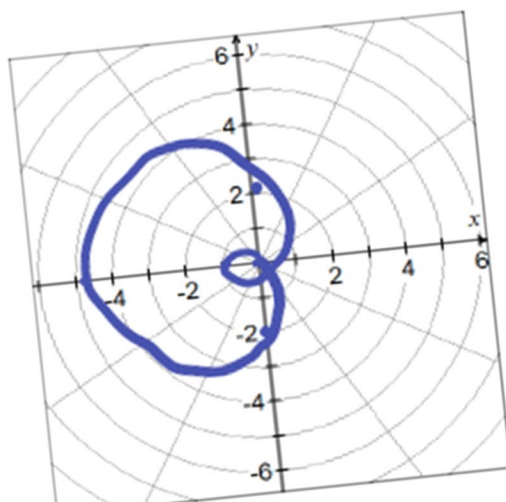


11. $r = 2 - 3 \cos \theta$

limaçon
w/inner
loop

$$\frac{a}{b} = \frac{3}{2}$$

left



Guided Notes

6

Completely Editable

PRECALCULUS CURRICULUM

Scope and Sequence

Unit 6: Polar Coordinates & Vectors

Lesson	Skills & Learning Objectives
[6.1] Polar Coordinates	☐ Convert points and equations from polar to rectangular form and vice versa
[6.2] Polar Graphs and Equations	☐ Graph Polar Equations ☐ Convert polar equations to rectangular form and vice versa ☐ Identify types of polar graphs from equations
[6.3] More About Polar Graphs	☐ Use auxiliary graphs to sketch polar curves ☐ Find negative r -values that create inner loops ☐ Find points of intersection in two polar graphs
[6.4] The Complex Plane; De Moivre's Theorem	☐ Represent complex numbers in the complex plane ☐ Write complex numbers in trigonometric form ☐ Find the conjugate of a complex number and moduli ☐ Find product and quotients of complex numbers ☐ Apply De Moivre's Theorem to find n th root of complex numbers
[6.5] Vectors in the Plane	☐ Apply the arithmetic of vectors ☐ Find component form, magnitude, and solve real-world problems ☐ Use vectors to solve real-world problems ☐ Find projection of vectors, angles between vectors
[6.6] The Dot Product	☐ Calculate dot products ☐ Find the angle between two vectors ☐ Finding vector components ☐ Find work done by a constant force

Lesson 6.I: Polar Coordinates

In this lesson we will define polar coordinates and learn how they are related to rectangular equations. Students will convert from rectangular to polar and vice versa. Let's explore how to set up this system.

The Language of Polar Graphs

A **polar coordinate system** is a system to plot points in a plane with a point O in the center of a circular grid.

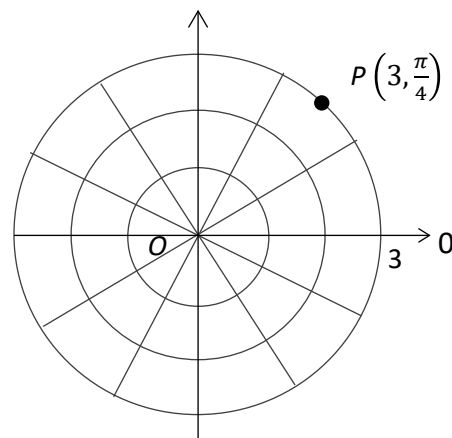
The **Pole** is the point O , the origin

Polar Axis the horizontal ray from O pole (origin) to the right.

Polar Coordinates of each point in the plane $P = (r, \theta)$

$r =$ **Directed distance** from O to P .

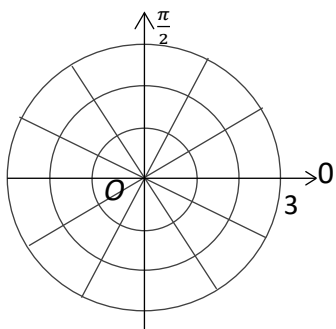
$\theta =$ **Directed angle** in a counter-clockwise direction from polar axis to OP



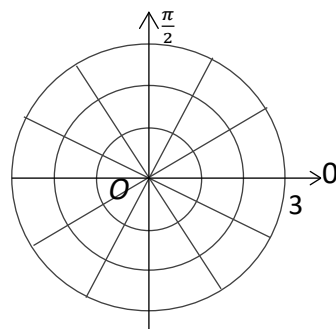
Plotting Points in a Polar System

EX #1: Plot the points with the given polar coordinates.

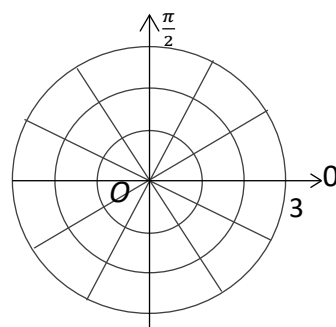
A. $P\left(2, \frac{\pi}{4}\right)$



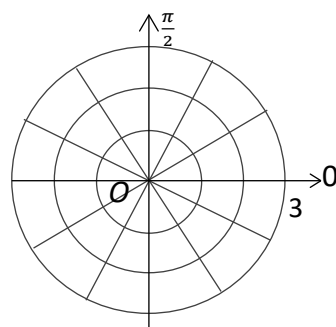
B. $Q\left(-1, \frac{\pi}{3}\right)$



C. $R(3, -30^\circ)$



D. $S\left(-2, -\frac{7\pi}{6}\right)$



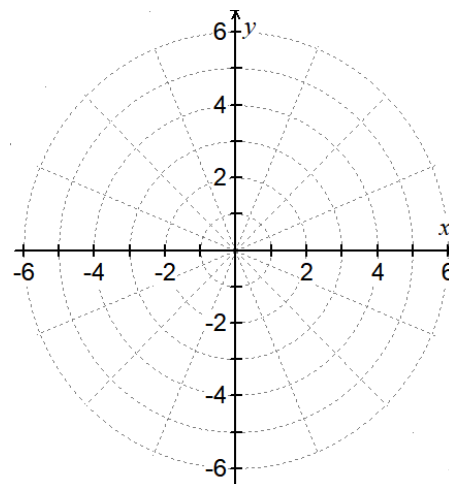
Zeros and Maximum R-Values

EX #5: Find zeros of $r^2 = 16 \sin 2\theta$

Zeros of r occur at:

Max of $|r| =$

r	θ

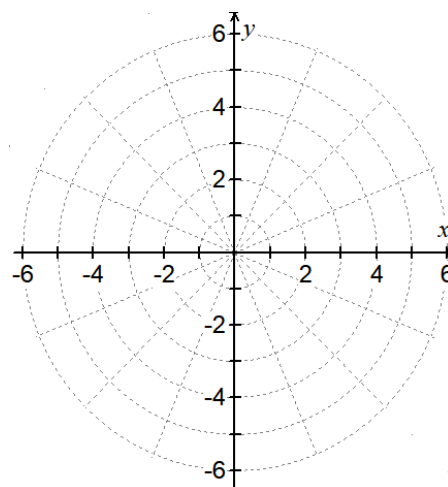


Graphing a Logarithmic Spiral

EX #6: Use a calculator to complete the table below. Then, graph the spiral $r = e^{\frac{\theta}{4}}$. We call this a logarithmic spiral because the equation may be written as $\theta = 4 \ln(r)$.

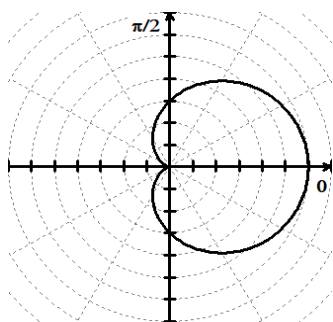
θ	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r								

Notice that r is positive for all θ . Also as $\theta \rightarrow -\infty, r \rightarrow$ _____.
 As $\theta \rightarrow \infty, r \rightarrow$ _____. Is there any symmetry to this polar graph? Explain.

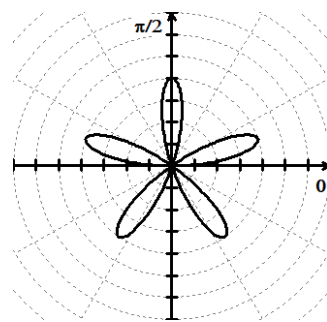


EX #2: Determine the domain restrictions for sketching the polar curve through one cycle. Use the auxiliary graph of the corresponding rectangular function to assist you.

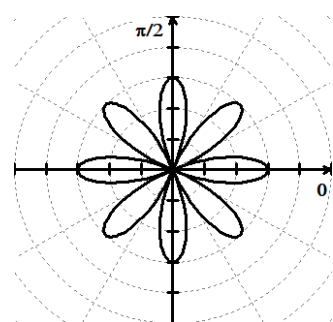
A. $r = 3 + 3 \cos(\theta)$



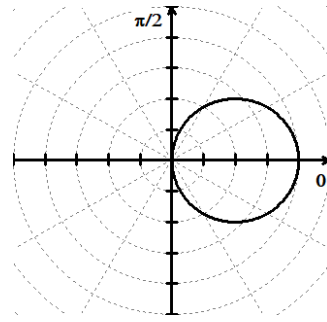
B. $r = 4 \sin(5\theta)$



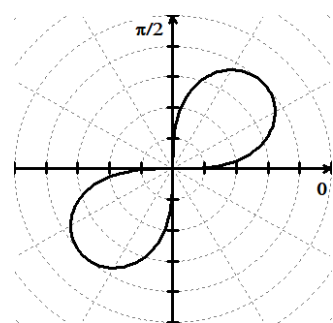
C. $r = 3 \cos(4\theta)$



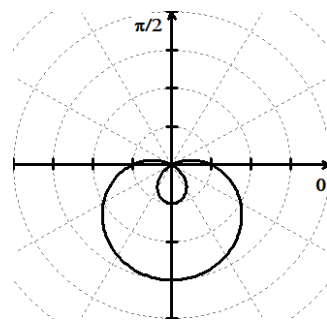
D. $r = 4 \cos(\theta)$



E. $r^2 = 16 \sin(2\theta)$



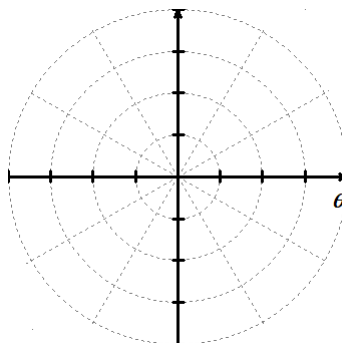
F. $r = 1 - 2 \sin(\theta)$



Intervals where r-values are Negative

EX #3: Let's look at the table from EX #1 again. Find the range of positive θ -values on the given domain for which the r value is negative. Confirm your answer by plotting this part of the polar graph. What do you notice?

$$r = 1 + 2 \sin \theta$$



r	θ

Lesson 6.4: The Complex Plane; DeMoivre's Theorem

This lesson will show how to represent complex numbers in polar (trigonometric) form. We first need to learn how to represent complex numbers graphically. Complex numbers $z = a + bi$ have two components, the real part, a ; and, the imaginary part, bi . We will use a **complex plane** with two axes in order to graph complex numbers, the **real axis** and the **imaginary axis**. To graph the complex number $a + bi$, we will plot the ordered pair of numbers (a, b) in this plane.

Graphing Complex Numbers

EX #1: Sketch each point on grid at right.

$$A = 3 + 4i$$

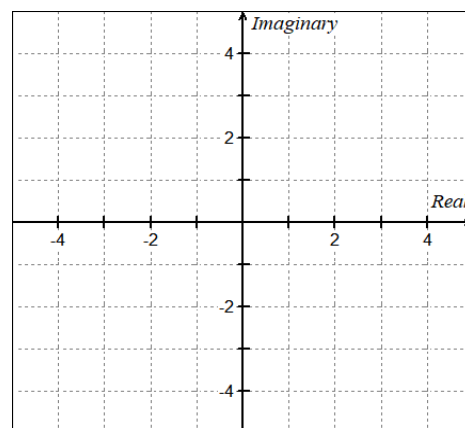
$$D = -4 + 0i$$

$$B = 0 + 2i$$

$$F = -1 - 2i$$

$$C = -2 + 3i$$

$$G = 3 - i$$



Absolute Value of Complex Numbers

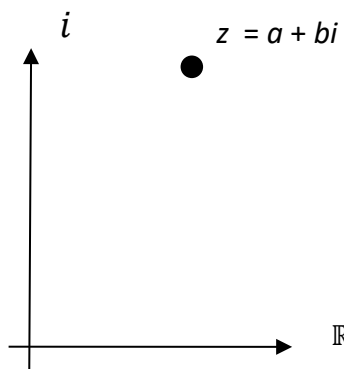
EX #2: OP has magnitude (length) and direction

The **modulus** (or **absolute value**) of the complex number $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$

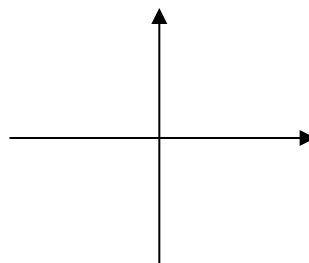
$$|a + bi| = \sqrt{a^2 + b^2}$$

$$\tan \theta = \frac{b}{a} \quad a \neq 0$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right) \text{ if } a > 0$$



NOTE: If $a < 0$, then $\theta = \tan^{-1} \left(\frac{b}{a} \right) + \pi$



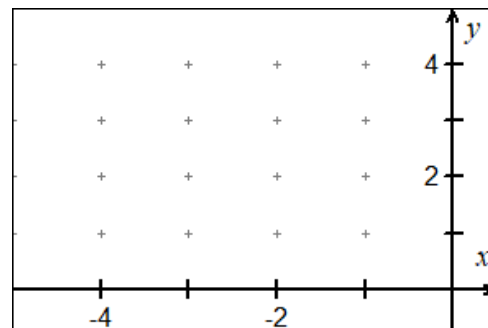
Magnitude of Vectors

The magnitude (length) of a vector $\mathbf{v} = \langle a, b \rangle$ is $\|\mathbf{v}\| = \sqrt{a^2 + b^2}$

If vector $\mathbf{v}$ is represented by the arrow from (x_1, y_1) to (x_2, y_2) , then

$$\|\mathbf{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ also if } \mathbf{v} = \langle a, b \rangle \text{ then } \|\mathbf{v}\| = \sqrt{a^2 + b^2}$$

EX #3: Sketch and find the magnitude of $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$



Properties of Vectors

The zero vector is the vector $\mathbf{0} = \langle 0, 0 \rangle$. This vector behaves the same for addition of vectors as the number 0 does for addition properties of real numbers.

Vector Addition	Multiplication by a scalar
Length of a vector	$0 \cdot \mathbf{u} = \mathbf{0}$
$ c \cdot \mathbf{u} = c \mathbf{u} $	$c \cdot \mathbf{0} = \mathbf{0}$
$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$	$1 \cdot \mathbf{u} = \mathbf{u}$
$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	$c \cdot (\mathbf{u} + \mathbf{v}) = c \cdot \mathbf{u} + c \cdot \mathbf{v}$
$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$	$(c + d)\mathbf{u} = c \cdot \mathbf{u} + d \cdot \mathbf{u}$
$\mathbf{u} + \mathbf{0} = \mathbf{u}$	$(cd)\mathbf{u} = c(d \cdot \mathbf{u}) = d(c \cdot \mathbf{u})$

EX #4: Given $\mathbf{v} = \langle -3, 5 \rangle$ and $\mathbf{w} = \langle 2, 4 \rangle$

A. Find $2\mathbf{v} + \mathbf{w}$

B. Find $\mathbf{w} - \mathbf{v}$

Lesson 6.6: The Dot Product

In this section, we explore one of the ways vectors can be multiplied. The **dot product of two vectors** results in a scalar. So, the dot product of two vectors is not a vector but a real number.

Another method of multiplying vectors, known as the cross product, is a vector that is perpendicular to the plane of the two vectors being multiplied. This takes us into three dimensions, so it is outside of this chapter's scope.

The Language of Vectors

A **dot product** of $\mathbf{u} = \langle a_1, b_1 \rangle$ and $\mathbf{v} = \langle a_2, b_2 \rangle$ is $\mathbf{u} \cdot \mathbf{v} = a_1 a_2 + b_1 b_2$

Properties of Dot Products

Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors and let c be a scalar

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

4. $\mathbf{v} \cdot \mathbf{v} = \mathbf{v}^2$

2. $\mathbf{0} \cdot \mathbf{v} = \mathbf{0}$

5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$

3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

Angle Between Two Vectors:

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \quad 0 \leq \theta \leq \pi$$

Alternate Form Of Dot Product:

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

Orthogonal Vectors

Orthogonal (perpendicular or normal) vectors have a dot product of zero

Two vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if and only if $\mathbf{u} \cdot \mathbf{v} = 0$

EX #1: Find the dot product $\mathbf{u} \cdot \mathbf{v}$,
given: $\mathbf{u} = \langle 5, 12 \rangle$ and $\mathbf{v} = \langle -3, 2 \rangle$

EX #2: Use the dot product to find magnitude
of $\mathbf{u} = \langle 2, -4 \rangle$

EX #3: Show that the vectors are orthogonal. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$