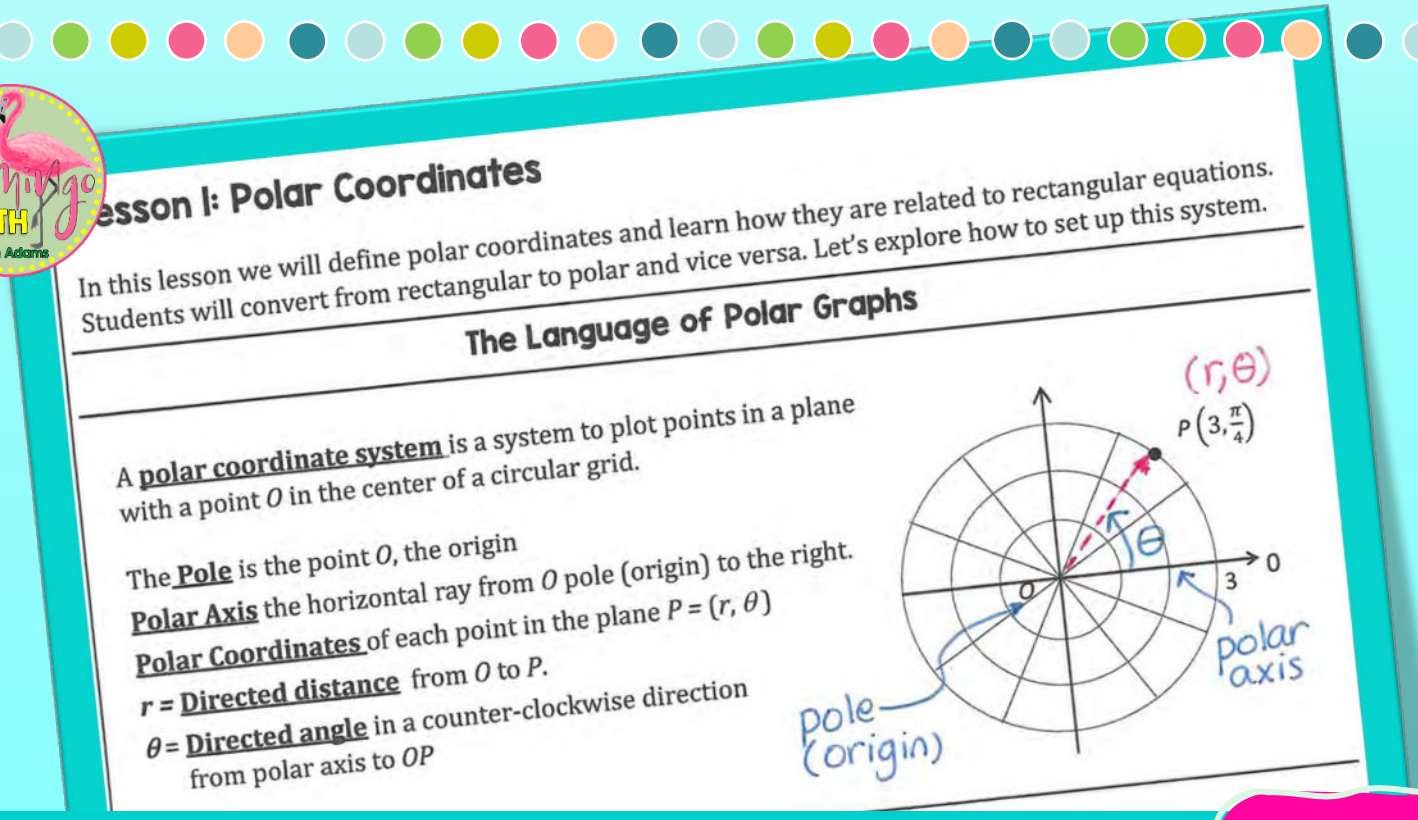
Precalculus Honors

# Polar Coordinates




Completely Editable

6.1

## Lesson 6.I: Polar Coordinates

In this lesson we will define polar coordinates and learn how they are related to rectangular equations. Students will convert from rectangular to polar and vice versa. Let's explore how to set up this system.

### The Language of Polar Graphs

A **polar coordinate system** is a system to plot points in a plane with a point $O$ in the center of a circular grid.

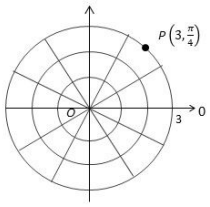
The **Pole** is the point $O$, the origin

**Polar Axis** the horizontal ray from $O$ pole (origin) to the right.

**Polar Coordinates** of each point in the plane $P = (r, \theta)$

$r$ = **Directed distance** from $O$ to $P$.

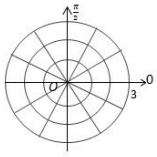
$\theta$ = **Directed angle** in a counter-clockwise direction from polar axis to $OP$



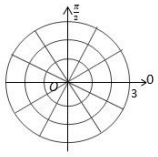

### Plotting Points in a Polar System

**EX #1:** Plot the points with the given polar coordinates.

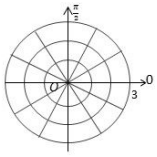
**A.** $P\left(2,\frac{\pi}{4}\right)$



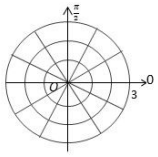

**B.** $Q\left(-1,\frac{\pi}{3}\right)$




**C.** $R\ (3, -30°)$




**D.** $S\left(-2,-\frac{7\pi}{6}\right)$




### Helpful Tips for Converting from Rectangular to Polar Form

1. Plot the point $(x, y)$, noting the quadrant or coordinate axis the point lies on.
2. If either $x = 0$ or $y = 0$, use your sketch to find $r$. Otherwise, use $r = \sqrt{x^2 + y^2}$
3. Find $\theta$. When the point lies in . . .

**Quadrant I or IV:** $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

**Quadrant II or III:** $\theta = \pi + \tan^{-1}\left(\frac{y}{x}\right)$

### Convert a Rectangular Equation to Polar Form

**EX #5:** Express the given equation in polar form.

**A.** $y = \frac{1}{2}x^2$

**B.** $4x^2y = 1$

### Convert a Polar Equation to Rectangular Form

**EX #6:** Convert each equation to rectangular form and identify the graph. Support your answer with a polar graphing utility.

**A.** $r = 4$

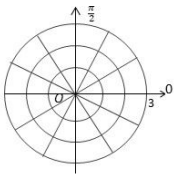
**B.** $\theta = \frac{2\pi}{3}$

### Finding All Polar Coordinates of a Single Point

**EX#2:** Plot the point $\left(2,\frac{2\pi}{3}\right)$, find three additional polar representations of this point on $[-2\pi, 2\pi]$

**Formula:** $(r, \theta + 2\pi n)$ or $(-r, \theta + (2n+1)\pi)$

**A.**




**B.** Write the rest of the possibilities:

### Relationships between Polar and Rectangular Coordinates

1. **To change from polar to rectangular coordinates, use the formulas:**

   $x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

2. **To change from rectangular to polar coordinates, use the formulas:**

   $x^2 + y^2 = r^2$ $\qquad$ $\tan\theta = \frac{y}{x}\ (x \neq 0)$

**EX #3:** Find the rectangular coordinates of the points with the given polar coordinates.

**A.** $P\left(3,\frac{5\pi}{6}\right)$

**B.** $Q(2, -200°)$

**EX #4:** Find two polar coordinate pairs for the points with given rectangular coordinates.

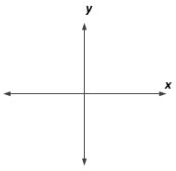
**A.** $P(1, -1)$

**B.** $Q(2, 0)$

**C.** $r = 2\csc\theta$

**D.** $r = 6\cos\theta$

**EX #7:** Convert $(x-3)^2 + (y-2)^2 = 13$ to polar form.




NOTES

**6.1 Polar Coordinates Homework**

Name _____

Date _____ Period _____

**Problems 1 – 4,** the polar coordinates of a point are given. Find its rectangular coordinates.

1. $\left(3, -\frac{\pi}{3}\right)$

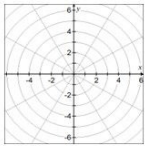
2. $\left(-2, \frac{7\pi}{6}\right)$

3. $\left(-5, -\frac{2\pi}{3}\right)$

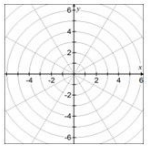
4. $\left(-4, -\frac{11\pi}{6}\right)$

**Problems 5-10,** plot each point given in polar coordinates.

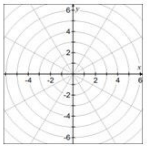
5. $\left(4, \frac{4\pi}{3}\right)$



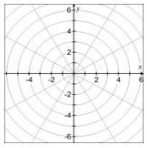
6. $\left(-2, \frac{5\pi}{6}\right)$



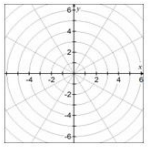
7. $\left(3, -\frac{3\pi}{4}\right)$



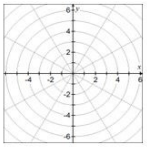
8. $\left(-5, -\frac{5\pi}{3}\right)$



9. $\left(6, \frac{3\pi}{2}\right)$



10. $(-4, -\pi)$



**Problems 11 – 14,** given the polar coordinates, name two other equivalent representations.

11. $\left(3, \frac{\pi}{6}\right)$

12. $\left(-4, \frac{2\pi}{3}\right)$

13. $\left(2, -\frac{\pi}{2}\right)$

14. $\left(-1, -\frac{\pi}{4}\right)$

**Problems 15 – 18,** the rectangular coordinates of a point are given. Find polar coordinates for each point.

15. $(\sqrt{3}, 1)$

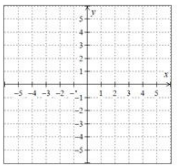
16. $(-2,2)$

17. $(4, -4\sqrt{3})$

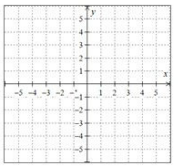
18. $(2.3, -1.5)$

**Problems 19 – 22,** convert the polar equation to rectangular form. Graph the equation.

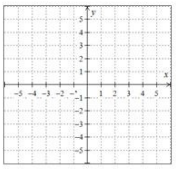
19. $r = 3$



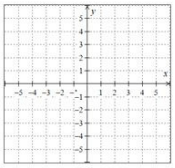
20. $r = 4\cos\theta$



21. $r = 2\sec\theta$

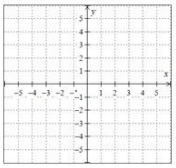


22. $r = 2\sin\theta - 4\cos\theta$

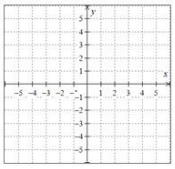


**Problems 23 - 26,** convert the rectangular equation to polar form. Graph the equation.

23. $x = 4$



24. $2x - y = 3$



25. $y^2 = 3x$



26. $x^2 + (y-2)^2 = 4$



# Practice Sets

# Lesson Warm-Ups

| | |
|---|---|
| **3.** Convert $r = 2\sin(\theta)$ to rectangular form. Identify the graph. | **4.** Name two different representations of the point on the interval of $[-2\pi, 2\pi]$ given $P = \left(3, \frac{\pi}{6}\right)$. |

Name _____ Date _____ Period _____

**Lesson 6.I: Polar Coordinates** **Warm-Up # 2**

| | |
|---|---|
| **1.** Find the rectangular coordinates for the polar point $P\left(2, \frac{\pi}{6}\right)$ | **2.** Given the point $Q(-3,0)$, find the polar coordinates. |
| **3.** Convert $r = 6\cos(\theta)$ to rectangular form. Identify the graph. | **4.** Name t on the |

**6.1 Polar Coordinates** **Form A**

Name _____

Date _____ Period _____

| |
|---|
| **1.** Find the rectangular coordinates for the polar point $P\left(3, \frac{\pi}{6}\right)$. |
| **2.** Given the point $Q\ (0, -3)$, find the polar coordinates. |
| **3.** Convert $r = -4\sin\theta$ to rectangular form. Identify the graph. |

**6.1 Polar Coordinates** **Form B**

Name _____

Date _____ Period _____

| |
|---|
| **1.** Find the rectangular coordinates for the polar point $P\left(2, \frac{\pi}{4}\right)$. |
| **2.** Given the point $Q\ (-4, 0)$, find the polar coordinates. |
| **3.** Convert $r = 3\cos\theta$ to rectangular form. Identify the graph. |

# Daily Lesson Quiz