

Precalculus Honors

Piecewise Functions



Applications from Real Life

In the land of Leibniz, a person filing taxes as single pay taxes based on the following table.

If your taxable income is at least	But not more than	Then, the tax rate is	Of the amount over
\$0	\$30,000	15%	\$0
\$30,000	\$65,000	\$4500 + 20%	\$30,000
\$65,000	\$125,000	\$11,500 + 24%	\$65,000

- A. Write a piecewise function $T(x)$ giving the tax owed by a person whose taxable income is x , when x is less than \$125,000.

$$T(x) = \begin{cases} 0.15x; & 0 \leq x \leq 30,000 \\ 4500 + 0.2x; & 30,000 < x \leq 65,000 \\ 11,500 + 0.24x; & 65,000 < x \leq 125,000 \end{cases}$$

...single person whose taxable income is \$68,750.

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1.4

Lesson 4: Piecewise Functions

Piecewise functions are functions defined by different formulas for different intervals of the _____ variable. Standard notation is given in the following format:

$$f(x) = \begin{cases} \text{formula 1 if (use this domain for formula 1)} \\ \text{formula 2 if (use this domain for formula 2)} \\ \text{formula 3 if (use this domain for formula 3)} \end{cases}$$

Examples of Piecewise Functions

The Absolute Value Function

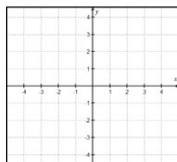
Is a piecewise function because

$$|x| = \begin{cases} \end{cases}$$

Is the function above continuous?

State the domain and range.

Where is the function increasing? decreasing?



The Ceiling Function is defined as

$f(x) = \lfloor x \rfloor$ where $\lfloor x \rfloor$ is the least

integer _____

or _____ to x .

Sketch the graph on the interval from $[-3, 3]$.

The Floor Function is defined as

$f(x) = \lceil x \rceil$ where $\lceil x \rceil$ is the greatest

integer _____

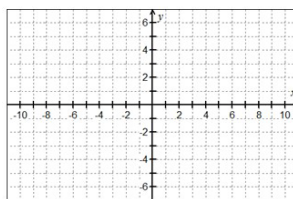
or _____ to x .

Sketch the graph on the interval from $[-3, 3]$.

Graphing Piecewise Functions

EX #5: Graph the piecewise function.

$$f(x) = \begin{cases} -\frac{1}{2}(x+4)^2 + 3, & x \leq -1 \\ 3|x-2| - 4 & -1 < x < 4 \\ (x-6)^2 + 2 & x \geq 4 \end{cases}$$



Converting Absolute Value Functions into Piecewise Functions

In an earlier lesson we used a calculator to investigate intervals of increase and decrease on the function $g(x) = |x+2| + |x-1| - 2$. Let's develop a technique so that you will have a graphing strategy for converting the absolute value form of g into a piecewise-defined function.

1. Find the zeros of each argument.
2. Use these values to divide a number line into intervals.
3. Pick test values in each interval.
4. Using parentheses instead of absolute value bars, determine the sign of each expression based on your test value.
5. Simplify the expression for each interval to find equations for the piecewise function.
6. Graph your function.

Evaluating Piecewise Functions

EX #2: Evaluate. $f(x) = \begin{cases} -2|x+1|, & x < 1 \\ 2, & 1 \leq x < 3 \\ 5 - \frac{1}{2}x, & x \geq 3 \end{cases}$

A. $f(12)$

B. $f(2)$

C. $f(-5)$

D. $f(0)$

EX #3: Evaluate. $f(x) = \begin{cases} 3x - 7; & x > 2 \\ x^2 - 1; & x \leq 2 \end{cases}$

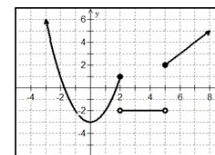
A. $f(6)$

B. $f(-3)$

That Graph

ents the graph.

B. $f(x) = \begin{cases} \text{_____ if _____} \\ \text{_____ if _____} \\ \text{_____ if _____} \end{cases}$



EX #6: Write the function as a piecewise-defined function.
 $f(x) = |x+1| + 2|x-1|$

Applications from

EX #7: In the land of Leibniz, a person filing taxes as

If your taxable income is at least	But not more than	Tax	Tax
\$0	\$30,000	15%	\$0
\$30,000	\$65,000	\$4500 + 20%	\$30,000
\$65,000	\$125,000	\$11,500 + 24%	\$65,000

A. Write a piecewise function $T(x)$ giving the tax owed by a person whose taxable income is x , when x is less than \$125,000.

More Applications

EX #8: In August, Orlando Utilities Commission supplied electricity to residents for a monthly customer charge of \$8.00 plus 10.143¢ per kilowatt-hour (KWH) for the first 1000 KWH used in the month and 12.143¢ per kilowatt-hour for all electricity over 1000 KWH in the month.

A. What is the charge for using 920 KWH in a month?

B. What is the charge for 1500 KWH?

C. If C is the monthly charge for x KWH, write a piecewise-defined function for C in terms of x .

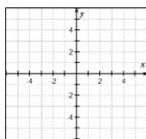
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1.4 Piecewise Functions Homework

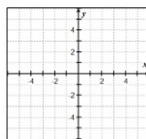
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Date _____ Period _____

Problems 1–6, (A) Sketch the graph of f . (B) Find the domain and range of f .

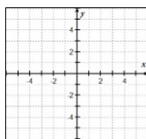
1. $f(x) = \begin{cases} 2x & x \neq 1 \\ -3 & x = 1 \end{cases}$



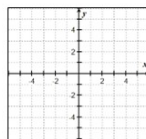
2. $f(x) = \begin{cases} x + 3 & x < -2 \\ -2x - 3 & x \geq -2 \end{cases}$



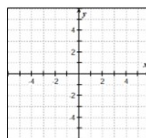
3. $f(x) = \begin{cases} \frac{1}{2}x + 1 & -4 < x \leq 2 \\ 2 - 2x & x > 2 \end{cases}$



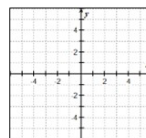
4. $f(x) = \begin{cases} \sqrt{1-x} & -3 < x \leq 1 \\ x - 4 & 1 < x \leq 5 \end{cases}$



5. $f(x) = \begin{cases} 3x + 5 & -3 \leq x < -1 \\ \sqrt{x+1} & x \geq -1 \end{cases}$

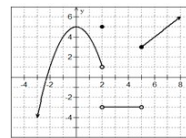


6. $f(x) = \begin{cases} x^2 - 5 & -3 \leq x < 2 \\ x - 3 & x \geq 2 \end{cases}$

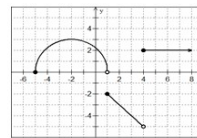


Problems 7–8, Write a piecewise-defined function for each function shown below.

7.



8.



9. An SUV was purchased for \$35,750. The value of the vehicle decreases by \$2800 per year for the first four years and \$1700 per year for the next six years.

A. Write a function that describes the value, V , of the SUV after x years, where $0 \leq x \leq 10$

B. Find $V(3)$

C. Find $V(8)$

10. Filing income taxes, $T(x)$, in dollars, on adjusted gross income can be modeled by the function

$$T(x) = \begin{cases} 0.15x & 0 \leq x \leq 18,000 \\ 0.28(x - 18,000) + 2700 & 18,000 < x \leq 44,250 \\ 0.32(x - 44,250) + 10,050 & x > 44,250 \end{cases}$$

A. Find and interpret $T(40,000)$

B. Find and interpret $T(75,000)$

11. In the 1995 tax form a tax rate schedule is given for people whose filing status is single. Part of the table is shown below:

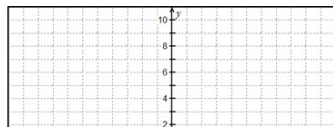
If the taxable income is over....	But not over...	Then the tax is...	Of the amount over...
\$0	\$23,350	15%	\$0
\$23,350	\$56,550	\$3,502.50 + 28%	\$23,350
\$56,550	\$117,950	\$12,798.50 + 31%	\$56,550

A. Write the defining rule for a piecewise function $T(x)$ giving the tax owed by a person whose taxable income is x , when x is less than \$117,950.

B. Evaluate the function to find the tax owed by a single person whose taxable income in 1995 was \$31,950.

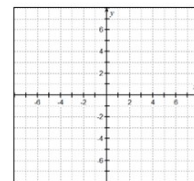
12. Graph the following piecewise function.

$$f(x) = \begin{cases} -\frac{1}{3}(x+4)^2 + 3 & x \leq -1 \\ 3|x-2| - 4 & -1 < x < 4 \\ (x-6)^3 + 3 & x \geq 4 \end{cases}$$

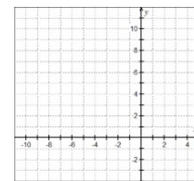


Problems 13–14, Write the absolute value function as a piecewise defined function, then graph.

13. $f(x) = -2|x-3| + 5$



14. $g(x) = |x+2| + |2x+6|$



Problems 15–16, For what value(s) of a and b will the function be continuous everywhere?

15. $f(x) = \begin{cases} ax^2 - 3x & x < 2 \\ 2a + 1 & x \geq 2 \end{cases}$

16. $g(x) = \begin{cases} 2x + 5 & x < -3 \\ ax + 5b & -3 \leq x < 2 \\ x^2 - 2x + 3 & x \geq 2 \end{cases}$

Practice Sets

Lesson Warm-Ups

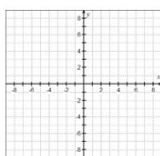
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Lesson I.4: Piecewise Functions

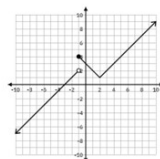
Warm-Up # 1

1. Graph. $f(x) = \begin{cases} 2x - 4; x \geq 1 \\ x^2 - 1; x < 1 \end{cases}$

2. State the domain and range for $f(x)$



3. Write a piecewise function for the graph shown.



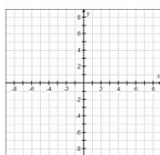
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Lesson I.4: Piecewise Functions

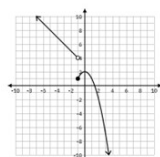
Warm-Up # 2

1. Graph. $g(x) = \begin{cases} -x - 2; x \geq 1 \\ -x^2 + 1; x < 1 \end{cases}$

2. State the domain and range for $g(x)$



3. Write a piecewise function for the graph shown.



1.4 Piecewise Functions

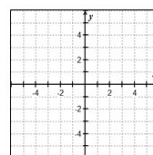
Form A

Name _____

Date _____ Period _____

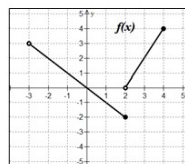
1. Graph

$$f(x) = \begin{cases} \frac{1}{2}x + 1 & x > 0 \\ x^2 - 1 & -2 \leq x \leq 0 \end{cases}$$



2. State the domain for the function above.

3. Write a piecewise function for the graph shown.



Grade:

1.4 Piecewise Functions

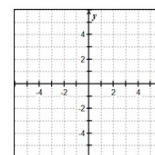
Form B

Name _____

Date _____ Period _____

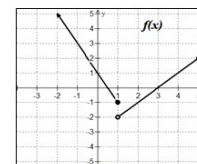
1. Graph

$$g(x) = \begin{cases} 4 - x^2 & -3 \leq x < 0 \\ 2x - 3 & x \geq 0 \end{cases}$$



2. State the domain for the function above.

3. Write a piecewise function for the graph shown.



Grade:

Daily Lesson Quiz