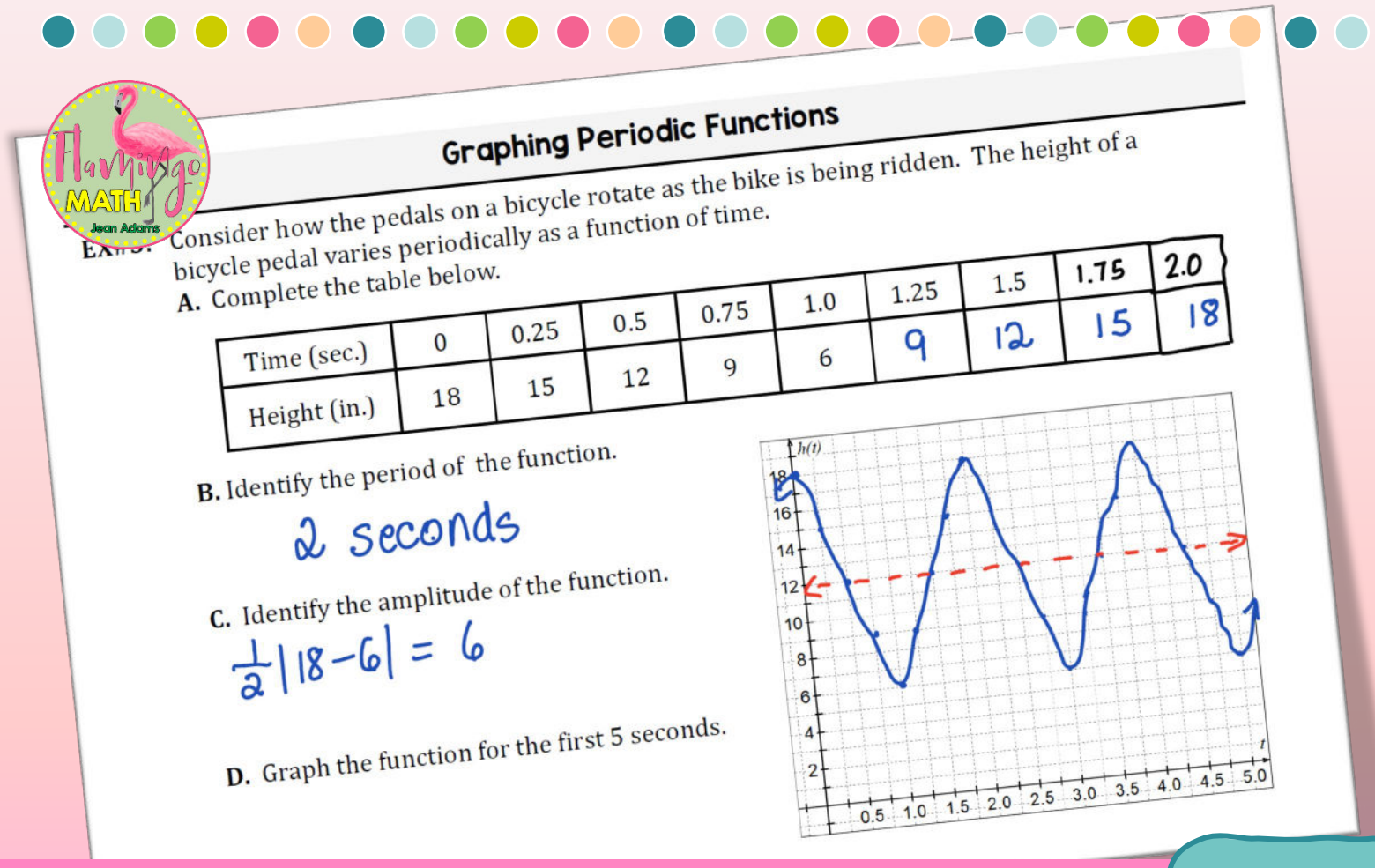
AP® Precalculus

Periodic Phenomena

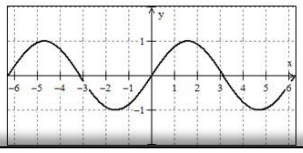


Completely Editable

3.1

Lesson 3-A: Periodic Phenomena

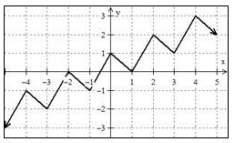
A periodic function is any function that repeats its values on regular intervals or "periods." Think about your breathing , your heartbeat, or the waves in the ocean. These phenomena repeat the same activity as a steady pattern. The most famous periodic functions are the trigonometric functions. We will see sine, cosine, and tangent in our second lesson. Other examples of periodic functions in nature include light waves, sound waves and phases of the moon.

Verbal Vocabulary	Visual Vocabulary
Periodic function – any function that repeats a pattern of y-values at regular intervals.	Use the graph below to label and identify the vocabulary terms.
Cycle – one complete pattern of the y-values for a periodic function.	
Period – the horizontal length of one cycle for a periodic or repeating function.	

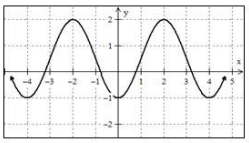
Identifying Periodic Functions

EX #2: Determine whether each function is or is not periodic. If it is, find the period.

A.



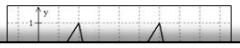
B.



Finding Amplitude of a Periodic Function

EX #3: Find the amplitude of each function.

A.



B.



Graphing Periodic Functions

EX#5: Consider how the pedals on a bicycle rotate as the bike is being ridden. The height of a bicycle pedal varies periodically as a function of time.

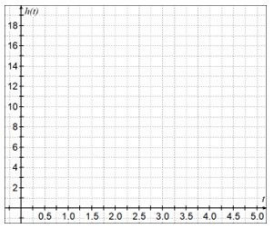
A. Complete the table below.

Time (sec.)	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
Height (in.)	18	15	12	9	6				

B. Identify the period of the function.

C. Identify the amplitude of the function.

D. Graph the function for the first 5 seconds.



Putting It All Together

EX #6: Answer each of the following:

A. A periodic function goes through 5 complete cycles in 4 minutes. What is the period of the

Lesson I-B: Periodic Phenomena

Definition of a Periodic Function

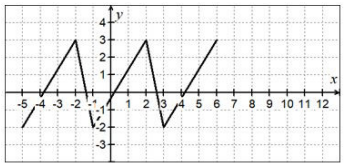
The function f is a **periodic function** of x if and only if there is a number k for which $f(x+nk) = f(x)$ for all values of x in the domain.
If k is the smallest such number, then k is called the **period** of the function.

EX #7: Consider the periodic function shown below.

A. What is the period, k of the function?

B. Complete the function behavior on the grid at right.

C. Show that $f(x+nk) = f(x)$ is valid for $n = 1, 2, 3, \ldots$ for any minimum value on the graph.



EX #8: In Austin, Texas, the average high temperature for each month is shown in the table.

Average High Temperatures			
Month	Temp (°F)	Month	Temp (°F)



Characteristics of Periodic Functions

Periodic functions take on characteristics of other functions, such as intervals of increase and decrease, concavities, and various rates of change. However, with periodic functions, all characteristics found in one period of the function will be in every period of the function.

EX #9: Identify the following characteristics.

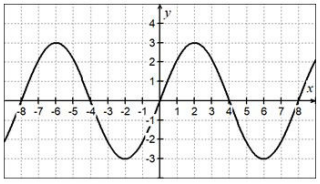
A. period

B. amplitude

C. maximum and minimum values

D. intervals of concave up

E. intervals of concave down

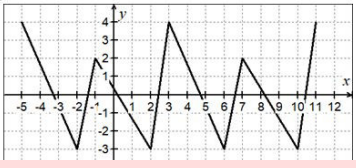


EX #10: Identify the following characteristics.

A. period

B. amplitude

C. maximum and minimum values

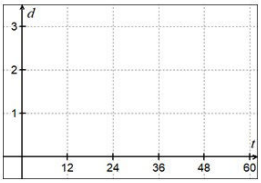


F. Find average rates of change on the intervals $[2, 3], [3, 6] [6, 7]$ and $[7, 10]$.

EX #11: A storm siren makes 2.5 rotations per minute and the sound waves have a radius of 1 mile. Mr. Tyler's house is 1 mile away from the siren. The distance of the sound wave from his house varies periodically as a function of time.

A. What is the period of the function?

B. Let the x-axis represent the time from $t = 0$ to $t = 60$. Let the y-axis represent the distance d the sound wave is from Mr.. Tyler's house at any time t.

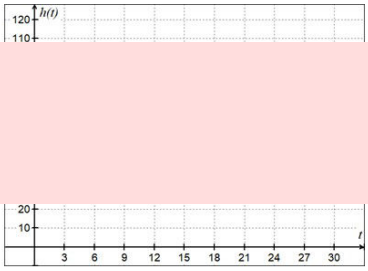


EX #12: A Ferris Wheel at the Orlando Eye attraction is 120 meters in diameter. The height of a compartment h is a function of time t. It takes about 30 minutes to make one complete revolution. Let the height of the car where you enter the ride represent the height at time 0.

A. Find the amplitude.

B. Sketch a graph of the function.

C. At what time does the Ferris wheel reach a maximum?



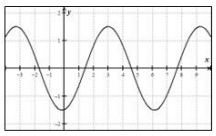
Guided Notes

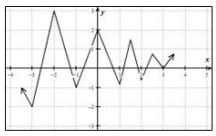
3.1 **Periodic Phenomena Practice Set 1**

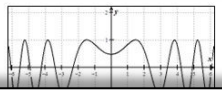
Name _____

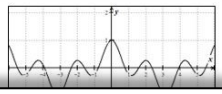
Date _____ Period _____

Problems 1 – 4, determine whether each function *is* or *is not* periodic. If it is, find the period. If not, explain why.

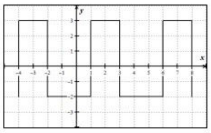
1. 

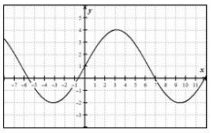
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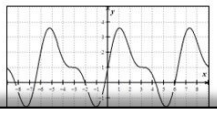
3. 

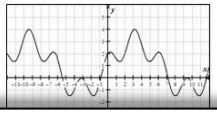
4. 

Problems 7 – 10, find the period and amplitude of each periodic function. State the maximum and minimum values for each function.

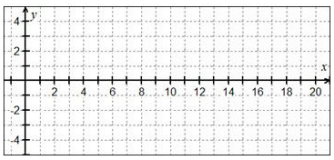
7. 

8. 

9. 

10. 

12. Your boat is bobbing slowly in the ocean over the waves while you are fishing. The range of the swells are 4 feet from the top of a swell to the bottom. If it takes you 10 seconds to travel from the top of one wave to the top of the next wave, sketch two complete cycles.



13. A periodic function goes through 14 complete cyc
function?

14. In a physics lab you studied the oscillations of a p
cycles in 12 seconds. Find the period.

3.1 **Periodic Phenomena Practice Set 2**

Name _____

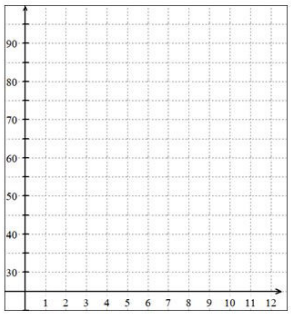
Date _____ Period _____

Problems 1-2, sketch a graph to complete each question.

1. In Atlanta, Georgia, the average temperature per month is shown in the table below. (www.worldcimate.com)

A. Plot the data on the grid provided.

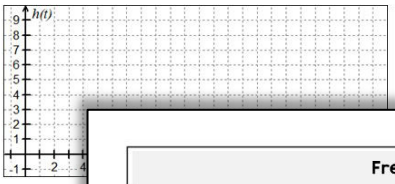
Average High Temperatures			



3. A buoy in the Gulf of Mexico sends a signal beacon to a Coast Guard station. The behavior of the buoy can be modeled by a periodic function, where h is measured in feet above sea level. During a recent tropical storm, the height varies from 1 foot to 8 feet with a 6 second interval between one 8-foot height to the next.

A. Sketch a graph showing how the height of the buoy oscillates over a 24-second time interval.

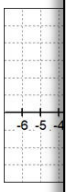
B. Find the amplitude and period of the buoy.



C. What is the midline of the graph?

D. Explain the meaning of the points of inflection i

5. For the graph at right:



A. Find the amplitude and period.

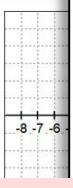
B. Find the midline of the graph.

C. Find the interval(s) where the function is increasing.

D. Find the intervals where the function is decreasi

E. Given $f(3) = 1$, show that $f(x + nk) = f(3)$ wh

6. For the graph at right:



A. Find the amplitude and period.

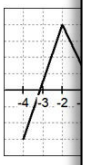
B. Find the midline of the graph.

C. Find the point(s) of inflection.

F. Given $f(0) = 1$, show that $f(x + nk) = f(0)$ where $n = -1$. Does this value fall on the graph?

Problems 4 – 6, Given the periodic function, complete the characteristics for each function.

4. For the graph at right:



A. Find the amplitude and period.

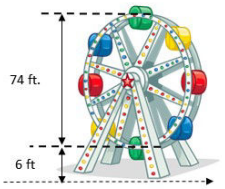
B. Find the rate(s) of change where the function is increasing.

C. Find the rate(s) of change where the function is d

D. Given $f(-2) = 4$, show that $f(x + nk) = f(-2)$

Free Response Question

Kevin and Alice are the last two seated on the Ferris wheel. Once the ride begins, the wheel moves at a constant speed. It takes 48 seconds to complete one revolution.

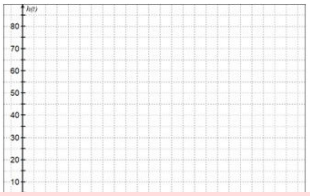


A. How high are the two friends when the ride starts?

B. At what height are they at 12 seconds? 24 seconds? 36 seconds?

C. What is their height at 168 seconds? How many revolutions have they made?

D. Sketch a graph showing the relationship between the height above the ground and the time since the ride began. Use $0 \le t \le 144$ for the domain, where $t = 0$ is the time at which the ride starts.



Practice Sets

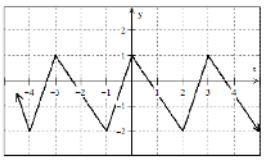
3.1 Periodic Phenomena Set 1 **Form A**

Name _____

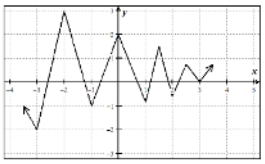
Date _____ Period _____

Determine whether each function *is or is not periodic*. If it is, find the period. If not, explain why not.

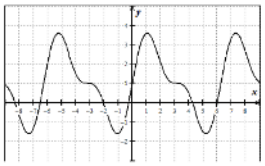
1.



2.



3. Find the period and amplitude of the periodic function below. State the maximum and minimum values for the function.



Grade:

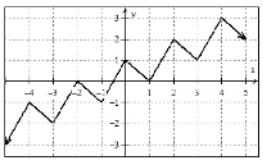
3.1 Periodic Phenomena Set 1 **Form B**

Name _____

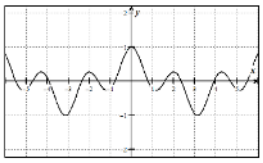
Date _____ Period _____

Determine whether each function *is or is not periodic*. If it is, find the period. If not, explain why not.

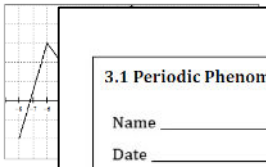
1.



2.



3. Find the period and amplitude of the periodic function below. State the maximum and minimum values for the function.



Grade:

3.1 Periodic Phenomena Set 2 **Form A**

Name _____

Date _____ Period _____

The depth of the water can be modeled with a periodic function. Suppose one day high tide occurs at 5:30 am and the depth of the water is 12 feet, then low tide occurs at 3:30 pm and the depth of the water is 8 feet.

1. What is the amplitude of the function?

2. What is the period of the function?

3. What is the midline of the graph?

4. What will be the depth at 10:30 am?

5. When will the high tide occur the following

Grade:

3.1 Periodic Phenomena Set 2 **Form B**

Name _____

Date _____ Period _____

The depth of water can be modeled with a periodic function. Suppose one day high tide occurs at 6:00 am and the depth of the water is 10 feet, then low tide occurs at 5:00 pm and the depth of the water is 4 feet.

1. What is the amplitude of the function?

2. What is the period of the function?

3. What is the midline of the graph?

4. What will the be depth at 11:30 am?

5. When will the high tide occur the following

Grade:

Daily Lesson Quiz