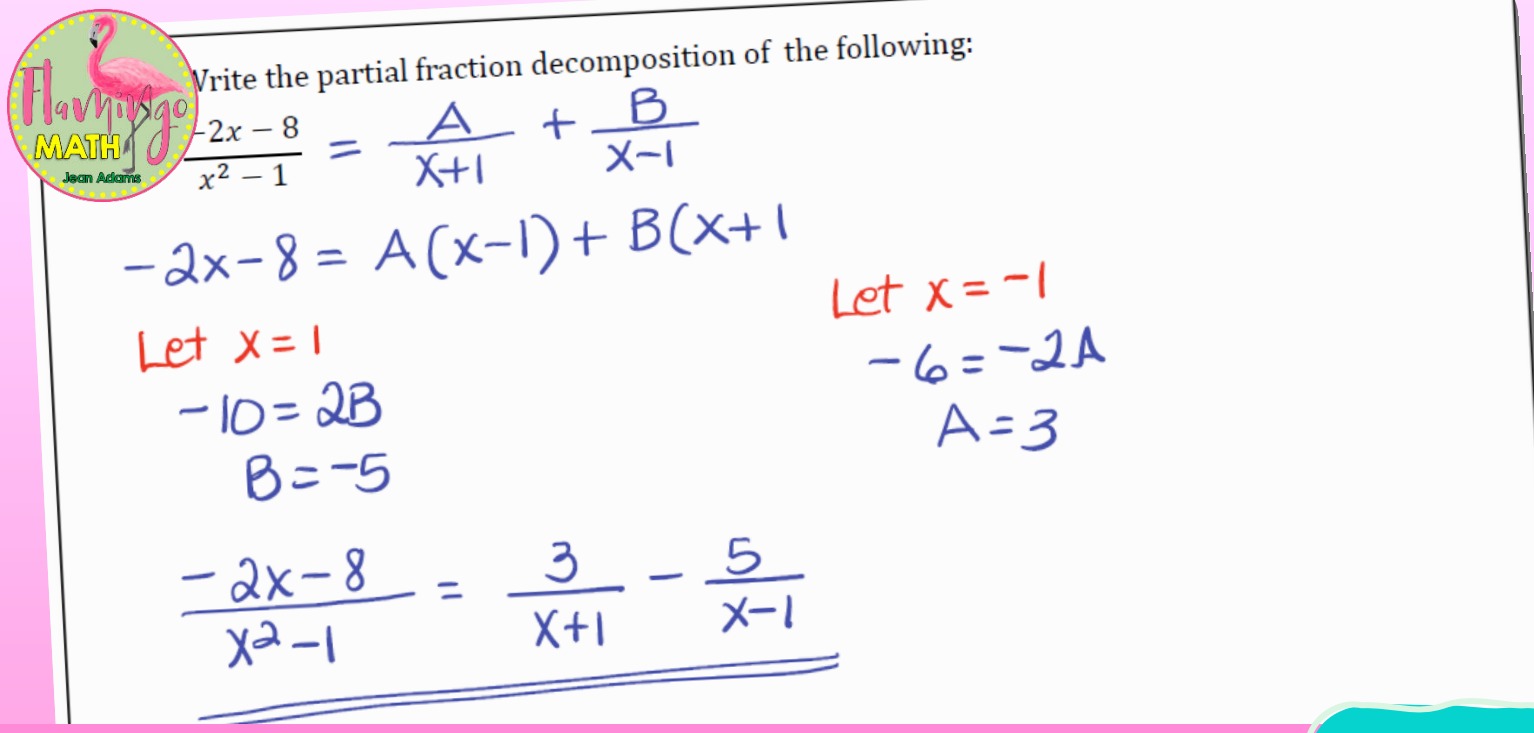
Precalculus Honors

Partial Fractions



Completely Editable

7.7

Lesson 7.7: Partial Fractions

In this lesson, we will learn to write a rational expression as the sum of two or more simpler rational expressions. In this example, each fraction on the right side of the equation is a **partial fraction**. Together, they make up the **partial fraction decomposition** of the left side:

$$\frac{6x+2}{x^2+3x-10} = \frac{4}{x+5} + \frac{2}{x-2}$$

Decomposition of N(x)/D(x) into Partial Fractions

1. ***Divide if Improper.*** If the degree of $N(x) \geq$ degree of $D(x)$, divide the denominator into the numerator to obtain proper form.

$$\frac{N(x)}{D(x)} = (polynomial) + \frac{R(x)}{D(x)}$$

2. ***Factor denominator:*** Completely factor the denominator into factors of the form $(px+q)^m$ and $(ax^2+bx+c)^n$ where (ax^2+bx+c) is irreducible over the real numbers.

3. ***Linear factors:*** For *each* factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \ldots + \frac{A_m}{(px+q)^m}$$

4. ***Quadratic factors:*** For *each* factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of n fractions.

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \ldots + \frac{B_nx+C_n}{(ax^2+bx+c)^m}$$

The Basic Guidelines for Decompositions

1. Use long division to obtain proper form when the degree of the numerator degree is greater than the degree of the denominator.
2. Factor the denominator as a product of linear factors or irreducible quadratic factors.
3. Write a partial fraction with a constant numerator for each linear factor of the denominator.
4. Clear the denominator.
5. Equate coefficients to write a linear system.
6. Solve the system to find the partial fraction decomposition.

Note: If you go on to take a calculus course, this procedure will be helpful in the course for an operation called antidifferentiation.

Distinct Linear Factors

EX #1: Write the form of the partial fraction decomposition. Do not solve for the constants.

A. $\frac{x-2}{x^2+4x+3}$ **B.** $\frac{8}{x^3-6x^2}$

EX #2: Write the partial fraction decomposition of the following:

$$\frac{5x+4}{x^2+3x-10}$$

EX #3: Write the partial fraction decomposition of the following:

$$\frac{6x-14}{x^2-5x+6}$$

EX #4: Write the partial fraction decomposition of the following:

$$\frac{-2x-8}{x^2-1}$$

Repeated Linear Factors

EX #5: Write the partial fraction decomposition of the following:

$$\frac{3x^2+12x+4}{x^3+2x^2+x}$$

Non-Repeated Quadratic Factors

EX #6: Write the partial fraction decomposition of the following:

$$\frac{3x-9}{x^3-1}$$

Repeated Irreducible Quadratic Factors

EX #7: Write the partial fraction decomposition of the following:

$$\frac{x^3-2x^2+3x+1}{(x^2+1)^2}$$

NOTES

7.7 **Partial Fractions Homework**

Name _____

Date _____ Period _____

Problems 1 – 4, tell whether to given expression is proper or improper. If improper, rewrite it as the sum of a polynomial and a proper rational expression.

1. $\frac{2x}{x^2-4}$

2. $\frac{x^2+3}{x^2-4}$

3. $\frac{4x^3+2x-1}{x^2-1}$

4. $\frac{5x+4}{x^3+1}$

Problems 5 – 8, write the form of the partial fraction decomposition. Do not solve for the constants.

5. $\frac{x-2}{x^2+2x}$

6. $\frac{x^2-2x+3}{x(x^2+1)}$

7. $\frac{2x^2-x+2}{(x^2+1)^2}$

8. $\frac{3x^3+7x}{(x^2+2)^2}$

Problems 9 – 16, write the partial fraction decomposition for the rational

9. $\frac{x+5}{x^2+4x+3}$

10. $\frac{2x-3}{(x-1)^2}$

Problems 9 – 16, write the partial fraction decomposition for the rational expression.

11. $\frac{x-1}{x^3+x^2}$

12. $\frac{3x^2-16}{x^2-4x}$

13. $\frac{5x^2-4}{x^3+2x^2}$

14. $\frac{4x^2+3x+14}{x^3-8}$

15. $\frac{32-2x}{x^2-4}$

Practice Sets

Lesson Warm-Ups

$\frac{}{x^2 - 4x}$

3. Write the partial fraction decomposition:

$$\frac{x^2 + 2x - 3}{x^3 + x}$$

Name _____ Date _____ Period _____

Lesson 7.7 Partial Fractions **Warm-Up # 2**

1. Write the form of the partial fraction decomposition. Do not solve for constants.

$$\frac{7}{x^2 - 8x}$$

2. Write the partial fraction decomposition.

$$\frac{12}{x^2 + 2x - 8}$$

3. Write the partial fraction decomposition:

$$\frac{x^2 + 6x + 4}{x^3 + 2x}$$

7.7 Partial Fractions **Form A**

Name _____

Date _____ Period _____

1. Write the form of the partial fraction decomposition. Do not solve for the constants.

$$\frac{3}{x^2 - 6x}$$

2. Write the partial fraction decomposition.

$$\frac{5}{x^2 + x - 6}$$

3. Write the partial fraction decomposition.

$$\frac{x^2 + 3x + 2}{x^3 + 2x}$$

Grade:

7.7 Partial Fractions **Form B**

Name _____

Date _____ Period _____

1. Write the form of the partial fraction decomposition. Do not solve for the constants.

$$\frac{5}{x^2 - 10x}$$

2. Write the partial fraction decomposition.

$$\frac{6}{x^2 - 2x - 8}$$

3. Write the partial fraction decomposition.

$$\frac{x^2 - 2x - 1}{x^3 + x}$$

Grade:

Daily Lesson Quiz