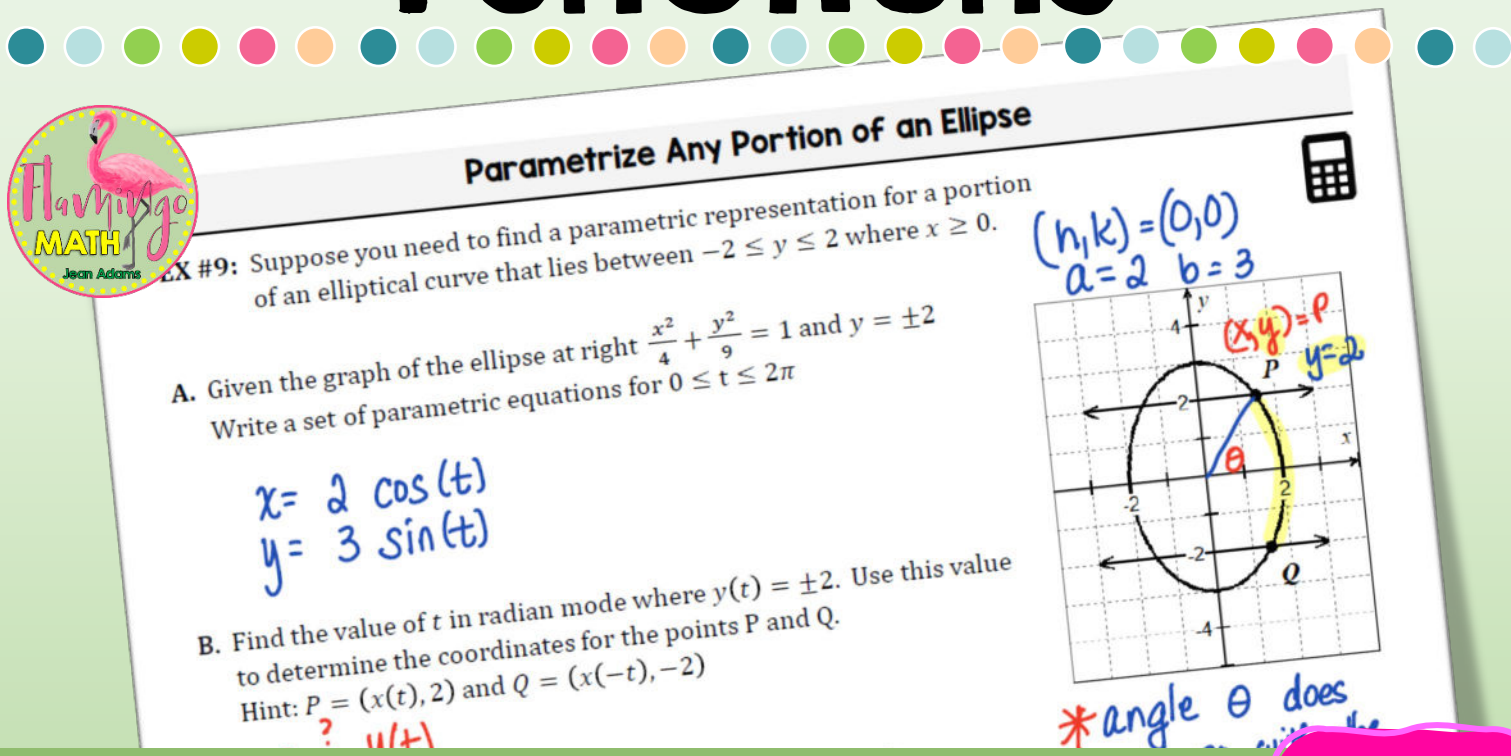
AP® Precalculus

Parametrization of Implicitly Defined Functions



Completely Editable

4.7

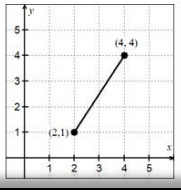
Lesson 7-A: Parametrization of Implicitly Defined Functions

In this lesson we want to revisit parametric equations for implicitly defined functions, including conic sections and inverse functions that can be parametrized. Recall that a curve in 2-dimensions requires a parameter, commonly we use t, and two functions $x(t)$ and $y(t)$. The interval of the parameter determines the direction and endpoints of the curve.

Parametrization of Implicitly Defined Functions

A parametrization $\big(x(t), y(t)\big)$ for an implicitly defined function will, when $x(t)$ and $y(t)$ are substituted for x and y, respectively, satisfy the corresponding equation for every value of t in the domain.

EX #1: Given the implicit curve $3x - 2y = 4$ from $[2, 4]$



A. Parametrize the curve so that the orientation is left and down.

Use the following with the two points shown on the graph.

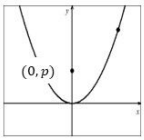
$$f(t) = \begin{cases} x(t) = x_1 + (x_2 - x_1)t \\ y(t) = y_1 + (y_2 - y_1)t \end{cases} \quad \text{for } 0 \le t \le 1$$

Parametric Equations for Parabolas

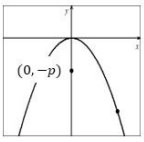
EX #4: Given the results from EX #3, find the general parametric form for the three remaining orientations of a parabola on the interval $-2\pi \le t \le 2\pi$

A. $x^2 = 4py$

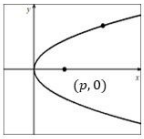
$\begin{cases} x(t) = 2pt \\ y(t) = pt^2 \end{cases}$



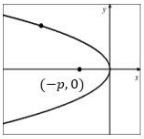
B. $x^2 = -4py$



C. $y^2 = 4px$



D. $y^2 = -4px$

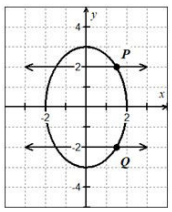


EX #5: The parametric equation of a parabola is given below. Find the vertex, focus and directrix.

Parametrize Any Portion of an Ellipse

EX #9: Suppose you need to find a parametric representation for a portion of an elliptical curve that lies between $-2 \le y \le 2$ where $x \ge 0$.

A. Given the graph of the ellipse at right $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $y = \pm 2$
Write a set of parametric equations for $0 \le t \le 2\pi$

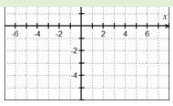


B. Find the value of t in radian mode where $y(t) = \pm 2$. Use this value to determine the coordinates for the points P and Q.
Hint: $P = (x(t), 2)$ and $Q = (x(-t), -2)$

Parametric Form of a Hyperbola

A **hyperbola** can be parametrized using trigonometric functions.

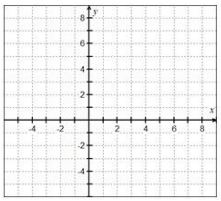
Major Axis is Horizontal: Hyperbola opens left and right	Major Axis is Vertical: Hyperbola opens up and down



Graphing Inverses of Parametric Equations

EX #2: Given $y = 2\sqrt{x+4}$ the parametric equations below represent the function.

$$f(t) = \begin{cases} x(t) = t^2 - 4 \\ y(t) = 2t \end{cases} \quad 0 \le t \le 3$$



A. Complete the table below for $y = f(t)$

t	0	1	2	3
$x(t)$				
$y(t)$				

B. Sketch the graph and the line $y = x$ on the grid at right.

C. Complete the table below for the inverse $y = f^{-1}(t)$

$$f^{-1}(t) = \begin{cases} x(t) = 2t \\ y(t) = t^2 - 4 \end{cases} \quad 0 \le t \le 3$$

t	0	1	2	3
$x(t)$				
$y(t)$				

D. Sketch the inverse function on the grid.

Lesson 7-B: Parametrization of Implicitly Defined Functions

Let's finish this lesson with more parametrizing for curves that represent conic sections. We will begin with the parametric form for an ellipse (including circles) and finish with parametric form of hyperbolas. Since a circle is just a special ellipse where $a = b$. You can write the standard equation for a circle as $(x-h)^2 + (y-k)^2 = r^2$

Parametric Form for an Ellipse

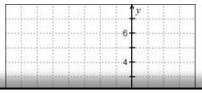
The parametric form for an ellipse is $F(t) = \big(x(t), y(t)\big)$. An **ellipse** can be parametrized using the trigonometric functions $x(t) = h + a\cos(t)$ and $y(t) = k + b\sin(t)$ for $0 \le t \le 2\pi$.

To parametrize a **circle** use $x(t) = h + r\cos(t)$ and $y(t) = k + r\sin(t)$ for $0 \le t \le 2\pi$

EX #7: Given the relation $x^2 + 4x + y^2 - 2y - 4 = 0$

A. Determine the conic represented by finding standard form of the equation.

B. Write the parametric equations that represent the curve.

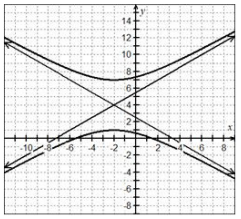


Parametric Form of a Hyperbola

EX #11: Given the graph of the hyperbola at right

A. Write a Cartesian equation for the conic section.

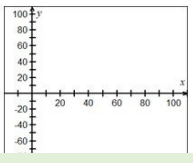
B. Write parametric equations for the conic section



C. Confirm your answer using a graphing utility.

EX 12: Astronomers are tracking an asteroid that is on a hyperbolic path heading toward the moon. They have determined its path is given by the hyperbola below as t ranges from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. If the center of the moon is located at the origin, and x and y are in thousands of miles:

$x(t) = -40 + 50\sec(t)$
$y(t) = 30\tan(t)$



A. Plot the branch of the hyperbola.

B. When $t = -1$ radian, what are the x- and y-coordinates of the asteroid? How far is this from the center of the moon?

Guided Notes

4.7 **Parametrization of Implicitly Defined Functions Practice Set 1**

Name _____

Date _____ Period _____

Problems 1-4, given the implicit curve, parametrize over the given interval and orientation and $0 \le t \le 1$.

1. $3x - 4y = 8$ where $4 \le x \le 8$ oriented right and up

2. $3x + 6y = 18$ where $-2 \le x \le 6$ oriented right and down.

3. $3x - 4y = 8$ where $4 \le x \le 8$ oriented left and down

4. $3x + 6y = 18$ where $-2 \le x \le 6$ oriented left and up.

Problems 5-8, use the given relations and their inverses to answer each question.

5. Given $x = 2t + 1$ and $y = t^2 - 4$. Does the point $(9, 12)$ lie on the function or its inverse?

6. Given $x = t^2$ and $y = 4t - 4$ find where the relation intersects with its inverse.

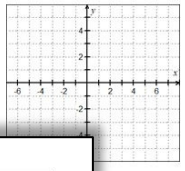
7. Given $x = 3t + 14$ and $y = t^2 - 2t$ fi
the parametric function intersects wi
inverse.

Problems 9-14, complete the following, given the parametric equations for parabolas.

9. Find the vertex, focus and directrix for the parabola:
$\begin{cases} x(t) = t^2 - 4 \\ y(t) = 4t + 2 \end{cases}$ for $-2\pi \le t \le 2\pi$

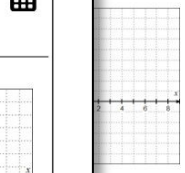
10. Determine when the parabola reaches the point $(-2, -2.25)$, given:
$\begin{cases} x(t) = 3 - 2t \\ y(t) = 4 - t^2 \end{cases}$ for $-2\pi \le t \le 2\pi$

11. Sketch the parabola and its inverse
$\begin{cases} x(t) = -6t^2 \\ y(t) = 3t + 2 \end{cases}$ for $-2\pi \le t \le 2\pi$



12. Write parametric equations for a parabola that opens right, with vertex at (3,2) and the point (4,0) lies along the curve.

parabola
ite a set of
ith a graphing



14. Write an equation in standard form for a parabola that opens downward, with vertex (4,5) and focus (4,2). Then, write the parametric equations for the function.

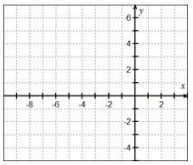
4.7 **Parametrization of Implicitly Defined Functions Practice Set 2**

Name _____

Date _____ Period _____

Problems 1-3, given the relations, determine the conic represented by
A. finding standard form of the equation.
B. Write the parametric equations that represent the curve.
C. Sketch the curve and show the orientation on the graph.

1. $x^2 + 4x + 4y^2 - 16y - 16 = 0$



2. $x^2 + y^2 + 8x - 6y - 15 = 0$

3. $16x^2 - 96x + 4y^2 - 16y + 96 = 0$

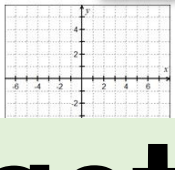
Problems 4-7, write the parametrizatic

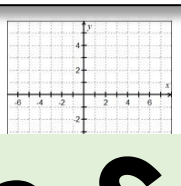
4. $\frac{(x+4)^2}{16} + \frac{(y+2)^2}{25} = 1$

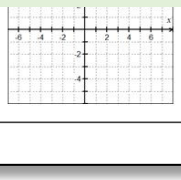
6. $\frac{(y+2)^2}{4} - \frac{(x+3)^2}{16} = 1$

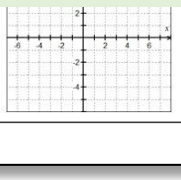
Problems 8-11, Given the parametric e
A. Name the conic secti
B. Sketch the graph an
C. Identify key charact
D. Show orientation of

8. $\begin{cases} x(t) = 3\cos(t) \\ y(t) = 4\sin(t) \end{cases}$



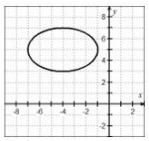




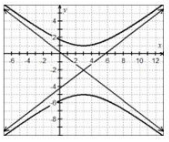


Problems 12-15, for each graph shown, complete the following
A. Write a Cartesian equation for the conic section
B. Write parametric equations for the conic section
C. Confirm your answer by using a graphing utility.

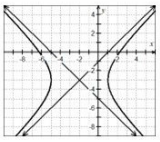
12.



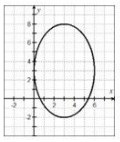
13.



14.



15.



Problem 16, solve the application problem.

16. A planet travels in an elliptical orbit modeled by the equation $\frac{x^2}{64} + \frac{y^2}{36} = 1$. Assume the planet makes one orbit every 2π units of time. Astronomers found an asteroid belt extends along the line $y = \frac{1}{2}x - 4$. Find the times when the planet passes through the asteroid belt during its first orbit.

Practice Sets

4.7 Parametrization of Implicitly Defined Functions Set 1 **Form A**

Name _____

Date _____ Period _____

1. Parametrize the given implicit curve over the given interval and orientation.
$2x - 3y = 7$ where $-2 \leq x \leq 5$ oriented right and up.

2. Use the given relation and its inverse to answer the question. Given $x = 3t + 2$ and $y = t^2 - 3$. Does the point $(-2, 5)$ lie on the function or its inverse?

3. Find the vertex, focus and directrix for the parabola:

$$\begin{cases} x(t) = t^2 - 2 \\ y(t) = 3t + 1 \end{cases}$$

Grade:

4.7 Parametrization of Implicitly Defined Functions Set 1 **Form B**

Name _____

Date _____ Period _____

1. Parametrize the given implicit curve over the given interval and orientation.
$2x + 3y = 7$ where $-2 \leq x \leq 5$ oriented right and down.

2. Use the given relation and its inverse to answer the question. Given $x = 3t + 2$ and $y = t^2 - 3$. Does the point $(2, -3)$ lie on the function or its inverse?

3.

G

4.7 Parametrization of Implicitly Defined Functions Set 2 **Form A**

Name _____

Date _____ Period _____

For problems 1 and 2, write the parametrization for each conic section.

1. $\frac{(x-3)^2}{9} + \frac{(y+2)^2}{16} = 1$

2. $\frac{(y+1)^2}{25} - \frac{(x+2)^2}{16} = 1$

For problems 3 and 4, name the conic section by looking at the parametric equations.

3. $\begin{cases} x(t) = 2 + 4\cos t \\ y(t) = -3 + 4\sin t \end{cases}$

Grade:

4.7 Parametrization of Implicitly Defined Functions Set 2 **Form B**

Name _____

Date _____ Period _____

For problems 1 and 2, write the parametrization for each conic section.

1. $\frac{(x+4)^2}{16} + \frac{(y-2)^2}{9} = 1$

2. $\frac{(y-1)^2}{9} - \frac{(x+3)^2}{25} = 1$

For problems 3 and 4, name the conic section by looking at the parametric equations.

3. $\begin{cases} x(t) = 4\cos t \\ y(t) = 6\sin t \end{cases}$

Grade:

Daily Lesson Quiz