Parametric Functions and Rates of Change

LEARNING OBJECTIVES:

★ Find and analyze the direction of planar motion as the parameter increases.
★ Identify key characteristics of parametric functions related to direction and rate of change.
★ Compute the average rate of change over an interval $[t_1, t_2]$

What is included in this resource?

- Guided Student Notes
- Homework/Practice assignment
- Google Slides™
- Answer keys and full solution sets

Product Overview

GUIDED NOTES and GOOGLE™ SLIDES:
Lesson 4.3 is a two-day lesson with notes and practice sets clearly marked. The unit is organized into fifteen lessons. Teachers can use GOOGLE Slides for presentation of the lesson.

HOMEWORK/PRACTICE:
Each lesson includes a practice set as an alternative to textbook homework assignments, with an emphasis on promoting rigor in the questioning techniques expected by College Board standards for the course exam.

STANDARDS ALIGNMENT:

4.3.A Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.

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Guided Notes

Lesson 3-A: Parametric Functions and Rates of Change

While a rectangular equation can tell you where a particle in motion is located, it cannot reveal when the object is in any given position. If you introduce a time parameter, parametric equations give the location of the moving object and tell when the object is located at each of its positions along the path. In this lesson, you will explore the direction of a particle in motion as it relates to the parameter. You will also analyze the average rate of change related to both horizontal and vertical motion over an interval of time.

Motion of a Particle Moving Along a Line

EX #1: Consider a particle is moving along a line described by the equations.

$$\begin{cases} x(t) = 5 - 3t \\ y(t) = 4 + 2t \end{cases}$$

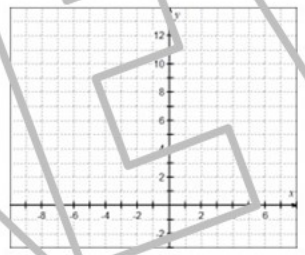
As t increases motion of the particle along the line can be analyzed in terms of x and y independently. The motion along the horizontal axis is right or left and motion along the vertical axis is up or down.

t	0	1	2	3	
$x(t)$					
$y(t)$					

A. Complete the table at right.

B. Sketch the graph, show orientation.

C. Describe the motion of the particle related to the parameter of time on the interval $0 \le t \le 4$.



Defining Motion Along a Curve

As the parameter increases, the direction of planar motion of a particle can be analyzed in terms of x and y independently. Let's summarize the exploration in EX #1.

1. If $x(t)$ is increasing, the direction of motion is to the _____.

2. If $x(t)$ is decreasing, the direction of motion is to the _____.

3. If $y(t)$ is increasing, the direction of motion is _____.

4. If $y(t)$ is decreasing, the direction of motion is _____.

Particle Motion Changing Direction

EX #2: A particle is moving in an elliptical orbit along a curve over the time interval $[0, 2\pi]$. Use the table below to complete the following.

t	0	1	$\frac{\pi}{2}$	2	π	4	$\frac{3\pi}{2}$	5	2π
$x(t)$	5	2.7	0	−2.1	−5	−3.3	0	1.4	5
$y(t)$	0	3.4	4	3.6	0	−3.0	−4	−1.8	0

A. Analyze the particle motion by completing the table below.

	intervals of increase	intervals of decrease
$x(t)$		
$y(t)$		

B. At what time, t does the particle change directions from moving left to right? Justify.

C. Is there a time interval where the particle is moving to the right and up? Explain how you know.

D. What is the direction to particle moves initially? How do you know.

E. When $t = \pi$ describe the behavior of the particle at this time.

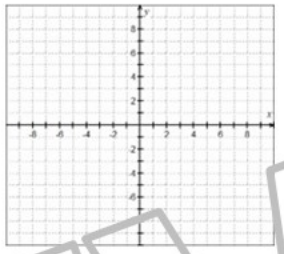
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Guided Notes

Parametrization of a Line

EX #3: Write the equation of the line in slope-intercept form whose slope is -3 that passes through the point $(-2, 4)$.

A. Sketch the equation on the restricted interval $-2 \leq x \leq 2$



B. Write three different sets of parametric equations and parameter restrictions that will give the path described above.

I. $x = t$	II. $x = -2t$	III. $x = e^t - 3$

C. Use the new parametric equations with restrictions to describe similarities and differences about the orientation and direction for each set of equations.

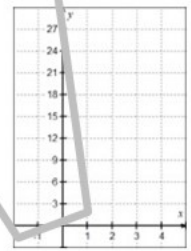
D. Suppose each set of parametric equations models the motion of an object along the defined curve. Describe the motion of the particle at the initial value of t on the restricted domain.

Lesson 3-B: Parametric Functions and Rates of Change

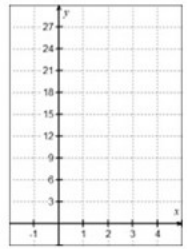
Did you realize that the same curve in the plane can be parametrized in different ways which can lead to traversing the curve in different directions simply by using different parametric functions? In this lesson we will explore that topic more, along with computing the average rate of change between given points along a parametric curve.

EX #4: Given $y = \frac{1}{2}x^3 - 2x + 3$ on the interval from $0 \leq x \leq 4$ can be used to represent motion of an object by creating parametric equations.

A. If you allow $x = t$ as a parameter of time, describe the direction and orientation of the parametric equation on $0 \leq t \leq 4$. Sketch this path on the grid at right. Show arrows for the orientation.



B. Let $x = -3t$ and write a set of parametric equations for the same rectangular equation. Explain how the time parameter must be adjusted to trace out the same path as **Part A**. Sketch this path on the grid at right. Show arrows for the orientation.



C. Use your findings to discuss the motion of the particle in both sets of equations. Explain your findings and justify how you know.

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Guided Notes

Average Rate of Change

Over a given interval $[t_1, t_2]$ within the domain, the average rate of change can be computer for $x(t)$ and $y(t)$ independently. The ratio of the average rate of change of y to the average rate of change of x gives the slope of the graph between the points on the curve corresponding to t_1 and t_2, so long as the average rate of change of $x(t) \neq 0$.

EX #5: A potato cannon launches a potato straight up from a height of 5 feet with an initial velocity of 128 ft/sec. The horizontal distance from the cannon launch and the vertical height from the ground, in feet, are modeled by a set of parametric equations.

A. Write a set of parametric equations that will model the height of the potato as a function of time, t.

B. Find the average rate of change in horizontal positi... over the first two se...ds. Show your work and units.

C. What is the aver...ge rate o... change i... verti...l motion ...ve... the first ...wo seconds? Show your work and ...nits.

D. Use your find...ngs to determ...ine th... ratio of averag... rate of chan...e of y to t...e average ...a...e of ch...nge of x ...etween the points o...n the curve corre...ponding to ... = 0 and ... 2. What ...oes this re...present graphicall...?

E. Determine th... rate at ...hich the height of th... potato is changing between $t = 5$ and $t = 6$. Describe what ...s happe...ing to the potato during this time interval.

Thinking Deeper About the Average Rate of Change

EX #6: Brandon throws a 16-pound shot from a height of 5 feet with an initial velocity of 40 feet per second at an angle of $\frac{\pi}{...}$ radians. The motion of the shot put can be modeled by the parametric equations given below.

$$\begin{cases} x(t) = 20\sqrt{2}\,t \\ y(t) = -16t^2 + 20\sqrt{2}\,t + 5 \end{cases}$$

A. Find the time, t in seconds that the shot put is at a maximum. Show your work.

B. What is the maximum vertical height of the shot, in feet, to the nearest tenth?

C. Consider a very small interval of time between this maximum height such that you are ± 0.01 second on either side of this maximum from Part A. Calculate the average rate of change for $x(t)$ and $y(t)$ on this small interval of time. Show your work.

D. Use your findings in Part C to explain the average rates of change in terms of the motion of the shot put.

E. What is the ratio of the average rate of change of y to the average rate of change of x?

F. Based on your results, is the shot put ever at rest? How do you know?

PREVIEW

Homework

4.3 **Parametric Functions and Rates of Change Practice Set 1**

Name _____

Date _____ Period _____

Problems 1 – 4, Given the parametric equations below, each describes the motion of a particle along the curve. Describe the motion of the particle related to the parameter of time. Justify your answer.

1. $x(t) = 2t - 3$
$y(t) = 8 - 4t; \quad 0 \le t \le 4$

2. $x(t) = t + 2$
$y(t) = t^2 - 3; \quad -2 \le t \le 3$

3. $x(t) = t^2$
$y(t) = t - 2; \quad -2 \le t \le 7$

4. $x(t) = \sqrt{t}$
$y(t) = 3 - t; \quad 0 \le t \le 6$

Problems 5-8, Determine the behavior and orientation of the curves. Explain any similarities and differences from each other.

5. A. $x(t) = t$
$y(t) = 2t + 1$

B. $x(t) = e^{-t}$
$y(t) = 2e^{-t} + 1$

6. A. $x(t) = -2t^2$
$y(t) = 4 + t^2$

B. $x(t) = 2\sqrt{t}$
$y(t) = 4 - \sqrt{t}$

7. A. $x(t) = t$
$y(t) = 3 + 2t - t^2$

B. $x(t) = e^t$
$y(t) = 3 + 2e^t - e^{2t}$

8. A. $x(t) = \sin(t)$
$y(t) = 3\sin(t) - 2$

B. $x(t) = \sqrt{t}$
$y(t) = 3\sqrt{t} - 2$

Problems 9 – 10, A particle moves in an elliptical orbit along a curve over the time interval $[0, 2\pi]$.
A. Analyze the motion for intervals of increase and intervals of decrease.
B. State interval(s) where the particle moves right/left and up/down.
C. Describe the behavior of the particle from $0 \le t \le 1$.

9. $x = 3\sin(t), y = 3\cos(t)$

10. $x = 1 + 3\cos(t), y = 2 + 4\sin(t)$

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Homework

4.3 **Parametric Functions and Rates of Change Practice Set 2**

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Date _____ Period _____

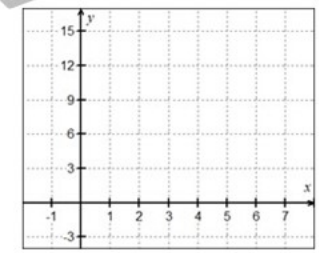
Problems 1 – 2, Write the equation of the line in slope-intercept form with given information. Write a set of parametric equations using the parameter $x = t$. Then, write another parametric equation using $x = \sqrt{t}$ with restrictions .Describe similarities and differences about the orientation and direction for each set of equations.

1. A line through $(1, -3)$ with slope 2 on the interval $-3 \leq x \leq 3$

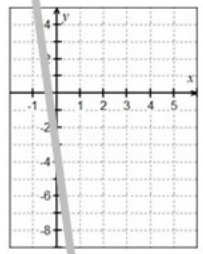
2. A line through $(-4, 6)$ with slope $-\frac{1}{2}$ on the interval $-2 \leq x \leq 10$

Problems 3 – 4, Determine the motion of the particle using the given information.

3. Given $y = -x^2 + 6x + 3$ on the interval $0 \leq x \leq 6$. Let $x = t$ to write a parametric equation. Sketch the curve, show orientation. Explain the motion on the adjusted interval.



4. Given $y = \frac{1}{2}x^2 - 4x + 2$ on the interval $0 \leq x \leq 10$. Let $x = \sqrt{t}$ to write a parametric equation. Sketch the curve, show orientation. Explain the motion on the adjusted interval.



Problems 5-7, Find average rate of change of an object in motion from each scenario.

5. The path of a bullet fired from a gun into the air from a height of 5 feet can be modeled by the equations below for $0 \leq t \leq 32$. The initial speed of the bullet is known to be 1024 ft/sec.

$$x(t) = 512\sqrt{3}t$$
$$y(t) = -16t^2 + 512t + 5$$

A. Find the average rate of change from $t = 0$ to $t = 5$ of both $x(t)$ and $y(t)$ independently.

B. Explain the behavior of the bullet's path in context to part A.

C. Find the ratio of the average rate of change of y to the average rate of change of x. Explain the meaning of this rate in context to the scenario.

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Homework

6. A Ferris wheel is turning counterclockwise at a rate of one revolution every 10 seconds. This behavior can be modeled by the equations below where units are given in ft/sec.

$$x(t) = 30\cos\left(\frac{\pi}{5}t\right)$$
$$y(t) = 40 + 30\sin\left(\frac{\pi}{5}t\right) \quad t \geq 0$$

A. Find the average rate of change from $t = 3$ to $t = 5$ of both $x(t)$ and $y(t)$ independently.

B. Explain the behavior of a passenger on the Ferris wheel on the interval from $3 \leq t \leq 5$ in context to the scenario.

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7. You throw a baseball from a height of 6 feet, with an initial velocity of 90 feet per second and the angle of inclination from the horizontal is $2\pi/9$. The table of values tells where the ball is located and when the ball is at a given point (x, y) on the path.

t	0	1	1.5	1.7[illegible]	1.8	[illegible].85	2	2.5	3	3.7
$x(t)$	[illegible]	68.9	103.4	120.7	[illegible]4.1	127.5	137.9	172.4	206.8	256.2
$y(t)$	6	47.9	[illegible].8	58.23	58.3	58.3	57.7	50.6	35.6	1

A. Find the average rate of change from $1.8 \leq t \leq 1.85$ of both vertical and horizontal motion.

B. What is the ratio of the two rates on this interval? Explain the meaning in context.

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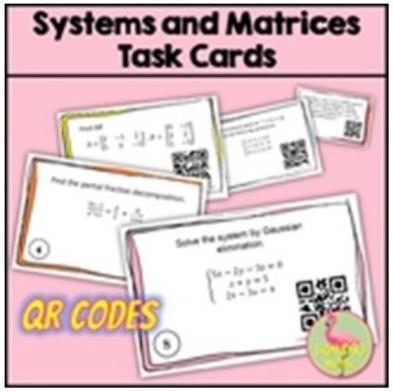
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